C1A TYPES OF MOTION

Motion of an object can be of three kinds :

(a) Translational Motion

(b) Rotational Motion

(c) Rolling Motion

C1B ROTATIONAL MOTION

Here we examine the rotation of a rigid body (a body with a definite and unchanging shape and size) about a fixed axis (an axis that does not move), called the axis of rotation or the rotational axis. Every point of the body moves in a circle whose centre lies on the axis of rotation, and every point moves through the same angle during a particular time. In pure translation, every point of the body moves through the same linear distance during a particular time interval in a straight line. Hence we can see the angular equivalent of the linear quantities position, displacement, velocity and acceleration.

Angular displacement : $\Delta\theta = \theta_2 - \theta_1$.

Angular velocity : Average angular velocity, $\langle\omega\rangle = \dfrac{\theta_2 - \theta_1}{t_2 - t_1} = \dfrac{\Delta\theta}{\Delta t}$.

Instant angular velocity, $\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$.

Both $\langle\omega\rangle$ and ω are vectors, with the direction given by the right hand rule. The magnitude of the body's angular velocity is the angular speed.

Angular acceleration : Average angular acceleration, $\langle\alpha\rangle = \dfrac{\omega_2 - \omega_1}{t_2 - t_1} = \dfrac{\Delta\omega}{\Delta t}$.

Instant angular acceleration, $\alpha = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt} = \omega\dfrac{d\omega}{d\theta}$.

Both $\langle\alpha\rangle$ and α are vectors.

Practice Problems :

1. The angular position of a reference line on a spinning wheel is given by $\theta = t^3 - 27t + 4$, where t is in seconds and θ is in radians. Find angular speed and angular acceleration.
2. A child's top is spun with angular acceleration $\alpha = 5t^3 - 4t$ where the coefficients are in units compatible with seconds and radians. At t = 0, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad. Find angular velocity and angular position of the top.

C1C ROTATIONAL MOTION WITH CONSTANT ANGULAR ACCELERATION

The kinematics equations for constant angular acceleration

$\omega = \omega_0 + \alpha(t - t_0)$

$\theta = \theta_0 + \omega_0(t - t_0) + (1/2)\alpha\,(t - t_0)2$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$. Here the symbols have the following meaning :

$\theta_0 \rightarrow$ Angular positon at t_0

$\theta \rightarrow$ Angular position at t

$\omega_0 \rightarrow$ Angular velocity at t_0

$\omega \rightarrow$ Angular velocity at t

$\alpha \rightarrow$ Angular acceleration.

Practice Problems :

1. A wheel is making revolutions about its axis with uniform angular acceleration. Starting from rest, it reaches 100 rev/sec in 4 seconds. Find the angular acceleration. Find the angle rotated during these four seconds.
2. A wheel rotating with uniform angular acceleration covers 50 revolutions in the first five seconds after the start. Find the angular acceleration and the angular velocity at the end of five seconds.
3. A wheel starting from rest is uniformly accelerated at 4 rad/s^2 for 10 seconds. It is allowed to rotate uniformly for the next 10 seconds and is finally brought to rest in the next 10 seconds. Find the total angle rotated by the wheel.
4. A body rotates about a fixed axis with an angular acceleration of one $radian/second^2$. Through what angle does it rotate during the time in which its angular velocity increases from 5 rad/s to 15 rad/s.
5. Find the angular velocity of a body rotating with an acceleration of 2 rev/s^2 as it completes the 5th revolution after the start.
6. The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 seconds. (i) What is its angular acceleration, assuming the acceleration to be uniform ? (ii) How many revolutions does the engine make during this time ?

[Answers : (1) 25 rev/s^2, 400 π rad (2) 4 rev/s^2, 20 rev/s (3) 800 rad (4) 100 rad (5) $2\sqrt{5}$ rev/s (6) (i) 4π rad/s^2 (ii) 576]

C1D RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

A point in a rigid rotating body at a perpendicular distance r from the rotation axis moves in a circle with radius r. If the body rotates through an angle θ, the point moves along the arc with length s is given by $s = \theta r$ where θ is in radians.

The linear velocity $\vec{v}$ of the point is tangent to the circle and the point's linear speed v is given by $v = \omega r$, where ω is the angular speed of the body.

The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The tangential component is $a_t = \alpha r$, where α is the magnitude of the angular acceleration of the body. The tangential acceleration $\left(a_t = \frac{dv}{dt}\right)$ represents only the part of linear acceleration that is responsible for change in the magnitude of the linear velocity $\vec{v}$. Like $\vec{v}$, that part of the linear acceleration is tangent to the path of the point in question.

