OSCILLATIONS

Syllabus :

Periodic motion – period, displacement as a function of time. Period functions. Simple harmonic motion (S.H.M.) and its equation; phase; oscillations of a spring - restoring force and force constant; energy in S.H.M. - kinetic and potential energies; Simple pendulum - derivation of expression for its time period; Free, forced and damped oscillations, resonance.

CONCEPTS

C1 Simple Harmonic Motion

If a particle moves to and fro about a fixed point (equilibrium position) under the application of a force or torque (called restoring force or torque) which is directly proportional to the displacement (linear or angular), directed towards the fixed point, is called simple harmonic motion.

C2 Equation of SHM

The necessary and sufficient condition for the motion to be simple harmonic (linear) is that the force should be directly proportional to the displacement i.e. $F \propto x$

$F = -kx$ or $m\frac{d^2x}{dt^2} = -kx$ or $\frac{d^2x}{dt^2} + \omega^2 x = 0$ with $\omega^2 = \frac{k}{m}$

The solution of the above differential equation representing SHM is given by : $x = A \sin(\omega t + \phi)$

where A is the amplitude of the motion, ω is the angular (circular) frequency = $2\pi/T$ (T = time period) and ϕ is phase constant.

Practice Problems :

1. **The equation of S.H.M. of a particle is $\frac{d^2y}{dt^2} + ky = 0$, where k is a positive constant. The time period of motion is given by**

(a) $\frac{2\pi}{\sqrt{k}}$ (b) $\frac{2\pi}{k}$ (c) $\frac{k}{2\pi}$ (d) $\frac{\sqrt{k}}{2\pi}$

2. **A particle is executing S.H.M. of period 4 s. Then the time taken by it to move from the extreme position to half the amplitude is**

(a) **1/3 s** (b) **2/3 s** (c) **3/4 s** (d) **4/3 s**

[Answers : (1) a (2) b]

C3 Velocity of SHM is given by : $v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$ or $v = \omega\sqrt{A^2 - x^2}$

The maximum and minimum velocity of the particle are $v_{max} = A\omega$ and $V_{min} = 0$.

Practice Problems :

1. **A particle is vibrating in S.H.M. If its velocities are v_1 and v_2 when the displacements from the mean position are y_1 and y_2, respectively, then its time period is**

(a) $2\pi\sqrt{\frac{y_1^2 + y_2^2}{v_1^2 + v_2^2}}$ (b) $2\pi\sqrt{\frac{v_1^2 + v_2^2}{y_1^2 + y_2^2}}$ (c) $2\pi\sqrt{\frac{v_2^2 - v_1^2}{y_1^2 - y_2^2}}$ (d) $2\pi\sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$

2. **A particle is executing S.H.M. Then the graph of velocity as a function of displacement is**

(a) **straight line** (b) **circle** (c) **ellipse** (d) **hyperbola**

[Answers : (1) d (2) c]

C4 The acceleration in SHM is $a = \frac{d^2y}{dt^2} = -\omega^2 y$. Thus $|a_{max}| = \omega^2 A$ and $|a_{min}| = 0$

Practice Problems :

1. **A particle is executing S.H.M. Then the graph of acceleration as a function of displacement is**

(a) **straight line** (b) **circle** (c) **ellipse** (d) **hyperbola**

[Answers : (1) a]

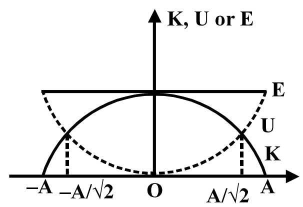
C5 Energy in SHM

(i) Kinetic Energy (K) $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

(ii) Potential Energy (U) $= \frac{1}{2}m\omega^2x^2$

(iii) Total energy (E) $= \frac{1}{2}m\omega^2A^2$ = constant

Graph for the variation of K, U and E with the position 'x' is given by :



Practice Problems :

1. **A body executes S.H.M. with an amplitude A. Its energy is half kinetic and half potential when the displacement is**

(a) **A/3** **(b)** **A/2** **(c)** **$A/\sqrt{2}$** **(d)** **$A/2\sqrt{2}$**

(c)

2. **When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is**

(a) **a/4** **(b)** **a/3** **(c)** **a/2** **(d)** **2a/3**

(c)

[Answers : (1) c (2) c]

C6 Simple Pendulum

The time period of a simple pendulum of length L is given by $2\pi\sqrt{\frac{L}{g}}$.

