

**GRAVITATION****C1 NEWTON'S LAW OF GRAVITATION**

Every particle of matter in the universe attracts every other particle with a force, known as gravitational force. Newton's Law of Gravitation states that two particles with masses  $m_1$  and  $m_2$ , a distance  $r$  apart,

attract each other with gravitational forces of magnitude  $F = \frac{Gm_1m_2}{r^2}$ .

**Principle of Superposition :**

If  $n$  particles interact, the net force  $\vec{F}_{1,\text{net}}$  on a particle labeled as particle 1 is the sum of the forces on it from all the other particles taken one at a time.

$$\vec{F}_{1,\text{net}} = \sum_{i=2}^n \vec{F}_{1i}$$

In which the sum is a vector sum of the forces  $\vec{F}_{1i}$  on particle 1 from particles 2, 3, ...,  $n$ . The gravitational force  $\vec{F}_1$  on a particle from an extended body is found by dividing the body into units of differential mass  $dm$ , each of which produces a differential force  $d\vec{F}$  on the particle, and then integrating to find the sum of those forces :

$$\vec{F}_1 = \int d\vec{F}$$

**Gravitation Behaviour of spherically symmetric solid :**

The gravitational effect outside any spherical symmetric mass distribution is the same as though all the mass of the sphere were concentrated at its center.

**Practice Problems :**

1. A point mass of mass  $M$  are broken into two parts and separated by certain distance  $d$ . The maximum force between the two parts is given by

(a)  $\frac{GM^2}{2d^2}$       (b)  $\frac{GM^2}{4d^2}$       (c)  $\frac{GM^2}{3d^2}$       (d)  $\frac{GM^2}{8d^2}$

2. Three point masses each of mass  $m$  are placed at the corner of an equilateral triangle of side length  $d$ . The force on one of the mass is

(a)  $\frac{GM^2}{d^2}$       (b)  $\frac{GM^2}{3d^2}$       (c)  $\frac{\sqrt{3}GM^2}{d^2}$       (d)  $\frac{2\sqrt{3}GM^2}{d^2}$

[Answers : (1) b (2) c]

**C2 GRAVITATION FIELD AND INTENSITY**

The magnitude gravitational intensity due to a mass  $m$  at point  $P$  is  $E = \frac{Gm}{r^2}$ , directed toward the mass  $m$ .

**Principle of Superposition :**

In the presence of  $n$  particles, the net gravitational intensity  $\vec{E}$  at a particular point is the vector sum of the intensities due to individual particles at that particular point :  $\vec{E} = \sum_{i=1}^n \vec{E}_i$ .

The gravitational intensity due to an extended body is found by dividing the body into units of differential

mass  $dm$ , each of which produces a differential intensity  $d\vec{E}$ , and then integrate to find the gravitational intensity :  $\vec{E} = \int d\vec{E}$ .

**Practice Problems :**

- Two uniform solid spheres of equal radii  $R$ , but mass  $M$  and  $4M$  have a centre to centre separation  $6R$ . The two spheres are held fixed. Find the distance from  $M$  at which the gravitational field intensity will be zero ?
- What is the gravitational field intensity due to uniform ring at the centre ?
- Find the gravitational field intensity due to a uniform ring of mass  $M$  and radius  $R$  at the distance  $x$  from the centre of the ring on its axis ?

[Answers : (1)  $2R$  (2) zero (3)  $\frac{GMx}{(x^2 + R^2)^{3/2}}$  ]

**C3 GRAVITATIONAL FIELD INTENSITY DUE TO EARTH AND ACCELERATION DUE TO GRAVITY**

The gravitational field intensity due to earth and acceleration due to gravity are the same.

**Outside or on the Earth :** The value of  $g$  outside the earth at the distance  $r$  from the center is  $g = \frac{GM}{r^2}$ .

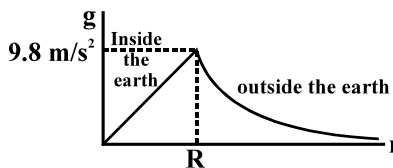
On the surface of earth,  $g_0 = \frac{GM}{R^2}$  where  $R$  is the radius of earth.

Above the surface of height  $h$ ,  $g$  is given by,  $g = g_0 \frac{R^2}{(R+h)^2} = g_0 \left(1 + \frac{h}{R}\right)^{-2}$

Using the approximation  $(1+x)^n \approx 1+nx$  for  $|x| \ll 1$ , we get  $g \approx g_0 \left(1 - \frac{2h}{R}\right)$  if  $h \ll R$ .

