PROPERTIES OF SOLIDS AND LIQUIDS

CONCEPTS

C1A All real "rigid" bodies are to some extent elastic, which means that we can change their dimensions slightly by pulling, pushing, twisting or compressing them.

Hooke's law states that in elastic deformations, stress (force per unit area) is proportional to strain (relative deformation) :

$$\frac{\textbf{Stress}}{\textbf{Strain}} = \textbf{Elastic modulus}$$

Three elastic moduli are used to describe the elastic bahaviour (deformations) of objects as they respond to forces that act on them.

1. Longitudinal stress and longitudinal strain : Longitudinal stress is defined as $\frac{F_\perp}{A}$, where $F_\perp$ is the force perpendicular to the plane of cross sectional A. There are two types of longitudinal stress :

   (a) Tensile longitudinal stress, and

   (b) Compresive longitudinal stress

   Tensile stress is tensile force per unit area, $F_\perp / A$. Tensile strain is fractional change in length, $\Delta l/l_0$. Young's modulus Y is the ratio of tensile stress to tensile strain :

$$Y = \frac{F_\perp / A}{\Delta l / l_0} = \frac{F_\perp}{A}\frac{l_0}{\Delta l}$$

   Compressives stress and strain are defined the same way as tensile stress and strain. For many materials, Young's modulus has the same value for both tension and compression.

2. Bulk stress or volume stress or hydraulic stress :

   The bulk modulus B is the negative of the ratio of pressure change Δp (bulk stress) a fractional volume change $\Delta V/V_0$:

$$B = -\frac{\Delta p}{\Delta V / V_0}$$

   Compressibility k is the reciprocal of bulk modulus : k = 1/B.

3. Shear stress is force per unit area $F_\parallel/A$ for a force applied parallel to a surface. Shear strain is the angle ϕ. The shear modulus S is the ratio of shear stress to shear strain :

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_\parallel / A}{x / h} = \frac{F_\parallel}{A}\frac{h}{x} = \frac{F_\parallel / A}{\phi}$$

   The proportional limit is the maximum stress for which stress and strain are proportional. Beyond the proportional limit, Hooke's law is not valid. The elastic limit is the stress beyond which irreversible deformation occurs. The breaking stress, or ultimate strength, is the stress at which the material breaks.

Practice Problems :

1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied.

(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm

(c) length = 200 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm

2. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(a) 0.4 cm^3 (b) 4×10^{-5} cm^3 (c) 0.025 cm^3 (d) 0.004 cm^3

[Answers : (1) a (2) a]

C1B Energy stored in a stretched wire per unit volume equals to $\frac{1}{2}$ × stress × strain

Practice Problems :

1. **Young's modulus of steel is 2×10^{11} N/m^2. A steel wire has a length of 1 m and area of cross section 1 mm^2. The work required to increase its length by 1 mm is**

(a) 0.1 J (b) 1 J (c) 10 J (d) 100 J

[Answers : (1) a]

C2 Density : Density is mass per unit volume. If a mass m of material has volume V, its density ρ is $\rho = \frac{m}{V}$.

Specific gravity is the ratio of the density of a material to the density of water.

Practice Problems :

1. **If equal masses of two liquids of densities d_1 and d_2 are mixed together, the density of the mixture is**

(a) $d_1d_2/(d_1 + d_2)$ **(b)** $2d_1d_2/(d_1 + d_2)$ **(c)** $d_1d_2/(d_1 + d_2)$ **(d)** $(d_1 + d_2)/2$

2. **If equal volume of two liquids of density is d_1 and d_2 are mix together then the density of the mixture is**

(a) $d_1d_2/(d_1 + d_2)$ **(b)** $2d_1d_2/(d_1 + d_2)$ **(c)** $d_1d_2/(d_1 + d_2)$ **(d)** $(d_1 + d_2)/2$

[Answers : (1) b (2) b]

C3 Pressure : Pressure is normal force per unit area.

Pressure (a scalar quantity) on a surface is defined as

$$p = \lim_{\Delta s \to 0} \frac{\Delta F_\perp}{\Delta S} = \frac{dF_\perp}{dS}$$

The units for pressure are Nm^{-2} or pascal (Pa), or mm of mercury (or any other substance).

C4A Hydrostatic pressure distribution : Pressure in a fluid at rest increases with vertical height 'h' according to the relation $\frac{dp}{dh} = \rho g$.

If the density of the liquid is constant at each point then the pressure at a point A at a depth h below the free surface is given by $p_A = \rho gh + p_0$, where p_0 is the pressure at the free surface (atmospheric pressure). Absolute pressure is the total pressure in a fluid; gauge pressure is the difference between absolute pressure and atmospheric pressure.

