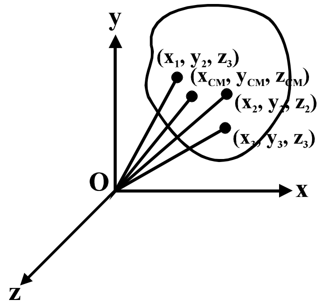
ROTATIONAL MOTION

CONCEPTS

C1 Centre of Mass

For a system of particles, that the distributed in three dimensions shown in the figure



The center of mass of the system of particles is given by

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

The position vector of centre of mass of the system of particles is given by $\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$

i.e., $$\vec{R}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Centre of Mass of Solid Bodies

Solid bodies are treated as continuous distribution of matter and the centre of mass for these bodies is given by

$$x_{CM} = \frac{\int x\,dm}{\int dm} \qquad y_{CM} = \frac{\int y\,dm}{\int dm} \qquad z_{CM} = \frac{\int z\,dm}{\int dm}$$

Here x, y, z are the centre of mass of the differential elements of the solid bodies and dm is the mass of the differential elements.

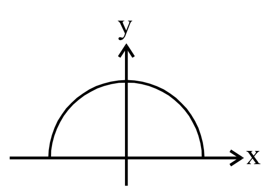
Practice Problems :

1. Four particles of masses m, 2m, 4m, 4m are placed at (*l*, *l*), (–*l*, *l*), (–*l*, –*l*) and (*l*, –*l*) respectively. The centre of mass will lie in

(a) First quadrant (b) Second quadrant

(c) Third quadrant (d) Fourth quadrant

2. A uniform half circular ring of radius r is placed on the x-y plane as shown in figure. The center of mass of uniform half circular ring is



(a) $\left(0, \frac{2r}{\pi}\right)$ **(b)** $\left(\frac{2r}{\pi}, \frac{2r}{\pi}\right)$ **(c)** $\left(\frac{2r}{\pi}, 0\right)$ **(d)** $\left(-\frac{2r}{\pi}, 0\right)$

[Answers : (1) c (2) a]

C2 Newton's Second Law For System of Particles

For a system of n particles, $\mathbf{M\vec{R}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}$

Differentiating with respect to time, $\mathbf{M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}$

where $\mathbf{\vec{V}_{CM}}$ is the velocity of centre of mass of the system of particles.

Differentiating with respect to time, $\mathbf{M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n}$

where $\mathbf{\vec{a}_{CM}}$ is the acceleration of the centre of mass of the system of particles.

From Newton's second law

$$\mathbf{M\vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n}$$

Among the forces that contribute to the right side of the above equation will be forces that the particles of the system exert on each other (internal forces) and forces exerted on the particles from outside the system (external forces). By Newton's third law, the internal forces cancel out in the sum that appears on the right side of the above equation, what remains is the vector sum of all the external forces that act on the system.

Hence $\mathbf{\vec{F}_{net} = M\vec{a}_{CM}}$.

Practice Problems :

1. **Two spheres of masses M and 2M are initially at rest at a distance R apart. Due to mutual force of attraction they approach each other.When they are at separation R/2, the acceleration of their centre of mass would be**

(a) 0 (b) g m/s^2 (c) 3g m/s^2 (d) 12g m/s^2

2. **An isolated particle of mass m is moving in a horizontal plane (x – y) along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses m/4 and 3m/4. An instant later, the smaller fragment is at y = + 15 cm. The larger fragment at this instant is at**

(a) y = –5 cm (b) y = + 20 cm (c) y = +5 cm (d) y = –20 cm

[Answers : (1) a (2) a]

C3 Rotational Motion

Here we examine the rotation of a rigid body (a body with a definite and unchanging shape and size) about a fixed axis (an axis that does not move), called the axis of rotation or the rotational axis. Every point of the body moves in a circle whose centre lies on the axis of rotation, and every point moves through the same angle during a particular time. In pure translation, every point of the body moves through the same linear distance during a particular time interval in a straight line. Hence we can see the angular equivalent of the linear quantities position, displacement, velocity and acceleration.

Angular displacement : $\Delta\theta = \theta_2 - \theta_1$.

Angular velocity : Average angular velocity, $\langle\omega\rangle = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$.

Instant angular velocity, $\omega = \lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$.

Both <ω> and ω are vectors, with the direction given by the right hand rule. The magnitude of the body's angular velocity is the angular speed.

Angular acceleration : Average angular acceleration, $\langle\alpha\rangle = \dfrac{\omega_2 - \omega_1}{t_2 - t_1} = \dfrac{\Delta\omega}{\Delta t}$.