If length of the pendulum is large, then time period is given by :

$$T = 2\pi\sqrt{\frac{l}{g\left(\frac{1}{L}+\frac{1}{R}\right)}}$$ where R = Radius of the earth

so if L $\to \infty$, $T = 2\pi\sqrt{\frac{R}{g}} \approx 84.6$ minutes

A second pendulum is the simple pendulum having a time period of 2s.

Practice Problems :

1. **The time period of a simple pendulum is T. If its length is increased by 2%, the new time period becomes**

(a) **0.98 T** **(b)** **1.02 T** **(c)** **0.99 T** **(d)** **1.01 T**

2. **The length of a simple pendulum is increased by 44%. The percentage increase in its time period will be**

   (a) 44% (b) 22% (c) 20% (d) 11%

3. **A girl is swinging on a swing in the sitting position. How will the period of swing be affected if she stands up ?**

   (a) The period will now be shorter

   (b) The period will now be longer

   (c) The period will remain unchanged

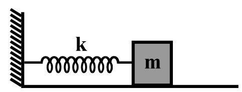
   (d) The period may become longer or shorter depending upon the height of the girl.

4. **For a simple pendulum the graph between length and time period will be a**

   (a) hyperbola (b) parabola (c) straight line (d) none of these

   [Answers : (1) d (2) c (3) a (4) b]

C7 Spring - Block System



The time period of SHM for a block of mass 'm' connected by a spring of spring constant 'k' as shown in figure is given by $\mathbf{T = 2\pi\sqrt{\frac{m}{k}}}$

Practice Problems :

1. **The bodies M and N of equal masses are suspended from two separate massless springs of constants k_1 and k_2, respectively. If the two oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is**

   (a) k_1/k_2 (b) k_2/k_1 (c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$

2. **The vertical extension in a light spring by a weight of 1 kg, in equilibrium, is 9.8 cm. The period of oscillation of the spring, in seconds, will be**

   (a) $\frac{2\pi}{10}$ (b) $\frac{2\pi}{100}$ (c) 20π (d) 200π

3. **A massless spring, having force constant k, oscillates with a frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2m is suspended from it. The frequency of oscillation will now be**

   (a) n (b) $n\sqrt{2}$ (c) $n/\sqrt{2}$ (d) 2n

   [Answers : (1) d (2) a (3) a]

C8 Physical Pendulum

The time period of a physical pendulum is given by : $\mathbf{T = 2\pi\sqrt{\frac{I}{Mdg}}}$

I : moment of inertia about the rotational axis passing through point of suspension

d : distance of the center of mass from the point of suspension

M : total mass of the body

Practice Problems :

1. **A rod of mass m and length L is suspended from one of the end point of the rod in vertical plane. The time period of the rod for small oscillation is given by**

(a) $2\pi\sqrt{\frac{2L}{3g}}$ (b) $2\pi\sqrt{\frac{L}{g}}$ (c) $2\pi\sqrt{\frac{3L}{g}}$ (d) $2\pi\sqrt{\frac{3L}{4g}}$

[Answers : (1) a]

C9 Damped Oscillation

Oscillations under the influence of frictional force are called damped oscillation. The amplitude and hence energy decreases with time exponentially and eventually the oscillator comes to rest.

C10 Forced Oscillation and Resonance

The oscillations of a system under the influence of an external periodic force are called forced oscillations. The amplitude of these oscillations remains constant.

If the frequency of the external applied force is equal to the natural frequency of the oscillator, resonance is said to occur. If damping is small, the amplitude of resonant oscillations will become very large. At resonance, the oscillator absorbs maximum energy supplied by the external force.