**Inside the Earth :** The value of  $g$  inside the earth at the distance  $r$  is given by  $g = g_0 \left(\frac{r}{R}\right)$

**Variation of  $g$  with position :**



**Effect of 'g' due to Earth's rotation :**  $g = g_0 - \omega^2 R \cos^2 \theta$ , where  $\theta$  is the angle of latitude and  $\omega$  is the angular velocity of the earth about its axis.

case I At the equator,  $\theta = 0$ ,  $g = g_0 - \omega^2 R$

case II At the pole,  $\theta = \frac{\pi}{2}$ ,  $g = g_0$ .

Hence there is no effect of earth's rotation on  $g$  at pole.

**Practice Problems :**

- If the radius of the earth were to shrink by one per cent, its mass remaining the same, the value of  $g$  on the earth's surface would
 

(a) increase by 0.5%	(b) increase by 2%
(c) decrease by 0.5%	(d) decrease by 2%

2. Two planets have radii  $R_1$  and  $R_2$  and densities  $\rho_1$  and  $\rho_2$  respectively. The ratio of the acceleration due to gravity at their surface is
- (a)  $\frac{\rho_1 R_1}{\rho_2 R_2}$       (b)  $\frac{\rho_1 R_2^2}{\rho_2 R_1^2}$       (c)  $\frac{R_1 R_2}{\rho_1 \rho_2}$       (d)  $\frac{\rho_2 R_1}{\rho_1 R_2}$
3. If the value of  $g$  at the surface of the earth is  $9.8 \text{ m/s}^2$ , then the value of  $g$  at a place  $480 \text{ km}$  above the surface of the earth will be (radius of earth =  $6400 \text{ km}$ )
- (a)  $4.2 \text{ m/s}^2$       (b)  $7.2 \text{ m/s}^2$       (c)  $8.5 \text{ m/s}^2$       (d)  $9.8 \text{ m/s}^2$
4. If  $R$  is the radius of the earth then the altitude at which the acceleration due to gravity will be  $25\%$  of its value at the earth's surface is
- (a)  $R/4$       (b)  $R$       (c)  $3R/8$       (d)  $R/2$
5. The radius of the earth is  $6400 \text{ km}$  and  $g = 10 \text{ m/s}^2$ . In order that a body of  $5 \text{ kg}$  weight zero at the equator, the angular speed of the earth should be
- (a)  $\frac{1}{80} \text{ rad/s}$       (b)  $\frac{1}{400} \text{ rad/s}$       (c)  $\frac{1}{800} \text{ rad/s}$       (d)  $\frac{1}{1600} \text{ rad/s}$
6. As we go from the equator to the poles, the value of  $g$
- (a) remains the same      (b) increases  
(c) decreases      (d) decrease up to a latitude of  $45^\circ$ .
- [Answers : (1) b (2) a (3) c (4) b (5) c (6) b]

#### C4 GRAVITATIONAL POTENTIAL ENERGY :

The gravitational potential energy  $U(r)$  of a system of two particles, with masses  $M$  and  $m$  and separated by a distance  $r$ , is given by  $U = -\frac{GMm}{r}$

#### Potential energy of the system of particles :

If a system contains more than two particles, its total gravitational potential energy  $U$  is the sum of terms representing the potential energies of all the pairs. As an example, for three particles, of masses  $m_1$ ,  $m_2$  and  $m_3$

$$U = -\left( \frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}} \right) \text{ where } r_{12} \text{ is } \dots\dots\dots$$

Relation between potential energy and force :  $\vec{F} = -\frac{dU}{dr} \hat{r}$

#### Practice Problems :

1. A body of mass  $m$  is taken from the earth's surface to a height equal to the radius  $R$  of the earth. If  $g$  is the acceleration due to gravity at the surface of the earth, then the change in the potential energy of the body is
- (a)  $\frac{1}{4}mgR$       (b)  $\frac{1}{2}mgR$       (c)  $mgR$       (d)  $2mgR$
2. Three point masses each of mass ' $m$ ' are placed on an equilateral triangle of side length  $l$ . Find the potential energy of this system of particles ?

[Answers : (1) b (2)  $\frac{-3Gm^2}{l}$ ]

**C5 GRAVITATION POTENTIAL (V) :**

It is defined as the Gravitational Potential Energy per unit mass. The Gravitational Potential (V) due to

mass  $m$  at point P is  $-\frac{Gm}{r}$ .