Instant angular acceleration, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$.

Both <α> and α are vectors.

C4 <u>Rotational Motion with Constant Angular Acceleration</u>

The kinematics equations for constant angular acceleration

ω	=	$\omega_0 + \alpha(t - t_0)$
θ	=	$\theta_0 + \omega_0(t - t) + (1/2)\alpha\,(t - t_0)2$
ω^2	=	$\omega_0^2 + 2\alpha(\theta - \theta_0)$. Here the symbols have the following meaning :
θ_0	→	Angular positon at t_0
θ	→	Angular position at t
ω_0	→	Angular velocity at t_0
ω	→	Angular velocity at t
α	→	Angular acceleration.

<u>Practice Problems :</u>

1. A wheel rotates with a constant acceleration of 2.0 rad/s². If the wheel starts from rest, the number of revolutions it makes in the first ten seconds will be approximately

(a) 8 (b) 16 (c) 24 (d) 32

[Answers : (1) b]

C5 <u>RELATION BETWEEN LINEAR AND ANGULAR VARIABLES</u>

A point in a rigid rotating body at a perpendicular distance r from the rotation axis moves in a circle with radius r. If the body rotates through an angle θ, the point moves along the arc with length s is given by s = θr where θ is in radians.

The linear velocity $\vec{v}$ of the point is tangent to the circle and the point's linear speed v is given by v = ωr, where ω is the angular speed of the body.

The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The tangential component is $a_t = \alpha r$, where α is the magnitude of the angular acceleration of the body. The tangential acceleration $\left(a_t = \frac{dv}{dt}\right)$ represents only the part of linear acceleration that is responsible for change in the magnitude of the linear velocity $\vec{v}$. Like $\vec{v}$, that part of the linear acceleration is tangent to the path of the point in question.

The radial component, responsible to change the direction of $\vec{v}$, is $a_r = \frac{v^2}{r} = \omega^2 r$. This component is directed radially inward.

For a body having constant velocity, $a_t = 0$ and $a_r = 0$. For a body having constant angular speed or linear speed, $a_t = 0$ and $a_r = \frac{v^2}{r}$. Body having variable angular speed or linear speed, $a_t = \frac{dv}{dt}$ and $a_r = \frac{v^2}{r}$.

Remember the following vector relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$\vec{a} = \vec{a}_t + \vec{a}_r$$
$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

C6 ROTATIONAL KINETIC ENERGY AND ROTATIONAL INERTIA

In terms of moment of inertia I, the rotational kinetic energy K of a rigid body is $\mathbf{K = \frac{1}{2} I\omega^2}$.

Remember the following :

Moment of inertia of point mass I = mr^2

Moment of inertia of a system of discret particles $\mathbf{I = \sum_{i=1} m_i r_i^2}$.

Here r and r_i in these expressions represents the perpendicular distance from the axis of rotation of each mass element in the body.

This quantity has the same significance in rotational motion as that of mass in linear motion. It is a measure of the resistance offered by a body to a charge in its rotational motion.

Practice Problems :

1. The moment of inertia of a body about an axis is 1.2 kg-m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad/s^2 must be applied about the axis for a duration of

(a) 2 s **(b)** 4 s **(c)** 8 s **(d)** 10 s

2. Two points masses m_1 and m_2 are joined by a massless rod of length r. The moment of inertia of the system about an axis passing through the center of mass and perpendicular to the rod is

(a) $(m_1 + m_2)\frac{r^2}{4}$ **(b)** $(m_1 - m_2)\frac{r^2}{4}$ **(c)** $\frac{m_1 m_2}{m_1 + m_2} r^2$ **(d)** $\frac{m_1 m_2}{m_1 - m_2} \frac{r^2}{4}$

[Answers : (1) a (2) c]

C7 THEOREM OF MOMENT OF INERTIA :

1. The parallel axis theorem

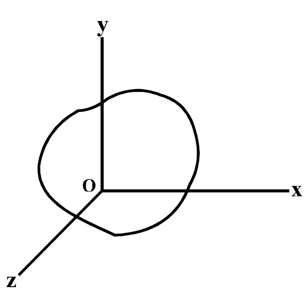
The parallel axis theorem relates the moment of inertia I of a body about any axis to that of the same body about a parallel axis through the centre of mass (also known as centroid axis) as $I = I_{com} + M h^2$

where I_{com} is the moment of inertia of the body about the centroidal axis and h is the perpendicular distance between the two axes. Note that the parallel axis may lie within or inside the body.