INITIAL STEP EXERCISE

1. A metallic sphere is filled with water and hung by a long thread. It is made to oscillate. If there is a small hole in the bottom through which water slowly flows out, the time period will

(a) go on increasing till the sphere is empty

(b) go on decreasing till the sphere is empty

(c) remain unchanged throughout

(d) first increase, then it will decrease till the sphere is empty and the period will not be the same as when the sphere was full of water.

2. A simple pendulum of length l suspended from the ceiling of a train is oscillating when the train is at rest. If the train starts moving with a constant acceleration α, the time period of the pendulum will be

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g+a}}$

(c) $2\pi\sqrt{\frac{l}{\sqrt{g^2-a^2}}}$ (d) $2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

3. A weakly damped harmonic oscillator of frequency n_1 is driven by an external periodic force of frequency n_2. When the steady state is reached, the frequency of the oscillator will be

(a) n_1 (b) n_2

(c) $(n_1 + n_2)/2$ (d) $\sqrt{n_1 + n_2}$

4. The period of a particle in S.H.M. is 8 s. At t = 0 it is at the mean position. The ratio of the distances travelled by it in the first and the second is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}-1}$

5. A body is on a rough horizontal surface which is moving horizontally in S.H.M. of frequency 2 Hz. The coefficient of static friction between the body and the surface is 0.5. The maximum value of the amplitude for which the body will not slip along the surface is approximately

(a) 9 cm (b) 6 cm

(c) 4.5 cm (d) 3 cm

6. A pan with a set of weight is attached to a light spring. The period of vertical oscillations is 0.5 s. When some additional weights are put in the pan, the period of oscillations increases by 0.1 s. The extension caused by the additional weights is

(a) 1.3 cm (b) 2.7 cm

(c) 3.8 cm (d) 5.5 cm

7. A pendulum clock, which keeps correct time at sea level, loses 15 s per day when taken to the top of a mountain. If the radius of the earth is 6400 km, the height of the mountain is

(a) 1.1 km (b) 2.2 km

(c) 3.3 km (d) 4.4 km

8. The minimum phase difference between the two simple harmonic oscillations

$y_1 = \frac{1}{2}\sin\omega t + \frac{\sqrt{3}}{2}\cos\omega t$ and $y_2 = \sin\omega t + \cos\omega t$ is

(a) $\pi/6$ (b) $-\pi/6$

(c) $\pi/12$ (d) $7\pi/12$

9. A simple pendulum is suspended from the roof of a trolley that moves freely down a plane of inclination θ. The time period of oscillation of the pendulum is

(a) $2\pi\sqrt{\frac{l}{g}}$ (b) $2\pi\sqrt{\frac{l}{g\cos\theta}}$

(c) $2\pi\sqrt{\frac{l}{g\sin\theta}}$ (d) $2\pi\sqrt{\frac{l}{g\tan\theta}}$

10. A body executes S.H.M. of period 3 s under the influence of one force, and S.H.M. of period 4 s under the influence of a second force. When both the forces act simultaneously in the same direction, the period of oscillation will be

(a) 7 s (b) 5 s

(c) $2\sqrt{3}$ s (d) 2.4 s

11. The time period of a simple pendulum measured inside a stationary lift is T. If the lift starts accelerating upwards with an acceleration g/3, the time will be

(a) $\sqrt{3}T$ (b) $\frac{\sqrt{3}}{2}T$

(c) $\frac{T}{\sqrt{3}}$ (d) $\frac{T}{3}$

12. A second's pendulum is placed in a space laboratory orbiting around the earth at a height 3 R from the earth's surface where R is the radius of the earth. The time period of the pendulum will be

(a) zero (b) 2/3 s
(c) 4 s (d) infinite

13. The percentage change in radius of the earth is 1% without change in its mass. The percentage change in time period of the simple pendulum is

(a) ½ % (b) 1 %
(c) 2 % (d) 2.3 %

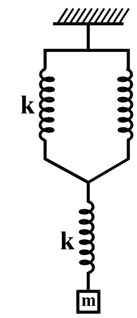
14. Let the coefficient of linear expansion of the massless string of simple pendulum is α. If the temperature is increased by Δt then the fractional change in time period of the simple pendulum is