Relation between Gravitational Potential (V) and intensity ( $\vec{E}$ ) :  $\vec{E} = -\frac{dV}{dr}\hat{r}$

**Practice Problems :**

1. The gravitational field due to a mass distribution is  $E = K/x^3$  in the x-direction, where  $K$  is a constant. Taking the gravitational potential to be zero at infinity, its value at a distance  $x$  is

(a)  $K/x$  (b)  $K/2x$  (c)  $K/x^2$  (d)  $K/2x^2$

[Answers : (1) d]

**C6 ESCAPE SPEED**

An object will escape from the gravitational pull of an astronomical body if the object is projected with a certain minimum speed from the body's surface and this minimum speed is known as escape speed. Escape

speed from the surface of the earth is  $V_e = \sqrt{\frac{2GM}{R}}$ , where  $M$  is the mass of the earth and  $R$  is the radius.

Let the astronomical body is of uniform density  $\rho$  then  $M = \frac{4}{3}\pi R^3\rho$  and hence  $V_e = \sqrt{\frac{8}{3}\pi GR^2\rho}$ . The

escape speed of an object at a given point in the field is independent of its mass and state of its motion, but is position-dependent. For earth the escape speed from the surface  $V_e$  is 11.2 km/s.

**Practice Problems :**

1. The escape velocity from the earth is 11 km/s. The escape velocity from a planet having twice the radius and the same mean density as those of the earth is

(a) 5.5 km/s (b) 11 km/s (c) 22 km/s (d) none of these

2. The mass of moon is  $\frac{1}{81}$  of earth's mass and its radius is  $\frac{1}{4}$  of that of the earth. If the escape velocity from the earth's surface is 11.2 km/s, its value for the moon is

(a) 0.14 km/s (b) 0.5 km/s (c) 2.5 km/s (d) 5.0 km/s

3. The escape velocity of a particle of mass  $m$  varies as  $Km^\alpha$ , where  $K$  is a constant. The value of  $\alpha$  is

(a) 0 (b) 1 (c)  $\frac{1}{2}$  (d)  $-\frac{1}{2}$

4. If a rocket is to be projected vertically upwards from the surface of the earth, it requires an escape velocity of 11 km/s. If the rocket is to be projected at an angle of  $60^\circ$  with the vertical, the escape velocity required will be about

(a) 5.5 km/s (b)  $11\sqrt{2}$  km/s (c) 11 km/s (d)  $5.5 \times \sqrt{3}$  km/s

[Answers : (1) c (2) c (3) a (4) c]

**C7 SATELLITES : ORBITS AND ENERGY**

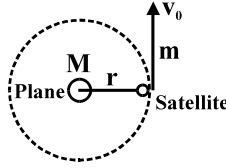
A satellite is any body revolving around a larger body, under the influence of the latter. For example the moon is a gravitational satellite of the earth. There are various artificial satellite of the earth, they are known as geo-static or geo stationary or geo-synchronous satellite. For this type of satellites, the conditions are

- (a) the orbit must be circular  
 (b) the orbit must be in equatorial plane of the earth  
 (c) the period of revolution of the satellites is 24 hour

- (d) the angular velocity of revolution of the satellite must be in the same direction as the angular velocity of rotation of the earth. If a satellite is to be always seen overhead, the observer should be on the equator of the Earth.

For a satellite in circular orbit :

$$\text{Orbital speed } v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$$



where  $r = R + h$ ,  $R$  is the radius of planet and  $h$  is height from the surface of planet at which a satellite is revolving.

$$\text{Time Period } T = \frac{2\pi r}{v_0} = \sqrt{\frac{4\pi^2}{GM} r^3}, \text{ Kinetic energy } K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$$

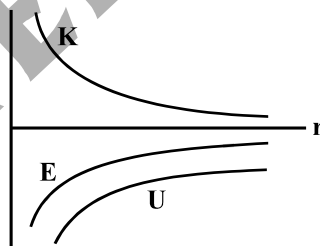
$$\text{Potential energy } U = -\frac{GMm}{r}, \text{ Total mechanical energy } = E = K + U = -\frac{GMm}{2r}$$

The negative sign indicates that the satellite is bound.

$$\text{Relation between } E, U \text{ and } K : E = -K, E = \frac{U}{2}, K = -\frac{U}{2}$$

$$\text{Binding energy of the satellite} = \frac{GMm}{2r}$$

Variation of  $K$ ,  $U$  and  $E$  with  $r$  is shown in figure.