2. The perpendicular axis theorem

This theorem is valid for a planar or laminar body (body in two dimensions like a thin disc, ring or thin plate etc.).

Let x and y be the two axes which lie in the plane of the body and pass through the point O, as shown in figure.

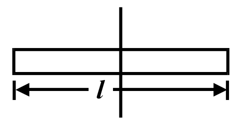


Then the moment of inertia about an axis (called z-axis) passing through O and perpendicular to the plane containing x and y axes is given by $I_z = I_x + I_y$, where I_x, I_y and I_z are the respective moment of inertia of the body about x, y and z axes.

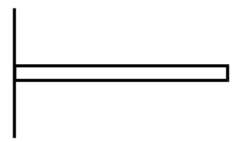
C8 MOMENT OF INERTIA OF IMPOTANT BODIES :

1. A thin rod

(a) About an axis through centre of mass perpendicular to length, $I = \frac{1}{12} Ml^2$



(b) About an axis through one of the end and perpendicular to length, $I = \frac{1}{3} ML^2$



2. A ring or Hoop

(a) About an central axis and perpendicular to the plane $I = MR^2$

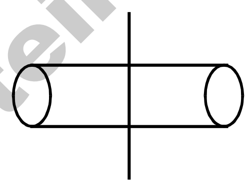
(b) About any diameter $I = \frac{MR^2}{2}$

(c) About a tangent in the plane of ring $I = \frac{3}{2} MR^2$

(d) About a tangent perpendicular to the plane of ring $I = 2\ MR^2$

3. Hollow cylinder

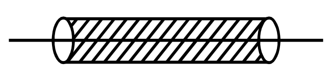
About central axis $I = MR^2$



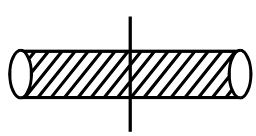
$$I = \frac{ML^2}{12} + \frac{MR^2}{2}$$

4. Solid cylinder

(a) About central axis or axis of symmetry $I = \frac{MR^2}{2}$



(b) About central diameter $I = \frac{ML^2}{12} + \frac{MR^2}{4}$



5. **Disc**

(a) About an axis through the centre and perpendicular to the plane $\mathbf{I = \frac{MR^2}{2}}$.

(b) About the diameter $\mathbf{I = \frac{MR^2}{4}}$

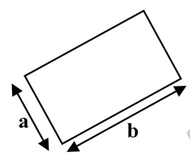
(c) About the tangent perpendicular to the plane of disc $\mathbf{I = \frac{3}{2}MR^2}$

(d) About the tangent in the plane of disc $\mathbf{I = \frac{5}{4}MR^2}$

6. **A rectangular plate**

About perpendicular axis through centre

$$\mathbf{I = \frac{1}{12}M(a^2 + b^2)}$$



7. **Solid sphere**

(a) About its diameter $\mathbf{I = \frac{2}{5}MR^2}$

(b) About its tangent $\mathbf{I = \frac{7}{5}MR^2}$

8. **Hollow sphere**

(a) About its diameter $\mathbf{I = \frac{2}{3}MR^2}$

(b) About its tangent $\mathbf{I = \frac{5}{3}MR^2}$

9. **Annular cylinder (or ring)**

About central axis $\mathbf{I = \frac{1}{2}M(R_1^2 + R_2^2)}$, here R_2 is the outer radius and R_1 is the inner radius.

<u>**Practice Problems :**</u>

1. **Which of the following has the highest moment of inertia if each has the same mass and the same radius ?**

(a) A ring about its axis perpendicular to the plane of the ring

(b) A solid sphere about one of its diameters

(c) A spherical shell about one of its diameters

(d) A disc about its axis perpendicular to the plane of its disc.

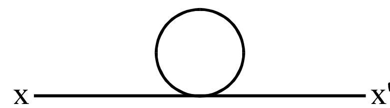
2. **The moment of inertia of a uniform circular disc about a diameter is *I*. Its momentum of inertia about an axis perpendicular to its plane and passing through a point on its rim is**

(a) 3 *I* **(b) 4 *I*** **(c) 5 *I*** **(d) 6 *I***

3. **The radius of gyration of a rod of mass m and length L about an axis of rotation perpendicular to its length and passing through the center is**

(a) $\frac{L}{2\sqrt{3}}$ (b) $\frac{L}{2\sqrt{2}}$ (c) $\frac{L}{2\sqrt{5}}$ (d) $\frac{L}{\sqrt{2}}$