(a) $\alpha\Delta t$ (b) $2\alpha\Delta t$
(c) $1/4\ \alpha\Delta t$ (d) $1/2\ \alpha\Delta t$

15. A spring of force constant k is divided in 1 : 2 : 3. The force constant of the longest spring is

(a) 6 k (b) 3 k
(c) 2 k (d) k

16. The frequency of vertical oscillations of the three spring-mass system, shown in the figure is



(a) $\frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{3m}}$

FINAL STEP EXERCISE (OBJECTIVE)

1. A simple pendulum has a time period T. The pendulum is completely immersed in a non-viscous liquid whose density is 1/10th of that of the material of the bob. The time period of the pendulum immersed in the liquid is

(a) T (b) $\sqrt{\frac{9}{10}}T$

(c) $\sqrt{\frac{10}{9}}T$ (d) $\frac{T}{10}$

2. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 Joule. Its

(a) maximum potential energy is 100 J
(b) maximum kinetic energy is 100 J
(c) maximum potential energy is 160 J
(d) both (b) and (c) are correct

3. When a body is suspended from two light springs separately, the periods of vertical oscillations are T_1 and T_2. When the same body is suspended from the two springs connected in series, the period will be

(a) $T_1 + T_2$ (b) $\sqrt{T_1T_2}$

(c) $\sqrt{(T_1^2 + T_2^2)/2}$ (d) $\sqrt{T_1^2 + T_2^2}$

4. In the above problem if the body is suspended from the two springs connected in parallel, the time period will be

(a) $\sqrt{T_1\,T_2}$ (b) $\frac{T_1T_2}{\sqrt{T_1^2+T_2^2}}$

(c) $\frac{\sqrt{T_1T_2}}{2}$ (d) $\frac{2T_1T_2}{\sqrt{T_1^2+T_2^2}}$

5. **The displacement y of a particle executing periodic motion is given by**

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

This expression may be considered to be a result of the superposition of

(a) two (b) three

(c) four (d) five

independent harmonic motions.

6. **A uniform cylinder of mass M and cross-sectional area A is suspended from a fixed point by a light spring of force constant k. The cylinder is partially submerged in a liquid of density ρ. If it is given a small downward push and released, it will oscillate with time period**

(a) $2\pi\sqrt{\frac{M}{k + A\rho g}}$ (b) $2\pi\sqrt{\frac{M}{k - A\rho g}}$

(c) $2\pi\sqrt{\frac{M}{k}}$ (d) $2\pi\sqrt{\frac{M}{A\rho g}}$

7. **Two particles execute S.H.M. of the same amplitude and frequency along the same straight line. They pass one another when going in opposite directions each time their displacement is half their amplitude. The phase difference between them is**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

8. **A person normally weighing 60 kg stands on a platform which oscillates up and down simple harmonically with a frequency 2 Hz and an amplitude 5 cm. If a machine on the platform gives the person's weight, then ($g = 10$ m/s^2, $\pi^2 = 10$),**

(a) the maximum reading of the machine will be 108 kg

(b) the maximum reading of the machine will be 90 kg

(c) the minimum reading of the machine will be 12 kg

(d) both (a) and (c) are correct

9. **A mass M is suspended from a massless spring. An additional mass m stretches the spring further by a distance x. The combined mass will oscillate on the spring with time period**

(a) $2\pi\sqrt{(M+m)x/mg}$

(b) $2\pi\sqrt{mg/(M+m)x}$

(c) $2\pi\sqrt{(M+m)/mgx}$

(d) $(\pi/2)\sqrt{mg/(M+m)x}$

10. **Electron in an oscilloscope are deflected by two mutually perpendicular oscillating electric fields such that at any time the displacements due to them are given by $x = A\cos\omega t$, $y = A\cos\left(\omega t + \frac{\pi}{6}\right)$. Then the path of the electron is**