The radial component, responsible to change the direction of $\vec{v}$, is $a_r = \frac{v^2}{r} = \omega^2 r$. This component is directed radially inward.

For a body having constant velocity, $a_t = 0$ and $a_r = 0$. For a body having constant angular speed or linear speed, $a_t = 0$ and $a_r = \frac{v^2}{r}$. Body having variable angular speed or linear speed, $a_t = \frac{dv}{dt}$ and $a_r = \frac{v^2}{r}$.

Remember the following vector relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$\vec{a} = \vec{a}_t + \vec{a}_r$$
$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

Practice Problems :

1. **A disc of radius 10 cm is rotating about its axis at an angular speed of 20 rad/s. Find the linear speed of**
   - **(a) a point on the rim,**
   - **(b) the middle point of a radius.**
2. **A disc rotates about its axis with a constant angular acceleration of 4 rad/s^2. Find the radial and tangential acceleration of a particle at a distance of 1 cm from the axis at the end of the first second after the disc starts rotating.**

[Answers : (1) 2 m/s, 1 m/s (2) 16 cm/s^2, 4 cm/s^2]

C2A ROTATIONAL KINETIC ENERGY AND ROTATIONAL INERTIA

Consider a body (rotating with angular velocity ω) as being made up a large number of particles with masses m_1, m_2,....at distances r_1, r_2,....from the axis of rotation. The total kinetic energy of the body is given by

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + ...$$

$$= \sum_{i=1} \frac{1}{2} m_i v_i^2 = \sum_{i=1} \frac{1}{2} m_i (\omega r_i)^2$$

$$= \sum_{i=1} \frac{1}{2} m_i r_i^2 \omega^2 = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

The quantity in parantheses on the right side of the above equation tells us how the mass of the rotating body is distributed about its axis of rotation. We call this quantity the rotational inertia (or moment of inertia) I of the body with respect to the axis of rotation. In terms of moment of inertia I, the rotational kinetic energy K of a rigid body is $K = \frac{1}{2} I\omega^2$.

Remember the following :

Moment of inertia of point mass $I = mr^2$

Moment of inertia of a system of discret particles $I = \sum_{i=1} m_i r_i^2$.

Moment of inertia of a system for constinuously mass distribution $I = \int r^2 dm$

Here r and r_i in these expressions represents the perpendicular distance from the axis of rotation of each mass element in the body.

This quantity has the same significance in rotational motion as that of mass in linear motion. It is a measure of the resistance offered by a body to a change in its rotational motion.

C2B THEOREM OF MOMENT OF INERTIA :

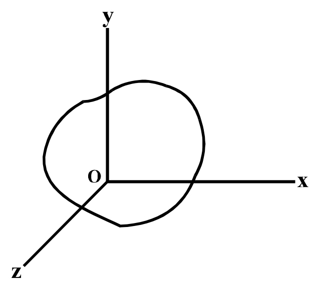
1. **The parallel axis theorem**

   The parallel axis theorem relates the moment of inertia I of a body about any axis to that of the same body about a parallel axis through the centre of mass (also known as centroid axis) as $I = I_{com} + M h^2$ where I_{com} is the moment of inertia of the body about the centroidal axis and h is the perpendicular distance between the two axes. Note that the parallel axis may lie within or inside the body.

2. **The perpendicular axis theorem**

   This theorem is valid for a planar or laminar body (body in two dimensions like a thin disc, ring or thin plate etc.).

   Let x and y be the two axes which lie in the plane of the body and pass through the point O, as shown in figure.

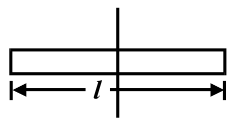


Then the moment of inertia about an axis (called z-axis) passing through O and perpendicular to the plane containing x and y axes is given by $I_z = I_x + I_y$, where I_x, I_y and I_z are the respective moment of inertia of the body about x, y and z axes.

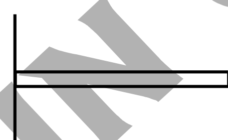
C2C MOMENT OF INERTIA OF IMPOTANT BODIES :

1. A thin rod

(a) About an axis through centre of mass perpendicular to length, $I = \frac{1}{12}Ml^2$



(b) About an axis through one of the end and perpendicular to length, $I = \frac{1}{3}ML^2$



2. A ring or Hoop

(a) About an central axis and perpendicular to the plane $I = MR^2$

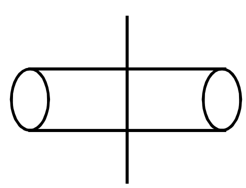
(b) About any diameter $I = \frac{MR^2}{2}$