4. **A wire of mass per unit length λ and length L is used to form a circular loop. The moment of inertia about the xx' is**



(a) $\frac{2\lambda L^3}{8\pi^2}$ (b) $\frac{4\lambda L^3}{8\pi^2}$ (c) $\frac{3\lambda L^3}{8\pi^2}$ (d) $\frac{5\lambda L^3}{8\pi^2}$

[Answers : (1) a (2) d (3) a (4) c]

C9 <u>Moment of Force or Torque</u>

Torque is a turning or twisting action on a body about a rotation axis due to a force $\vec{F}$. It has the same role in rotational motion as that of force in linear motion. Consider a force $\vec{F}$ is exerted at a point given by the position vector $\vec{r}$ relative to the axis, as shown in figure. Its torque about O is given by $\vec{\tau} = \vec{r} \times \vec{F}$.

Newton's Second Law for Rotation

The rotational analog of newton's second law is $\tau_{net} = I\alpha$

where τ_{net} is the net torque acting on a particle or rigid body, I is the rotational inertia of the particle or body about the rotation axis, and α is the resulting angular acceleration about that axis. Here τ_{net} and I are taken with respect to the same rotation axis.

Note the following points for $\tau_{net} = I\alpha$

Equilibrium : A rigid body is said to be in equilibrium if

(a) Net external force equal to zero. This is the condition of transiational equilibrium $\sum \vec{F} = 0$.

(b) Net external torque equal to zero. This is the condition of rotational equilibrium $\sum \vec{\tau} = 0$.

<u>Practice Problems :</u>

1. **A wheel of radius 10 cm and mass 12.5 kg rotates freely about an axis passing through the center and perpendicular to the plane of the wheel by applying a constant force F and it is found that its angular speed increases from zero to 2 rad/s in 1s. The force F acting on the wheel to do so**

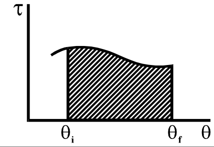
(a) 1.25 N (b) 2.5 N (c) 4.5 N (d) 6.25 N

[Answers : (1) a]

C10 <u>WORK IN ROTATIONAL MOTION</u>

When a torque τ acts on a rigid body that undergoes an angular displacement from θ_i to θ_f then work W done by the torque is $W = \int_{\theta_i}^{\theta_f} \tau d\theta$. If the torque is constant, then $W = \tau(\theta_f - \theta_i) = \tau\Delta\theta$.

Graphical interpreration of rotational work done is shown in figure.



Work - Energy Theorem for Rotational Motion

Work energy theorem for rotational motion of a rigid body is $\mathbf{W = \Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2}$.

Practice Problems :

1. **A uniform cylinder of radius R and mass M is spinned about its axis to the angular velocity ω_0 and then placed into a corner. The coefficient of friction between the walls and the cylinder is equal to μ. The total work done is**

(a) $-\frac{1}{2}MR^2\omega_0^2$ **(b)** $\frac{1}{2}MR^2\omega_0^2$ **(c)** $\frac{1}{4}MR^2\omega_0^2$ **(d)** $-\frac{1}{4}MR^2\omega_0^2$

[Answers : (1) d]

C11 POWER IN ROTATIONAL MOTION

When the body rotates with angular velocity ω, the power P (rate at which the torque does work) is

$\mathbf{P = \frac{dW}{dt}} = \tau\omega$.

Practice Problems :

1. **An electric motor exerts a constant torque of τ = 10 N-m on a gridstone mounted on it shaft. The moment of inertia of the grindstone is 2 kg-m². If the system starts from rest, the kinetic energy at the end of 8s is**

(a) **400 J** **(b)** **800 J** **(c)** **1600 J** **(d)** **2000 J**

2. **In the above problem, the instant power at t = 8s delivered by the motor is**

(a) **100 W** **(b)** **200 W** **(c)** **400 W** **(d)** **800 W**

[Answers : (1) c (2) c]

C12 ANGULAR MOMENTUM

1. **Angular momentum of a particle**

The angular momentum $\vec{L}$ of a particle, with linear momentum $\vec{p}$, mass m and linear velocity $\vec{v}$ is a vector quantity defined relative to a fixed point (usually an origin). It is $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$.

The magnitude of $\vec{L}$ is given by

$|\vec{L}| = mvr\sin\phi = rp_\perp = rmv_\perp = r_\perp p = r_\perp mv$.

where ϕ is the angle between $\vec{r}$ and $\vec{p}$, $p_\perp$ and $v_\perp$ are the components of $\vec{p}$ and $\vec{v}$ respectively, perpendicular to $\vec{r}$ and $r_\perp$ is the perpendicular distance between the fixed point and the line of extension of $\vec{p}$. The direction of $\vec{L}$ is given by the right hand rule for cross products.