Speeds and nature of orbits :

- If  $V < V_0$  ; the orbit is elliptical with the centre of the earth as the further focus,  $E$  is negative.
- If  $V = V_0$  ; the orbit is circular,  $E$  is negative
- If  $V_0 < V < V_e$  : the orbit is elliptical,  $E$  is negative
- If  $V = V_e$  : the body escapes,  $E$  is zero.
- If  $V > V_e$  : the body escapes along a hyperbolic Path,  $E$  is positive.

Here  $E$  stands for total mechanical energy of the body.

**Practice Problems :**

- A satellite of mass  $m$  is revolving around the earth at a height  $R$  above the surface of the earth. If  $g$  is the gravitational field intensity at the earth's surface and  $R$  is the radius of the earth, the kinetic energy of the satellite is

- (a)  $\frac{mgR}{4}$       (b)  $\frac{mgR}{2}$       (c)  $mgR$       (d)  $2 mgR$

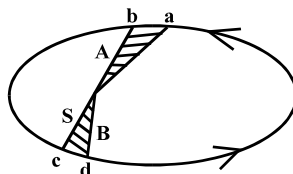
2. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is  $v$ . For a satellite orbiting at an altitude of half the earth's radius, the orbital velocity is
- (a)  $\frac{3}{2}v$       (b)  $\sqrt{\frac{3}{2}}v$       (c)  $\sqrt{\frac{2}{3}}v$       (d)  $\frac{2}{3}v$
3. Let the total energy, kinetic energy and potential energy of the artificial satellite are  $E_1$ ,  $E_2$  and  $E_3$  respectively. Then  $E_1 : E_2 : E_3$  is
- (a)  $1 : 1 : 2$       (b)  $-1 : 1 : -2$       (c)  $-1 : 1 : 2$       (d)  $1 : 1 : -2$
4. Time period of revolution of a satellite close to the surface of a spherical planet of radius  $R$  is  $T$ . The period of revolution close to the surface of another planet of radius  $3R$  and same density is
- (a)  $T$       (b)  $3T$       (c)  $3\sqrt{3}T$       (d)  $9T$
- [Answers : (1) a (2) c (3) b (4) a]

### C8 KEPLER'S LAW

- (i) First Law (law of orbit)  
The planets move around the sun in elliptical orbits with the sun at one focus.
- (ii) Second Law (law of area)  
The line joining the sun to a planet sweeps out equal areas in equal times. This law is based on conservation of angular momentum.
- (iii) Third Law (law of period)  
The square of the period of planet is proportional to the cube of its mean distance from the sun.  
The mean distance turns out to be the semi-major axis,  $a$ , i.e.,  $T^2 \propto a^3$

#### Practice Problems :

1. A planet moves around the sun. At a point P it is closest to the sun at a distance  $d_1$  and has a speed  $v_1$ . At another point Q, when it is farthest from the sun at a distance  $d_2$ , its speed will be
- (a)  $\frac{d_1^2 v_1}{d_2^2}$       (b)  $\frac{d_2 v_1}{d_1}$       (c)  $\frac{d_1 v_1}{d_2}$       (d)  $\frac{d_2^2 v_1}{d_1^2}$
2. The figure shows the motion of a planet around the sun in an elliptic orbit with the sun at one focus. The shaded areas A and B can be assumed to be equal. If  $t_1$  and  $t_2$  represent the times taken by the planet to move from a to b and from c to d respectively, then



- (a)  $t_1 < t_2$   
 (b)  $t_1 > t_2$   
 (c)  $t_1 = t_2$   
 (d) from the given information the relation between  $t_1$  and  $t_2$  cannot be determined.
3. Kepler's second law is based on
- (a) conservation of energy      (b) conservation of linear momentum  
 (c) conservation of angular momentum      (d) conservation of mass

[Answers : (1) c (2) c (3) c]

**SINGLE CORRECT CHOICE TYPE**

1. Time period of revolution of a satellite close to the surface of a spherical planet of radius  $R$  is  $T$ . The period of revolution close to the surface of another planet of radius  $3R$  and same density is

- (a)  $T$  (b)  $3T$   
 (c)  $3\sqrt{3} T$  (d)  $9T$

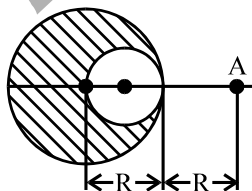
2. Imagine a light planet revolving around a very massive star in a circular orbit of radius  $R$  with period  $T$ . If the gravitational force of attraction between the planet and the star is proportional to  $R^{-5/2}$ , then  $T^2$  is proportional to