(a) a straight line having the equation x = y

(b) a circle having the equation $x^2 + y^2 = A^2$

(c) an ellipse having the equation $x^2 - \sqrt{3}xy + y^2 = A^2/4$

(d) an ellipse having the equation $x^2 - xy + y^2 = 3A^2/4$

11. **Two linear simple harmonic motions of equal amplitudes and frequencies ω and 2ω are impressed on a particle along x and y axes respectively. If the initial phase difference between them is π/2, the resultant path followed by the particle is given by**

(a) $y^2 = x^2\left(1 - \frac{x^2}{A^2}\right)$

(b) $y^2 = 2x^2\left(1 - \frac{x^2}{A^2}\right)$

(c) $y^2 = 4x^2\left(1 - \frac{x^2}{A^2}\right)$

(d) $y^2 = 8x^2\left(1 - \frac{x^2}{A^2}\right)$

12. **A piece of wood has dimensions a, b and c. Its relative density is d. It is floating in water such that the side a is vertical. If it is pushed down a little and then released, the time period of oscillation will be**

(a) $2\pi\sqrt{\frac{abc}{g}}$ (b) $2\pi\sqrt{\frac{bc}{dg}}$

(c) $2\pi\sqrt{\frac{ad}{g}}$ (d) $2\pi\sqrt{\frac{bcd}{g}}$

13. **A U tube of uniform bore of cross-sectional area a is set up vertically with open ends up. A liquid of mass m and density d is poured into it. The liquid column will oscillate with a period**

(a) $2\pi\sqrt{\frac{m}{g}}$ (b) $2\pi\sqrt{\frac{ma}{dg}}$

(c) $2\pi\sqrt{\frac{m}{adg}}$ (d) $2\pi\sqrt{\frac{m}{2adg}}$

14. **A body of mass m falls from a height h onto the pan of a spring balance. The masses of the pan and spring are negligible. The force constant of the spring is k. The body sticks to the pan and oscillates simple harmonically. The amplitude of oscillation is**

(a) mg/k

(b) $(mg/k)\sqrt{1+(2hk/mg)}$

(c) $(mg/k)(1+\sqrt{1+(2hk/mg)})$

(d) $(mg/k)(1+\sqrt{1+(2hk/mg)-1})$

ANSWERS (INITIAL STEP EXERCISE)

1.	d	9.	b
2.	d	10.	d
3.	b	11.	b
4.	d	12.	d
5.	d	13.	b
6.	b	14.	d
7.	a	15.	c
8.	c	16.	b

ANSWERS (FINAL STEP EXERCISE)

1.	c	8.	d
2.	d	9.	a
3.	d	10.	c
4.	b	11.	c
5.	b	12.	c
6.	a	13.	d
7.	c	14.	b

AIEEE ANALYSIS [2002]

1. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be

(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$

(c) nT (d) T

2. In a sample harmonic oscillator, at the mean position

(a) kinetic energy is minimum, potential energy is maximum

(b) both kinetic and potential energies are maximum

(c) kinetic energy is maximum, potential energy is minimum

(d) both kinetic and potential energies are minimum

3. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is

(a) 16 J (b) 8 J

(c) 32 J (d) 24 J

4. A child swinging on a swing in sitting position, stands up, then the time period of the swing will

(a) increase

(b) decrease

(c) remain same

(d) increase if the child is long and decreases if the child is short

5. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic (K.E.) and total energy (T.E.) are measured as a function of displacement x. Which of the following statements is true ?

(a) P.E. is maximum when x = 0

(b) K.E. is maximum when x = 0

(c) T.E. is zero when x = 0

(d) K.E. is maximum when x is maximum

AIEEE ANALYSIS [2003]

6. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. Then the ratio of m/M is

(a) 16/9 (b) 5/3

(c) 3/5 (d) 25/9

7. Two particles A and B of equal masses are suspended from two massless springs of spring constant k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitudes of A and B is

(a) $\sqrt{\frac{k_2}{k_1}}$ (b) $\frac{k_1}{k_2}$

(c) $\sqrt{\frac{k_1}{k_2}}$ (d) $\frac{k_2}{k_1}$

8. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is

(a) 42% (b) 10%

(c) 11% (d) 21%

9. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is

(a) $4\sqrt{2}$ (b) 8

(c) –4 (d) 4

10. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m³. What relationship between t and t_0 is true ?