2. **Angular momentum of a system of particles**

The angular momentum $\vec{L}$ of a system of particles is the vector sum of the angular momentum of individual particles, $\vec{L} = \sum_{i=1}^{n} \vec{L}_i$.

3. **Angular Momentum of a Rigid Body**

When a symmetric rigid body with moment of inertia I rotates with angular velocity $\vec{\omega}$ about a stationary axis of symmetry, its angular momentum is given by $\vec{L} = I\vec{\omega}$. If the body is not symmetric or the rotation axis is not an axis of symmetry, the component of angular momentum along the axis of rotation is equal to Iω.

1. **A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin**

   (a) is zero (b) remains constant

   (c) goes on increasing (d) goes on decreasing

   (b)

2. **When a mass is rotated in a plane about a fixed point, its angular momentum is directed along**

   (a) the radius

   (b) the tangent to the orbit

   (c) a line at an angle of 45^0 to the plane of rotation

   (d) the axis of rotation.

   (d)

   [Answers : (1) b (2) d]

C13 RELATION BETWEEN TORQUE AND ANGULAR MOMENTUM

The rate of change of angular momentum of a rigid body equals the net torque acting on it i.e., $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$.

Practice Problems :

1. **A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 seconds. The magnitude of this torque is**

   (a) $\frac{3A_0}{4}$ (b) A_0 (c) $4\,A_0$ (d) $12\,A_0$

   [Answers : (1) a]

C14 CONSERVATION OF ANGULAR MOMENTUM

The angular momentum $\vec{L}$ of a system remains constant if the net external torque acting on the system is zero i.e. $\vec{\tau} = \frac{d\vec{L}}{dt} = 0 = \vec{L} = \text{constant}$.

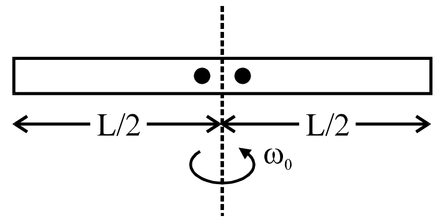
This is a law of conservation of angular momentum. It is one of the fundamental conservation laws of nature, having been verified even in situation (involving high speed particles or subatomic dimension) in which newton's laws are not applicable.

Practice Problems :

1. **A thin circular ring of mass M is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity.**

   (a) $\frac{\omega M}{M+m}$ (b) $\frac{\omega(M-2m)}{M+2m}$ (c) $\frac{\omega M}{M+2m}$ (d) $\frac{\omega(M+2m)}{M}$

2. **A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid-point of the rod. There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is**



(a) $\frac{M\omega_0}{M+3m}$ **(b)** $\frac{M\omega_0}{M+4m}$ **(c)** $\frac{M\omega_0}{M+5m}$ **(d)** $\frac{M\omega_0}{M+6m}$

[Answers : (1) c (2) d]

C15 ROLLING MOTION :

(i) The combined motion of translation and rotation is known as rolling motion.

(ii) Condition of pure rolling motion on fixed surface : $v_c = \omega r$ where v_c is the velocity of the centre of mass of the rolling body of radius r and ω is the angular velocity about the centre of mass.

(iii) Kinetic energy of rolling body = $\frac{1}{2}mv_c^2 + \frac{1}{2}I\omega^2$.

(iv) For the body rolling along the incline plane of inclination θ : $a_c = g\sin\theta/(1 + I/MR^2)$,

$v_c = \sqrt{2gh/(1+I/MR^2)}$

Practice Problems :

1. A solid cylinder of mass M and radius R rolls down an inclined plane from height h without slipping. The speed of its centre of mass when it reaches the bottom is

(a) $\sqrt{2gh}$ **(b)** $\sqrt{\frac{4}{3}gh}$ **(c)** $\sqrt{\frac{3}{4}gh}$ **(d)** $\sqrt{\frac{4g}{h}}$

2. A thin, uniform, circular disc is rolling down an inclined plane of inclination 30^0 without slipping. Its linear acceleration along the plane is

(a) g/4 **(b)** g/3 **(c)** g/2 **(d)** 2g/3

3. A solid sphere, a hollow sphere and a solid cylinder, all of the same radius, roll down an inclined plane from the same height, starting from rest. Which of them takes the least time in reaching the bottom of the plane ?