- (a)  $R^3$  (b)  $R^{7/2}$   
 (c)  $R^{3/2}$  (d)  $R^{3.75}$

3. The masses and radii of the earth and the moon are  $M_1, R_1$  and  $M_2, R_2$  respectively. Their centres are at a distance  $d$  apart. The minimum speed with which a particle of mass  $m$  should be projected from a point mid-way between the two centres so as to escape to infinite is

- (a)  $2\sqrt{\frac{G}{d}(M_1 + M_2)}$   
 (b)  $2\sqrt{\frac{G}{d}(M_1 - M_2)}$   
 (c)  $\sqrt{\frac{G}{d}(M_1 - M_2)}$   
 (d)  $\sqrt{2\frac{G}{d}(M_1 + M_2)}$

4. A solid sphere of uniform density and radius  $R$  applies a gravitational force of attraction equal to  $F_1$  on a particle placed at A, distant  $2R$  from the centre of the sphere. A spherical cavity of radius  $R/2$  is now made in the sphere as shown in the figure.



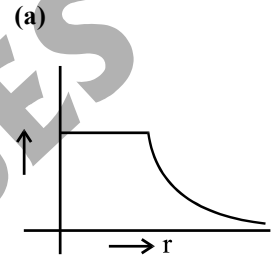
The sphere with cavity now applies a gravitational force  $F_2$  on the same particle placed at A. The ratio  $F_2/F_1$  will be

- (a)  $1/2$  (b)  $3$   
 (c)  $7/36$  (d)  $7/9$

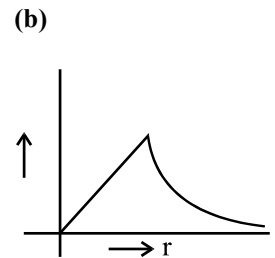
5. In table (A) there are some physical quantities. In table (B) the variation of that physical quantities with distance are given. Match the table (A) and table (B).

Table (A)	Table (B)
Quantity	Graph

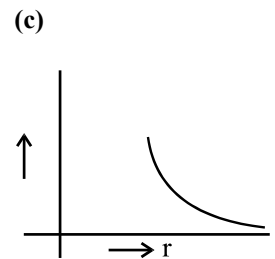
(i) Gravitational field due to solid sphere (earth) or the variation of acceleration due to gravity



(ii) Gravitational field due to hollow sphere



(iii) Gravitational potential due to hollow sphere



- (a) (i) – (b) (ii) – (c) (iii) – (a)  
 (b) (i) – (c) (ii) – (a) (iii) – (b)  
 (c) (i) – (b) (ii) – (a) (iii) – (c)  
 (d) (i) – (a) (ii) – (b) (iii) – (c)

6. The escape velocity from the center of the earth is given by

(a)  $\sqrt{\frac{GM_E}{R_E}}$  (b)  $\sqrt{\frac{2GM_E}{R_E}}$

(c)  $\sqrt{\frac{5GM_E}{R_E}}$  (d)  $\sqrt{\frac{3GM_E}{R_E}}$

7. A particle is projected from the surface of earth with speed such that it attains a height equals to the  $\eta$  times the radius of earth. The minimum value of the speed is

(a)  $\sqrt{\frac{2GM\eta}{(\eta+1)R}}$  (b)  $\sqrt{\frac{3GM\eta}{(\eta+1)R}}$

(c)  $\sqrt{\frac{4GM\eta}{(\eta+1)R}}$  (d)  $\sqrt{\frac{5GM\eta}{(\eta+1)R}}$

8. Three uniform spheres, each having mass  $m$  and radius  $r$ , are kept in such a way that each touches the other two. The magnitude of the gravitational force on any sphere due to the other two is

(a)  $\frac{\sqrt{3}Gm^2}{4r^2}$  (b)  $\frac{Gm^2}{4\sqrt{3}r^2}$

(c)  $\frac{Gm^2}{4r^2}$  (d)  $\frac{4Gm^2}{\sqrt{3}r^2}$

9. Two masses  $m_1$  and  $m_2$  are initially at rest at infinite distance apart. They approach each other due to gravitational interaction. Their speed of approach at the instant when they are distance  $d$  apart is