(a) $t = t_0$ (b) $t = t_0/2$

(c) $t = 2t_0$ (d) $t = 4t_0$

[2004]

11. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

[2004]

12. The total energy of a particle, executing simple harmonic motion is

(a) $\propto x$

(b) $\propto x^2$

(c) independent of x

(d) $\propto x^{1/2}$

where x is the displacement from mean position.

[2004]

13. A particle of mass m is attached to a spring (of spring constant k) and has a neutral angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The displacement of the oscillator will be proportional to

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

[2004]

14. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then

(a) $\omega_1 = \omega_2$

(b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(d) $\omega_1 < \omega_2$

[2004]

15. The function $\sin^2(\omega t)$ represents

(a) a simple harmonic motion with a period $2\pi/\omega$

(b) a simple harmonic motion with a period π/ω

(c) a periodic, but not simple harmonic, motion with a period $2\pi/\omega$.

(d) a periodic, but not simple harmonic, motion with a period π/ω.

[2005]

16. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is

(a) $\frac{-\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{-\pi}{6}$ (d) $\frac{\pi}{3}$

[2005]

17. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is

(a) $2\pi\alpha$ (b) $2\pi\sqrt{\alpha}$

(c) $\frac{2\pi}{\alpha}$ (d) $\frac{2\pi}{\sqrt{\alpha}}$

[2005]

18. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would

(a) remain unchanged

(b) increase towards a saturation value

(c) first increase and then decrease to the original value

(d) first decrease and then increase to the original value

[2005]

AIEEE ANALYSIS [2006]

19. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy ?

(a) $\frac{1}{3}$s (b) $\frac{1}{12}$s

(c) $\frac{1}{6}$s (d) $\frac{1}{4}$Hz

20. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1}. The period of oscillation is

(a) 0.1 s (b) 100 s

(c) 0.01 s (d) 10 s

21. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) for an amplitude of g^2/ω^2

(b) at the highest position of the platform

(c) at the mean position of the platform

(d) for an amplitude of g/ω^2.

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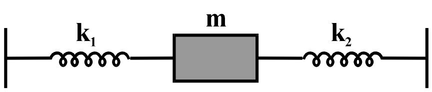
22. Two springs, of force constant k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes



(a) 2f (b) f/4

(c) f/2 (d) 4f

23. A particle of mass m executes simple harmonic motion with amplitude 'a' and frequency 'v'. The average kinetic energy during its motion from the position of equilibrium to the end is

(a) $2\pi^2 m a^2v^2$ (b) $\frac{1}{4} m a^2v^2$

(c) $\pi^2 m a^2v^2$ (d) $4\pi^2 m a^2v^2$

24. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^2 \cos \pi t$ metre. The time at which the maximum speed first occurs is

(a) 0.25 s (b) 0.75 s

(c) 0.5 s (d) 0.125 s

25. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A \cos(\omega t + \delta)$, then

(a) $A = x_0\omega^2, \delta = 3\pi/4$

(b) $A = x_0, \delta = \pi/4$

(c) $A = x_0\omega^2, \delta = \pi/4$

(d) $A = x_0 \omega^2, \delta = -\pi/4$

ANSWERS AIEEE ANALYSIS

1.	b	2.	c	3.	b	4.	b	5.	b	6.	a	7.	a
8.	b	9.	a	10.	c	11.	b	12.	c	13.	b	14.	a
15.	b	16.	c	17.	d	18.	c	19.	c	20.	c	21.	d
22.	a	23.	c	24.	c	25.	a						

TEST YOURSELF

1. A simple pendulum is oscillating in a lift. Let the time period of the oscillation is T_1. Consider the following cases :

	Case	Time Period
1.	Lift is moving with constant velocuty (upward or downward)	T_2
2.	Lift is moving with constant acceleration (upward)	T_3
3.	Lift is moving with constant acceleration (downward)	T_4

Which of the above time period is minimum :

(a) T_1 (b) T_2

(c) T_3 (d) T_4

2. **If a hole is bored along a diameter of the earth and a stone is dropped into the hole, it will**

   (a) reach the centre of the earth and stop there

   (b) reach the other side of the earth and stop there

   (c) execute S.H.M. about the centre of the earth

   (d) execute oscillatory, but not simple harmonic, motion about the centre of the earth.