(a) Solid sphere **(b)** Hollow sphere

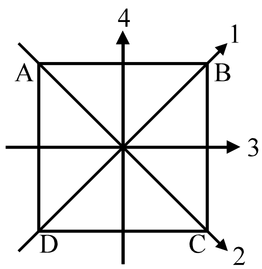
(c) Solid cylinder **(d)** All will take the same time

4. A ring is rolling without slipping on a horizontal surface. The velocity of centre of mass of the ring is v. The fraction of rotational kinetic energy of the total kinetic energy is

(a) 1/2 **(b)** 1/3 **(c)** 1/4 **(d)** 1/5

[Answers : (1) b (2) b (3) a (4) a]

1. The moment of inertia of a thin square plate ABCD of uniform thickness about an axis passing through the centre O and perpendicular to the plane is



(a) $I_1 + I_2$ (b) $I_3 + I_4$
(c) $I_2 + I_4$ (d) all the above

2. A particle performs uniform circular motion with angular momentum *l*. If the frequency of the motion of the particle is doubled and its kinetic energy halved, the angular momentum becomes

(a) $2l$ (b) $4l$
(c) $l/2$ (d) $l/4$

3. Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2/I_1 = 4$ then $\frac{r_2}{r_1}$ equals

(a) $4^{2/3}$ (b) $4^{1/3}$
(c) $4^{-2/3}$ (d) $4^{-1/3}$

4. The rotational kinetic energy of a body is E and its moment of inertia is I. The angular momentum of the body is

(a) E I (b) $2\sqrt{EI}$
(c) $\sqrt{2EI}$ (d) E/I

5. A false balance has equal arms. An object weighs x when placed in one pan and y when placed in the other pan. The true weight of the object is equal to

(a) $\sqrt{xy}$ (b) $\frac{x+y}{2}$
(c) $\frac{x^2+y^2}{2}$ (d) $\frac{\sqrt{x^2+y^2}}{2}$

6. A uniform cube of side *l* and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the center of the face, at a height $\frac{3l}{4}$ above the base. The minimum value of F for which the cube begins to tip about an edge is

(a) 1/3 mg (b) 2/3 mg
(c) 1/2 mg (d) 1/4 mg

7. A string is wrapped around a cylinder of mass m and radius r. The string is pulled vertically upward to prevent the center of mass to fall as the cylinder unwinds the string. The tension in the string is

(a) mg/4 (b) mg/2
(c) mg (d) 2mg

8. Two discs with moment of inertia I_1 and I_2 initially they are rotating with angular velocities ω_1 and ω_2 respectively in anticlockwise direction, are pushed with forces acting along the axis, so as not to apply any torque on either disks. The disks rub against each other and eventually reach a common final angular velocity ω. Which of the following quantity will be conserved ?

(a) Kinetic energy
(b) Angular Momentum
(c) Linear momentum
(d) All the above

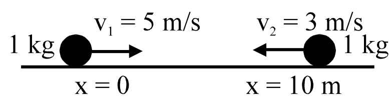
9. If the radius of earth contarcts to half of its present day value, the mass remaining unchanged, the duration of the day will be

(a) 48 hrs (b) 24 hrs
(c) 12 hrs (d) 6 hrs

FINAL STEP EXERCISE

1. Two skaters A and B, having masses 50 kg and 70 kg respectively, stand facing each other 6 m apart on a horizontal smooth surface. They pull on a rope stretched between them. How far does each move before they meet ?
   (a) both move 3 m
   (b) A moves 2.5 m and B moves 3.5 m
   (c) A moves 3.5 m and B moves 2.5 m
   (d) none of the above

2. At t = 0, the position and velocities of two particles are as shown in the figure. They are kept on a smooth surface and being mutually attracted by gravitational force. The position of centre of mass at t = 2s is



   (a) x = 5 m (b) x = 7 m
   (c) x = 3 m (d) x = 2 m

3. A hollow sphere and a solid sphere, having the same mass, are released from rest simultaneously from the top of a smooth inclined plane. Which of the two will reach the bottom first ?
   (a) solid sphere
   (b) hollow sphere
   (c) the one which has the greater density
   (d) both will reach the bottom simultaneously

4. A particle of mass m is projected with a velocity v making an angle of 45^0 with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is
   (a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$
   (c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{gh^3}$

5. Four spheres, each of mass M and diameter 2r, are placed with their centres on the four corners of a square of side a (> 2r). The moment of inertia of the system about one side of the square is
   (a) $\frac{2}{5}M(5r^2 + 4a^2)$
   (b) $\frac{2}{5}M(5r^2 + 2a^2)$
   (c) $\frac{2}{5}M(2r^2 + 5a^2)$
   (d) $\frac{2}{5}M(4r^2 + 5a^2)$