(a)  $2\sqrt{\frac{[G(m_1 + m_2)]}{d}}$

(b)  $\sqrt{\frac{[2G(m_1 + m_2)]}{d}}$

(c)  $\sqrt{\frac{[G(m_1 + m_2)]}{2d}}$

(d)  $\frac{1}{2}\sqrt{\frac{[G(m_1 + m_2)]}{d}}$

10. If a particle is fired vertically upwards with a speed of 15 km/s, the speed with which it will move in interstellar space is

(a) 10 km/s (b) 10.5 km/s

(c) 11 km/s (d) 11.5 km/s

11. A satellite of mass  $m$  moves in a circular orbit of radius  $r$  around a planet of radius  $R$ . The total energy required to put the satellite in the orbit assuming it initially to be at rest and acceleration due to gravity at the surface of planet 'g'

(a)  $mgR[1 - (R/3r)]$

(b)  $2mgR[1 - (R/2r)]$

(c)  $2mgR[1 - (R/3r)]$

(d)  $mgR[1 - (R/2r)]$

12. Three identical bodies of mass  $M$  are located at the vertices of an equilateral triangle with side  $L$ . The speed with which they must move if they all revolve under the influence of one another's gravitation in a circular orbit circumscribing the triangle while still preserving the equilateral triangle is

(a)  $\sqrt{\frac{Gm}{L}}$  (b)  $\sqrt{\frac{2Gm}{L}}$

(c)  $\sqrt{\frac{Gm}{2L}}$  (d)  $2\sqrt{\frac{Gm}{L}}$

13. A satellite of mass  $m$  moves in an elliptic orbit around a planet of mass  $M$ , so that its maximum and minimum distances from the planet are  $r_1$  and  $r_2$  respectively. The angular momentum  $L$  of this satellite relative to the centre of planet is

(a)  $m\sqrt{[2GMr_1r_2/(r_1 + r_2)]}$

(b)  $2m\sqrt{[GMr_1r_2/(r_1 + r_2)]}$

(c)  $4m\sqrt{[GMr_1r_2/(r_1 + r_2)]}$

(d)  $8m\sqrt{[GMr_1r_2/(r_1 + r_2)]}$

14. Halley's comet has a period of 76 years and in 1986, had a distance of closest approach to the sun equal to  $8.9 \times 10^{10}$  m. The comet's farthest distance from the sun if the mass of sun is  $2 \times 10^{30}$  kg is

(a)  $4.3 \times 10^{12}$  m (b)  $5.3 \times 10^{12}$  m

(c)  $6.3 \times 10^{12}$  m (d)  $7.3 \times 10^{12}$  m

15. If a satellite is revolving around a planet of mass  $M$  in an elliptic orbit of semi-major axis  $a$ , the orbital speed of the satellite when it is at a distance  $r$  from the focus is given by

(a)  $2\sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$  (b)  $\sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$

(c)  $\sqrt{\frac{GM}{2}\left(\frac{2}{r} - \frac{1}{a}\right)}$  (d) none



## EXERCISE BASED ON NEW PATTERN

COMPREHENSTION TYPEComprehension-1

A sky lab of mass  $2 \times 10^3$  kg is first launched from the surface of earth in a circular orbit of radius  $2R$  (from the centre of earth) and then it is shifted from this circular orbit to another circular orbit of radius  $3R$ . Given,  $R = 6400$  km and  $g = 10$  m/s<sup>2</sup>.

- The minimum energy required to place the lab in the first orbit is
  - $9.6 \times 10^{10}$  J
  - $10.6 \times 10^{10}$  J
  - $12.6 \times 10^{10}$  J
  - $12.6 \times 10^{11}$  J
- The minimum energy required to shift the lab from first orbit to the second orbit is
  - $4.1 \times 10^{10}$  J
  - $3.1 \times 10^{10}$  J
  - $2.1 \times 10^{10}$  J
  - $1.1 \times 10^{10}$  J
- If the energy given to the sky lab from the surface is  $10.6 \times 10^{10}$  J, then the
  - sky lab will be in the circular orbit
  - sky lab will be in the elliptical orbit
  - the sky lab will escape from the earth
  - nothing can be said