Practice Problems :

1. **The pressure in a water tap at the base of a building is 3×10^6 dynes/cm^2 and on its top it is 1.6×10^6 dynes/cm^2. The height of the building is approximately**

(a) 7 m (b) 14 m (c) 70 m (d) 140 m

[Answers : (1) b]

C5 **Pascal Law :** Pascal's law states that pressure applied to the surface of an enclosed fluid is transmitted undiminished to every portion of the fluid.

Practice Problems :

1. **A piston of cross-sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. The cross-sectional area of the other piston which supports a truck of mass 2000 kg is**

(a) 9.8×10^2**cm^2** **(b)** 9.8×10^3**cm^2** **(c)** 1.96×10^3**cm^2** **(d)** 1.96×10^4**cm^2**

[Answers : (1) d]

C6 **Archimede's Principle :** When a body is immersed partly or wholly in a fluid, there acts an upward force on it called the buoyancy and its magnitude is equal to the weight of the fluid displaced. The point of the application of buoyancy is at the centre of mass of the displaced fluid and is called the centre of buoyancy. Buoyancy exists because of pressure gradient. Thus in case of a free fall situation buoyancy is zero.

Principle of floatation

Weight of the object = Buoyancy

$$\rho_s V g = \rho_l V_s g$$

V : total volume of the object

V_s : submerged volume of the object

ρ_s : density of object

ρ_l : density of liquid

C7A **Fluid Dynamics :** An ideal fluid is incompressible and has no viscosity. A flow line is the path of the fluid particle; a streamline is a curve tangent at each point to the velocity vector at that point. A flow tube is a tube bounded at its sides by flow lines. In laminar flow, layers of fluid slide smoothly past each other. In turbulent flow there is great disorder and a constantly changing flow pattern.

C7B **Principle of Continuity :** Conservation of mass in an incompressible fluid is expressed by the equation of continuity; for two cross sections A_1 and A_2 in a flow tube, the flow speed v_1 and v_2 are related by $A_1v_1 = A_2v_2$. The product Av is the volume flow rate, dV/dt, the rate at which volume crosses a section of the tube : $\frac{dV}{dt} = Av$.

<u>Practice Problems :</u>

1. **Two large tanks a and b, open at the top, contains different liquids. A small hole is made in the side of each tank at the same depth h below the liquid surface, but the hole in a has twice the area of the hole in b. The ratio of the densities of the liquids in a and b so that the mass flux is the same for each hole should be**

(a) 2 **(b)** 0.5 **(c)** 4 **(d)** 0.25

(b)

2. **In the above problem the ratio of flow rates (volume flux) from the holes in a and b is**

(a) 2 **(b)** 0.5 **(c)** 4 **(d)** 0.25

(a)

[Answers : (1) b (2) a]

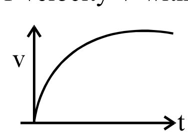
C7C Bernoulli's equation relates the pressure p, flow speed v, and elevation y for steady flow in an ideal fluid which is based on conservation of energy principle. For any two points, denoted by subscripts 1 and 2.

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

C8 **Viscosity :** The viscosity of a fluid characterizes its resistance to shear strain. In a Newtonian fluidthe viscous force is propotional to strain rate. The viscous force between two layers of a fluid of area A having a velocity gradient dv/dx is given by $F = -\eta A\frac{dv}{dx}$ where η is called the coefficient of viscosity. In SI unit of η is poiseuille (1 PI = 1 Ns m^{-2}) and the dimension of η is $ML^{-1}T^{-1}$.

C9 **Stoke's Law and Terminal Speed :** A sphere of radius r moving with speed v through a fluid having viscosity η experiences a viscous resisting force F given by Stoke's law : F = 6πηrv.

The following graph shows the variation of velocity v with time t for a small spherical body falling vertically in a long column of viscous liquid



The terminal speed acheived by a sphere is given by $v_t = \frac{2}{9}\frac{r^2 g}{\eta}(\sigma - \rho)$ where σ is the density of the sphere and ρ is the density of the fluid in which sphere is moving.

Practice Problems :

1. If a raindrop with a mass of 0.05 g falls with constant velocity, the retarding force of atmospheric friction is (neglect density of air)

(a) zero (b) 49 dynes (c) 490 dynes (d) none of these

2. A steel ball of radius 2 mm acquires a terminal velocity of 20 cm/s in a liquid. The teminal velocity of another steel ball of radius 1 mm in the same liquid will be

(a) 5 cm/s (b) 10 cm/s (c) 40 cm/s (d) 80 cm/s

3. An air bubble of radius 10 cm rises with a constant speed of 3.5 mm/s through a liquid of density $1.75 \times 10^3 kg/m^3$. Neglecting the density of air, the coefficient of viscosity of the liquid is (in kg m^{-1} s^{-1}).