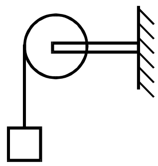
6. A cord is wound round the circumference of a wheel of radius r. The axis of the wheel is horizontal and its moment of inertia about this axis is I. A weight mg is attached to the end of the cord and allowed to fall from rest. The angular velocity of the wheel, when the weight has fallen through a distance h, is
   (a) $\left[\frac{2gh}{I + mr}\right]^{1/2}$ (b) $\left[\frac{2mgh}{I + mr^2}\right]^{1/2}$
   (c) $\left[\frac{2mgh}{I + 2mr^2}\right]^{1/2}$ (d) $(2gh)^{1/2}$

7. A body of mass M and radius r, rolling on a smooth horizontal floor with velocity v, rolls up an irregular inclined plane up to a vertical height $\frac{3v^2}{4g}$. The body may be
   (a) sphere (b) solid cylinder
   (c) disc (d) both (b) and (c)

8. A thin rod of length L and mass M is held vertically with one end on the floor and is allowed to fall. The velocity of the other end when it hits the floor, assuming that the end on the floor does not slip
   (a) $\sqrt{3gL}$ (b) $\sqrt{2gL}$
   (c) $\sqrt{gL}$ (d) $2\sqrt{gL}$

9. Three uniform rods each of mass m and length L, is used to form an equilateral triangle. The moment of inertia of this frame about an axis through the centroid and perpendicular to the plane of triangle is
   (a) mL^2 (b) $\frac{mL^2}{2}$
   (c) $\frac{mL^2}{3}$ (d) $\frac{mL^2}{4}$

10. A uniform disk, with mass M and radius R mounted on a fixed horizontal axle. A block with mass M hangs from a massless cord that is wrapped around the rim of the disk. The cord does not slip, and there is no friction at the axle. The acceleration of the falling block is



(a) g (b) g/2

(c) 2/3 g (d) g/3

11. A uniform ladder of mass 10 kg rests against a smooth vertical wall making an angle of 53^0 with it. The other end rests on a rough horizontal floor. The friction froce that the floor exerts on the ladder is

(a) 65 N (b) 98 N

(c) 75 N (d) 86 N

12. Figure shows a mass m placed on a frictionless horizontal table and attached to a string passing through a small hole in the surface. Initially, the mass moves in a circle of radius r_0 with a speed v_0 and a person holds the free end of the string. The person pulls on the string slowly to decrease the radius of the circle to r. Let the tension in the string when the mass moves depends on radius r as r^n. The value of n is

(a) –1 (b) –2

(c) –3 (d) –4

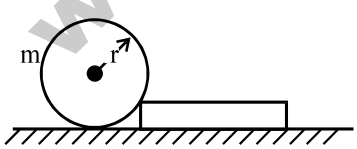
13. A sphere of mass m and radius R is rolling without slipping with angular speed ω on a horizontal plane.

The angular momentum of the sphere about any point lying on the surface is

(a) $2/5\ mR^2\omega$ (b) $3/5\ mR^2\omega$

(c) $7/5\ mR^2\omega$ (d) $8/5\ mR^2\omega$

14. A wheel of radius r and mass m stands in contact with step of height h. The least horizontal force F which should be applied to the axle of the wheel to force it climb onto the step is



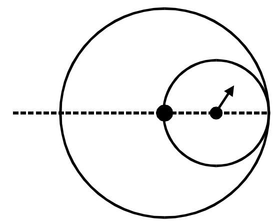
(a) $\dfrac{mg\sqrt{[h(2r-h)]}}{r-h}$

(b) $\dfrac{mgh(2r-h)}{r-h}$

(c) $\dfrac{mgh}{r-h}$

(d) None of these

15. A circular hole of radius R/2 is cut from a homogenous circular disc of a radius R. The centre of mass of the remaining disc is



(a) R/6 towards left

(b) R/6 towards right

(c) R/3 towards left

(d) R/3 towards right

ANSWERS (INITIAL STEP EXERCISE)

1.	d	8.	b
2.	d	9.	d
3.	b		
4.	c		
5.	b		
6.	b		
7.	c		

ANSWERS (FINAL STEP EXERCISE)