Comprehension-2

A particle of mass  $m$  moves in a circular orbit of radius  $a$  under the action of central force whose potential is given by

$$V(r) = kr^3, (k > 0)$$

- The energy of the particle be a circle of radius  $a$  about the origin is
  - $kma^3$
  - $\frac{3}{2}kma^3$
  - $\frac{5}{2}kma^3$
  - $3kma^3$
- The period of the circular motion is
  - $\frac{2\pi}{\sqrt{5ka}}$
  - $\frac{2\pi}{\sqrt{3ka}}$
  - $\frac{2\pi}{\sqrt{15ka}}$
  - $\frac{2\pi}{\sqrt{17ka}}$
- If the particle be slightly disturbed from this circular motion, the period of small radial oscillations about  $r = a$  is
  - $\frac{2\pi}{\sqrt{5ka}}$
  - $\frac{2\pi}{\sqrt{3ka}}$
  - $\frac{2\pi}{\sqrt{15ka}}$
  - $\frac{2\pi}{\sqrt{17ka}}$

Comprehension-3

A satellite of mass  $m$  is put in an orbit just above the earth's atmosphere with a velocity  $\sqrt{1.5}$  times the velocity for a circular orbit at that height. The initial velocity imparted is horizontal. The mass of the earth is  $M$  and the radius of earth is  $R$ .

- Choose the incorrect statement from the following
  - the orbit of the satellite is elliptical
  - the satellite will escape from the earth
  - the angular momentum of the satellite remains constant about the centre of the earth
  - the total energy of the satellite remains constant
- The maximum distance of the satellite from the earth is
  - $R$
  - $1.5 R$
  - $2 R$
  - infinite
- The total energy of the satellite is
  - $-\frac{GmM}{R}$
  - $-\frac{GmM}{2R}$
  - $-\frac{GmM}{3R}$
  - zero

MULTIPLE CORRECT CHOICE TYPE

- A uniform solid sphere of mass  $M$  and radius  $b$  is surrounded symmetrically by a uniform spherical shell of equal mass and radius  $2b$ . Then
  - The gravitational field at the common centre is zero.
  - The variation of gravitational field inside the solid sphere is linear.
  - The gravitational field at a distance  $\frac{3}{2}b$  is  $4GM/9b^2$
  - The gravitational field at a distance  $\frac{5}{2}b$  from the centre is  $8GM/25b^2$
- Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity depends on
  - mass of the planet
  - mass of the particle escaping
  - temperature of the planet
  - radius of the planet.

3. The magnitudes of gravitational field at distances  $r_1$  and  $r_2$  from the centre of a uniform sphere of radius  $R$  and mass  $M$  are  $F_1$  and  $F_2$ , respectively. Then

(a)  $\frac{F_1}{F_2} = \frac{r_1}{r_2}$  if  $r_1 < R$  and  $r_2 < R$

(b)  $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$  if  $r_1 > R$  and  $r_2 > R$

(c)  $\frac{F_1}{F_2} = \frac{r_1}{r_2}$  if  $r_1 > R$  and  $r_2 > R$

(d)  $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$  if  $r_1 < R$  and  $r_2 < R$

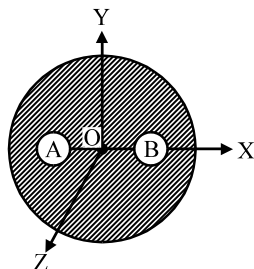
4. For a planet moving around the sun in an elliptical orbit of semi-major and semi-minor axes  $a$  and  $b$ , respectively, and period  $T$ , then

- (a) the torque acting on the planet about the sun is zero  
 (b) the angular momentum of the planet about the sun is constant  
 (c) the areal velocity is  $\pi ab/T$   
 (d) all are correct

5. Two objects of masses  $m$  and  $4m$  are at rest at an infinite separation. They move towards each other under mutual gravitational attraction. If  $G$  is the universal gravitational constant, then at a separation  $r$

- (a) the total energy of the two objects is zero  
 (b) their relative velocity is  $(10 Gm/r)^{1/2}$  in magnitude.  
 (c) the total kinetic energy of the objects is  $4 Gm^2/r$   
 (d) the ratio of their speed is 1 : 2

6. A solid sphere of uniform density and radius 4 units is located with its centre at the origin  $O$  of co-ordinates.



Two spheres of equal radii 1 unit, with their centres of  $A(-2, 0, 0)$  and  $B(2, 0, 0)$  respectively are taken out of the solid leaving behind spherical cavities as shown in figure.