(a) 54.5 (b) 109 (c) 163.5 (d) 218

[Answers : (1) b (2) a (3) b]

C10 Reynolds Number : The turbulence flow of a fluid is determined by a dimensionless parameter called the Reynolds number given by $R_e = \frac{\rho v d}{\eta}$ where ρ is the density of liquid, v its velocity, η its viscosity and d is the diameter of tube in which liquid will flow. For most cases $R_e < 1000$ signifies laminar flow; $1000 < R_e < 2000$ is unsteady flow and $R_e > 2000$ implies turbulent flow.

C11 Surface Tension : The surface of a liquid behaves like a membrane under tension; the force per unit length across a line on the surface is called the surface tension, denoted by T.

C12 Excess Pressure : Excess pressure inside a liquid drop of radius r is given by $\frac{2T}{r}$. Excess pressure inside a liquid bubble or air bubble of radius r is given by $\frac{4T}{r}$.

Practice Problems :

1. The surface tension of soap solution is 25×10^{-3} N/m. The excess pressure inside a soap bubble of diameter 1 cm is

(a) 5 Pa (b) 10 Pa

(c) 20 Pa (d) none of the above

[Answers : (1) c]

C13 Capillary Rise or Fall : The rise or fall of a liquid in a capillary tube is given by $h = \frac{2T\cos\theta}{\rho g r}$, where θ is the angle of contact, ρ is the density of liquid in the tube and r is the radius of the tube. For a clean glass plate in contact with pure water, θ = 0.

INITIAL STEP EXERCISE

1. A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius R > r. If Young's modulus of the metal is Y then the tenstion in the ring is

(a) $\frac{AYR}{r}$ (b) $\frac{AY(R-r)}{r}$

(c) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (d) $\frac{Yr}{AR}$

2. A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be

(a) 1.12 (b) 1.1

(c) 1.05 (d) 1.0

3. A sample of a metal weighs 210 g in air, 180 g in water and 120 g in an unknown liquid. Then

(a) the density of the metal is 3 g/cm^3

(b) the density of the metal is 7 g/cm^3

(c) the density of the metal is four times the density of the unknown liquid

(d) the metal will float on water

4. Choose the correct statement :

(a) principle of continuity is based on conservation of mass

(b) Bernoulli's principle is based on conservation of energy

(c) Both (a) and (b) are incorrect

(d) Both (a) and (b) are correct

5. Bernoulli's principle does not explain

(a) curved path of a spinning ball

(b) lift of a jet

(c) working of a paint sprayer

(d) automatic blowing off of the roofs of houses during blizzard in hilly areas.

6. Bernoulli's equation is applicable in the case of

(a) streamlined flow of compressible fluids

(b) stremlined flow of incompressible

(c) turbulent flow of compressible

(d) turbulent flow of incompressible fluids

7. If temperature rises, the coefficient of viscosity of a liquid

(a) decreases

(b) increases

(c) remains unchanged

(d) increases for some liquids and decreases for others.

8. A liquid rises to a height h in a capillary tube on the earth. The height to which the same liquid would rise in the same tube on the moon is about

(a) 6 h (b) $\sqrt{6}$ h

(c) h/6 (d) h/$\sqrt{6}$

9. Water flows out of two small holes P and Q in a wall of a tank and the two streams strike the ground at the same point. If the hole P is at a height h above the ground and the level of water stands at a height H above the ground, then the height of Q is

(a) $\frac{H+h}{2}$ (b) H – h

(c) H – h/2 (d) $\frac{H-h}{2}$

FINAL STEP EXERCISE (OBJECTIVE)

1. If a rubber ball is taken down to a 100 m deep lake, its volume decreases by 0.1%. If g = 10 m/s² then the bulk modulus of elasticity for rubber, in N/m², is

   (a) 10^8 (b) 10^9
   (c) 10^{11} (d) 10^{10}

2. A rubber cord of length L is suspended vertically. Density of rubber is D and Young's modulus is Y. If the cord extends by a length *l* under its own weight, then *l* is

   (a) L^2Dg/Y (b) $L^2Dg/2Y$
   (c) $L^2Dg/4Y$ (d) $\frac{2L^2Dg}{Y}$

3. The normal density of gold is ρ and its bulk modulus is K. The increase in density of a piece of gold when a pressure P is applied uniformly from all sides is