1.	c	9.	b
2.	b	10.	c
3.	d	11.	a
4.	b	12.	c
5.	d	13.	c
6.	b	14.	a
7.	d	15.	a
8.	a		

TEST YOURSELF

1. In the HCl molecule, the separation between the nuclei of hydrogen and chlorine atoms is 1.27Å. If the mass of a chlorine atom is 35.5 times that of a hydrogen atom, the centre of mass of the HCl molecule is at a distance of

(a) $\frac{35.5\times1.27}{36.5}$ Å from the hydrogen atom

(b) $\frac{35.5\times1.27}{36.5}$ Å from the chlorine atom

(c) $\frac{1.27}{36.5}$ Å from the chlorine atom

(d) both (a) and (c) are correct

2. The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis is

(a) $1 : \sqrt{2}$ (b) $\sqrt{5} : \sqrt{6}$

(c) $\sqrt{2} : \sqrt{3}$ (d) $\sqrt{2} : 1$

3. A solid sphere is rotating about its diameter. Due to increase in room temperature, its volume increases by 0.5%. If no external torque acts, the angular speed of the sphere will

(a) increase by nearly $\frac{1}{3}$%

(b) decrease by nearly $\frac{1}{3}$%

(c) increase by nearly $\frac{1}{2}$%

(d) decrease by nearly $\frac{2}{3}$%

4. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in second) applied tangentially. The force is then withdrawn. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is very nearly equal to

(a) $5\frac{1}{2}$ (b) $18\frac{1}{2}$

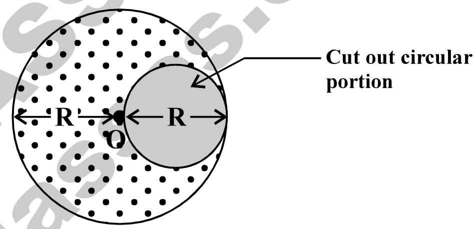
(c) $11\frac{1}{2}$ (d) $14\frac{1}{2}$

5. Moment of inertia of uniform horizontal solid cylinder of mass M about an axis passing through its edge and perpendicular to the axis of the cylinder when its length is 6 times its radius R is

(a) $\frac{39MR^2}{4}$ (b) $\frac{39MR^2}{2}$

(c) $\frac{49MR^2}{2}$ (d) $\frac{49MR^2}{4}$

6. A circular portion of diameter R is cut out from a uniform circular disc of mass M and radius R as shown in the figure. The moment of inertia of the remaining (shaded) portion of the disc about an axis passing through the centre O of the disc and perpendicular to its plane is



(a) $\frac{15}{32}MR^2$ (b) $\frac{7}{16}MR^2$

(c) $\frac{13}{32}MR^2$ (d) $\frac{3}{8}MR^2$

7. A small coin is placed at a distance r from the centre of the gramophone record. The rotational speed of the record is gradually increased. If the coefficient of friction between the coin and the record is μ, the minimum angular frequency of the record for which the coin will fly off is given by

(a) $\sqrt{\frac{2\mu g}{r}}$ (b) $\sqrt{\frac{\mu g}{2r}}$

(c) $\sqrt{\frac{\mu g}{r}}$ (d) $2\sqrt{\frac{\mu g}{r}}$

8. A disc is rotating with angular velocity $\vec{\omega}$. A force $\vec{F}$ acts at a point whose position vector with respect to the axis of rotation is $\vec{r}$. The power associated with the torque due to the force is given by

(a) $(\vec{r}\times\vec{F})\cdot\vec{\omega}$ (b) $(\vec{r}\times\vec{F})\times\vec{\omega}$

(c) $\vec{r}\cdot(\vec{F}\times\vec{\omega})$ (d) $\vec{r}\times(\vec{F}\cdot\vec{\omega})$

9. **When an explosive shell, travelling in a parabolic path under the effect of gravity explodes, the centre of mass of the fragments will move**

(a) first vertically upwards and then vertically downwards

(b) vertically downwards

(c) along the original parabolic path

(d) first horizontally and then along a parabolic path.

10. **Two blocks m_1 and m_2, having masses 10 kg and 5 kg respectively, are placed on a frictionless horizontal surface and are connected by a light spring of force constant 5 N/m. m_1 is in contact with a rigid wall. m_2 is pushed through a distance of 4 cm towards m_1 and then released. The velocity of the centre of mass of the system when m_1 breaks off the wall is**

(a) 2/3 cm/s (b) 4/3 cm/s

(c) 2 cm/s (d) 4 cm/s

ANSWERS

1.	d	6.	c
2.	b	7.	c
3.	b	8.	a
4.	a	9.	c
5.	d	10.	b