- (a) the gravitational field due to this object at the origins zero  
 (b) the gravitational potential is the same at all points on the circle  $y^2 + z^2 = 4$   
 (c) the gravitational potential is the same at all points of the circle  $y^2 + z^2 = 36$   
 (d) the gravitational field at the point  $A$  and  $B$  is non-zero

7. As the earth is a uniform solid sphere. Let the escape velocity for this earth from any point is  $u$ . Now assume that the earth will become hollow uniform sphere. The mass and radius of the earth does not change. The escape velocity of this earth from any point is  $v$ . Then

- (a)  $u = v$  for all values of  $r$   
 (b)  $u = v$  for  $r \geq R_E$   
 (c)  $u < v$  for  $r < R_E$   
 (d)  $u > v$  for  $r < R_E$

#### Assertion-Reason Type

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1  
 (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1  
 (C) Statement-1 is True, Statement-2 is False  
 (D) Statement-1 is False, Statement-2 is True
1. STATEMENT-1 : Four identical bodies of mass  $M$  are located at the vertices of a square with side  $L$ . If they will move then they can all revolve in a circular orbit circumscribing the square while still preserving the square.  
 STATEMENT-2 : The necessary centripetal force provided by the gravitational force.
2. STATEMENT-1 : If a satellite is revolving very near to the planet of density  $\rho$  with period  $T$ , then the entity  $\rho T^2$  is a universal constant.  
 STATEMENT-2 : The angular momentum of the satellite about the centre of the planet remains constant.
3. STATEMENT-1 : If a mass  $M$  is broken into two parts and placed at a certain distance then the force between them will be maximum when they are broken into two equal parts.  
 STATEMENT-2 : Gravitational force is directly proportional to product of masses for the given distance.

**ANSWERS (SINGLE CORRECT CHOICE TYPE)**

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4. STATEMENT-1 : Gravitational force does not depend upon the medium but electrostatic force depends

STATEMENT-2 : Both forces are conservative.

5. STATEMENT-1 : A satellite cannot be placed in a stable orbit in a plane not passing through the earth centre.

STATEMENT-2 : The gravitational force acted on the satellite by the earth passes through the centre of the earth.

6. STATEMENT-1 : If a body is released from the satellite outside the satellite into the space then it will fall to the earth.

STATEMENT-2 : Due to inertia of motion, the velocity of the body is equal to that of the satellite.

7. STATEMENT-1 : A satellite placed in an orbit very near to the surface of the earth can follow a spiral path and collide with the earth.

STATEMENT-2 : It is due to the presence of frictional force.

8. STATEMENT-1 : There are two satellites each in a circular orbit in the equatorial plane of the earth but revolving in opposite sense will have different time period as observed from the earth.

STATEMENT-2 : It is due to relative motion.

9. STATEMENT-1 : If a planet was suddenly stopped in its orbit supposed to be circular then it would fall onto the sun in a time  $(\sqrt{2}/8)$  times the period of the planet's revolution.

STATEMENT-2 : It is due to gravitational force of attraction on the planet by the sun.

10. STATEMENT-1 : Distance between the centres of two stars is  $10a$ . The masses of these stars are  $M$  and  $16M$  and their radii  $a$  and  $2a$ , respectively. A body of mass  $m$  is fired straight from the surface of the larger star towards the smaller star with a

speed of  $\sqrt{\frac{5GM}{a}}$ . The body will reach the other star.

STATEMENT-2 : The mechanical energy of the body remains constant.

- |       |       |
|-------|-------|
| 1. a  | 11. d |
| 2. b  | 12. a |
| 3. a  | 13. a |
| 4. d  | 14. b |
| 5. a  | 15. b |
| 6. d  |       |
| 7. a  |       |
| 8. a  |       |
| 9. b  |       |
| 10. a |       |

**(Answers) EXERCISE BASED ON NEW PATTERN**

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**COMPREHENSION TYPE**

- |      |      |      |      |      |      |
|------|------|------|------|------|------|
| 1. a | 2. d | 3. b | 4. c | 5. b | 6. c |
| 7. b | 8. c | 9. c |      |      |      |

**MULTIPLE CORRECT CHOICE TYPE**

- |               |               |         |               |
|---------------|---------------|---------|---------------|
| 1. a, b, c, d | 2. a, d       | 3. a, b | 4. a, b, c, d |
| 5. a, b, c    | 6. a, b, c, d | 7. b, d |               |

**ASSERTION-REASON TYPE**

- |      |      |      |       |      |      |
|------|------|------|-------|------|------|
| 1. A | 2. B | 3. A | 4. B  | 5. A | 6. D |
| 7. A | 8. A | 9. A | 10. D |      |      |