   (a) $\frac{\rho P}{2K}$ (b) $\frac{\rho P}{2P}$
   (c) $\frac{\rho P}{K-P}$ (d) $\frac{\rho K}{K-P}$

4. The length of rubber cord is l_1 metres when the tension 4 N and l_2 metres when the tension is 5 N. The length in metres when the tension is 9 N is

   (a) $5l_1 - 4l_2$ (b) $5l_2 - 4l_1$
   (c) $9l_1 - 8l_2$ (d) $9l_2 - 8l_1$

5. A piece of wood of relative density 0.36 floats in oil of relative density 0.90. The fraction of volume of wood above the surface of oil is

   (a) 0.3 (b) 0.4
   (c) 0.6 (d) 0.8

6. A large block of ice 10 m thick with a vertical hole drilled through it is floating in a lake. The minimum length of the rope required to scoop out a bucket full of water through the hole is (density of ice = 0.9 g/cm³)

   (a) 0.5 m (b) 1.0 m
   (c) 1.2 m (d) 1.8 m

7. A vessel contains oil (density 0.8 g/cm³) over mercury (density 13.6 g/cm³). A homogenous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in g/cm³ is

   (a) 3.3 (b) 6.4
   (c) 7.2 (d) 12.8

8. A streamlined body of relative density d_1 falls from a height h on the surface of a liquid of relative density d_2, where $d_2 > d_1$. The time for which the body will fall inside the liquid is

   (a) $\frac{d_1}{d_2}\sqrt{\frac{2h}{g}}$ (b) $\frac{d_2}{d_1}\sqrt{\frac{2h}{g}}$
   (c) $\frac{d_1}{d_2-d_1}\sqrt{\frac{2h}{g}}$ (d) $\frac{d_2-d_1}{d_2}\sqrt{\frac{2h}{g}}$

9. A small ball of density ρ is immersed in a liquid of density σ(σ > ρ) to a depth h and then released. The height above the surface of water up to which the ball will jump is

   (a) $\frac{\sigma h}{\rho}$ (b) $\left(\frac{\sigma}{\rho}-1\right)h$
   (c) $\left(1-\frac{\rho}{\sigma}\right)h$ (d) $\frac{\rho h}{\sigma}$

10. A small ball of density ρ is dropped from a height h into a liquid of density σ (σ > ρ). Neglecting damping forces, the maximum depth to which the body sinks is

    (a) $\frac{h\sigma}{\sigma-\rho}$ (b) $\frac{h\rho}{\sigma-\rho}$
    (c) $\frac{h(\sigma-\rho)}{\rho}$ (d) $\frac{h(\sigma-\rho)}{\sigma}$

11. A vessel of cross-sectional area A contains a liquid to a height H_1. If a hole having cross-sectional area a is made at the bottom of the vessel, then the time taken by the liquid level to decrease from H_1 and H_2 is

    (a) $\frac{A}{a}\sqrt{\frac{g}{2}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$
    (b) $\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$
    (c) $\frac{a}{A}\sqrt{\frac{g}{2}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$
    (d) $\frac{a}{A}\sqrt{\frac{2}{g}}\left[\sqrt{H_1}-\sqrt{H_2}\right]$

12. Two capillary tubes of the same length and radii r_1 and r_2 are fitted horizontally side by side to the bottom of a vessel containing water. The radius of a single tube that can replace the two tubes such that the rate of study flow through this tube equals the combined rate of flow through the two tubes, is

(a) $r_1 + r_2$ (b) $\sqrt{r_1 r_2}$

(c) $\left(r_1^2 + r_2^2\right)^{1/2}$ (d) $\left(r_1^4 + r_2^4\right)^{1/4}$

13. Two spherical soap bubbles of radii r_1 and r_2 in vacuum coalesce under isothermal conditions. The resulting bubble has a radius equal to

(a) $\frac{r_1 + r_2}{2}$ (b) $\frac{r_1 r_2}{r_1 + r_2}$

(c) $\sqrt{r_1 r_2}$ (d) $\sqrt{r_1^2 + r_2^2}$

14. n identical spherical drops of a liquid of surface tension T, each of radius r, coalesce to form a single drop. The surface energy

(a) decreases by $4\pi r^2(n - n^{1/3})T$

(b) increases by $4\pi r^2(n - n^{1/3})T$

(c) decreases by $4\pi r^2(n - n^{2/3})T$

(d) increases by $4\pi r^2(n - n^{2/3})T$

15. A long cylindrical glass vessel has a small hole of radius r at its bottom. The depth to which the vessel can be lowered vertically in a deep water bath (surface tenstion T, density d) without any water entering inside is