3. **Two masses m_1 and m_2 are suspended together by a massless spring of constant k. When the masses are in equilibrium, m_1 is gently removed. Then the angular frequency of oscillation of m_2 and the amplity of oscillation are respectively.**

   (a) $\sqrt{k/m_1}$, $\frac{m_2 g}{k}$

   (b) $\sqrt{k/m_2}$, $\frac{m_1 g}{k}$

   (c) $\sqrt{k/(m_1+m_2)}$, $\frac{m_1 g}{k}$

   (d) $\sqrt{k/(m_2-m_1)}$, $\frac{m_1 g}{k}$

4. **The ratio of the amplitudes of the simple harmonic oscillations given by $y_1 = A \sin \omega t$ and $y_2 = (A/2) \sin \omega t + (A/2) \cos \omega t$ is**

   (a) 1 (b) 2

   (c) $1/\sqrt{2}$ (d) $\sqrt{2}$

5. **A particle is subjected to two simple harmonic motions along x and y axes : $x = a \sin \omega t$ and $y = -a \sin \omega t$. The resultant trajectory is a**

   (a) sine curve (b) circle

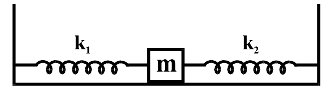
   (c) ellipse (d) straight line

6. **When a particle oscillates simple harmonically, its potential energy varies periodically. If the frequency of oscillation of the particle is n, the frequency of potential energy variation is**

   (a) n/2 (b) n

   (c) 2n (d) 4n

7. **A block of mass m is placed on a frictionless horizontal table. Springs of force constants k_1 and k_2 are attached on either side of it. The other ends of the springs are fixed as shown in the figure. If the block is displaced a little horizontally and left to oscillate, the angular frequency of oscillation will be**



   (a) $\left(\frac{k_1+k_2}{m}\right)^{1/2}$ (b) $\left(\frac{k_1 k_2}{(k_1-k_2)m}\right)^{1/2}$

   (c) $\left(\frac{k_1^2+k_2^2}{(k_1+k_2)m}\right)^{1/2}$ (d) $\left(\frac{k_1 k_2}{(k_1+k_2)m}\right)^{1/2}$

8. **Springs of constants k, 2k, 4k, 8k,..........., ∞ are connected in series. A mass m is attached to one end and the system is allowed to oscillate. The time period is**

   (a) $2\pi\sqrt{\frac{m}{2k}}$ (b) $2\pi\sqrt{\frac{2m}{k}}$

   (c) $2\pi\sqrt{\frac{m}{4k}}$ (d) $2\pi\sqrt{\frac{4m}{k}}$

9. **Two simple pendulums having lengths 1 m and 16 m are both given small displacements in the same directions at the same instant. They will again be in phase at the mean position after the shorter pendulum has competed n oscillations where n is**

   (a) 1/4 (b) 4

   (c) 5 (d) 16

10. **A flat horizontal board moves up and down in S.H.M. of amplitude A. Then the smallest permissible value of time period, such that an object on the board may not lose contact with the board, is**

   (a) $2\pi\sqrt{\frac{g}{A}}$ (b) $\pi\sqrt{\frac{g}{A}}$

   (c) $\pi\sqrt{\frac{A}{g}}$ (d) $2\pi\sqrt{\frac{A}{g}}$

ANSWERS

1.	c	6.	c
2.	c	7.	a
3.	b	8.	b
4.	d	9.	b
5.	d	10.	d