(a) $\frac{T}{rdg}$ (b) $\frac{2T}{rdg}$

(c) $\frac{3T}{rdg}$ (d) $\frac{4T}{rdg}$

16. A large tank, filled with water to a height h, is to be emptied through a small hole at the bottom. The ratio of the time taken for the level to fall from h to h/2 and than taken for the level to fall from h/2 to 0 is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2} - 1$ (d) $\frac{1}{\sqrt{2} - 1}$

ANSWERS (INITIAL STEP EXERCISE)

1.	b
2.	b
3.	b
4.	d
5.	b
6.	b
7.	a
8.	a
9.	b

ANSWERS (FINAL STEP EXERCISE)

1.	b	9.	b
2.	b	10.	b
3.	c	11.	b
4.	b	12.	d
5.	c	13.	d
6.	b	14.	c
7.	c	15.	b
8.	c	16.	c

TEST YOURSELF

1. Eight spherical rain drops of the same mass and radius are falling down with a terminal speed of 6 cm s^{-1}. If they coalesce to form one big drop, what will be its terminal speed ? Neglect the buoyancy due to air

   (a) 1.5 cms^{-1} (b) 6 cms^{-1}

   (c) 24 cms^{-1} (d) 32 cms^{-1}

2. If the surface tension of soap solution is σ, what is the work done in blowing soap bubble of radius ?

   (a) $\pi r^2\sigma$ (b) $2\pi r^2\sigma$

   (c) $4\pi r^2\sigma$ (d) $8\pi r^2\sigma$

3. A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of water in the capillary tube is 5g. Another capillary tube of radius 2r is immersed in water. The mass of water that will rise in this tube is

   (a) 2.5 g (b) 5.0 g

   (c) 10 g (d) 20 g

4. A liquid flows through a pipe of varying diameter. The velocity of the liquid is 2 ms^{-1} at a point where the diameter is 6 cm. The velocity of the liquid at a point where the diameter is 3 cm will be

   (a) 1 ms^{-1} (b) 4 ms^{-1}

   (c) 8 ms^{-1} (d) 16 ms^{-1}

5. Two springs of equal lengths and equal cross-sectional areas are made of materials whose Young's modulii are in the ratio of 3 : 2. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of

   (a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2

   (c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4

6. Two rods of different materials, having coefficient of linear expansion α_1 and α_2 and Young's modulii Y_1 and Y_2, are fixed between two rigid walls. The rods are heated to the same temperature. There is no bending of rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods will be equal provided $Y_1 : Y_2$ is equal to

   (a) 2 : 3 (b) 1 : 1

   (c) 3 : 2 (d) 4 : 9

7. A film of water is formed between two straight parallel wires, each 10 cm long and at a separation of 0.5 cm. The work that must be done to increase the separation between the wires by 1 mm is (surface tension of water = 7.0×10^{-2} Nm^{-1}).

   (a) 7.0×10^{-5} N (b) 1.4×10^{-5} N

   (c) 7.0×10^{-7} N (d) 1.4×10^{-7} N

8. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of the earth, the ratio $\frac{x}{y}$ is

   (a) $\left(1-\frac{d}{R}\right)$ (b) $\left(1+\frac{d}{R}\right)$

   (c) $\left(\frac{R-d}{R+d}\right)$ (d) $\left(\frac{R+d}{R-d}\right)$

9. Tanks A and B open at the top contain two different liquids upto certain height in them. A hole is made to the wall of each tank at a depth 'h' from the surface of the liquid. The area of the hole in A is twice that of in B. If the liquid mass flux through each hole is equal, then the ratio of the densities of the liquids respectively, is

   (a) $\frac{2}{1}$ (b) $\frac{3}{2}$

   (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

10. If a number of little droplets of a liquid of density ρ, surface tension T and specific heat c, each of radius r, coalesce to form a single drop of radius R, the rise in temperature will be

   (a) $\frac{3T}{\rho c}\left(\frac{1}{r}+\frac{1}{R}\right)$ (b) $\frac{3T}{\rho c}\left(\frac{1}{r}-\frac{1}{R}\right)$

   (c) $\frac{3T}{2\rho c}\left(\frac{1}{r}+\frac{1}{R}\right)$ (d) $\frac{3T}{2\rho c}\left(\frac{1}{r}-\frac{1}{R}\right)$

ANSWERS

1.	c	6.	c
2.	d	7.	d
3.	c	8.	a
4.	c	9.	d
5.	a	10.	b