(c) About a tangent in the plane of ring $I = \frac{3}{2}MR^2$

(d) About a tangent perpendicular to the plane of ring $I = 2\ MR^2$

3. Hollow cylinder

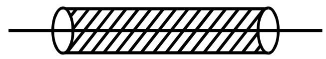
About central axis $I = MR^2$



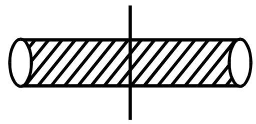
$$I = \frac{ML^2}{12} + \frac{MR^2}{2}$$

4. **Solid cylinder**

(a) **About central axis or axis of symmetry** $I = \frac{MR^2}{2}$



(b) **About central diameter** $I = \frac{ML^2}{12} + \frac{MR^2}{4}$



5. **Disc**

(a) **About an axis through the centre and perpendicular to the plane** $I = \frac{MR^2}{2}$.

(b) **About the diameter** $I = \frac{MR^2}{4}$

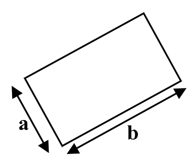
(c) **About the tangent perpendicular to the plane of disc** $I = \frac{3}{2}MR^2$

(d) **About the tangent in the plane of disc** $I = \frac{5}{4}MR^2$

6. **A rectangular plate**

About perpendicular axis through centre

$I = \frac{1}{12}M(a^2 + b^2)$



7. **Solid sphere**

(a) **About its diameter** $I = \frac{2}{5}MR^2$

(b) **About its tangent** $I = \frac{7}{5}MR^2$

8. **Hollow sphere**

(a) **About its diameter** $I = \frac{2}{3}MR^2$

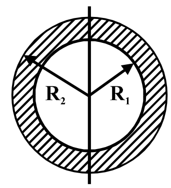
(b) **About its tangent** $I = \frac{5}{3}MR^2$

9. **Annular cylinder (or ring)**

About central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$, **here** R_2 **is the outer radius and** R_1 **is the inner radius.**

10. A spherical shell

About the diameter $I = \frac{2}{5} M \frac{(R_2^5 - R_1^5)}{(R_2^3 - R_1^3)}$.



<u>Practice Problems :</u>

1. The moment of inertia of a body about an axis is 1.2 kg-m². Initially the body is at rest. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad/s² must be applied about the axis for a duration of

(a) 2 s (b) 4 s (c) 8 s (d) 10 s

2. Which of the following has the highest moment of inertia if each has the same mass and the same radius ?

(a) A ring about its axis perpendicular to the plane of the ring

(b) A solid sphere about one of its diameters

(c) A spherical shell about one of its diameters

(d) A disc about its axis perpendicular to the plane of its disc.

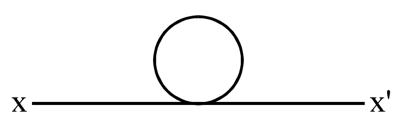
3. The moment of inertia of a uniform circular disc about a diameter is *I*. Its momentum of inertia about an axis perpendicular to its plane and passing through a point on its rim is

(a) 3 *I* (b) 4 *I* (c) 5 *I* (d) 6 *I*

4. The radius of gyration of a rod of mass m and length L about an axis of rotation perpendicular to its length and passing through the center is

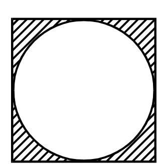
(a) $\frac{L}{2\sqrt{3}}$ (b) $\frac{L}{2\sqrt{2}}$ (c) $\frac{L}{2\sqrt{5}}$ (d) $\frac{L}{\sqrt{2}}$

5. A wire of mass per unit length λ and length L is used to form a circular loop. The moment of inertia about the xx' is



(a) $\frac{2\lambda L^3}{8\pi^2}$ (b) $\frac{4\lambda L^3}{8\pi^2}$ (c) $\frac{3\lambda L^3}{8\pi^2}$ (d) $\frac{5\lambda L^3}{8\pi^2}$

6. From a uniform square plate of mass M and length L, a circular plate is removed and the remaining part is shown in figure

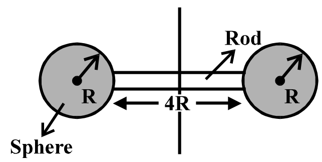


The moment of inertia of the remaining part passing through the centre and perpendicular to the plane is

(a) $ML^2\left(\frac{1}{6}-\frac{\pi}{32}\right)$ (b) $ML^2\left(\frac{1}{3}-\frac{\pi}{32}\right)$

(c) $ML^2\left(\frac{1}{2}-\frac{\pi}{16}\right)$ (d) $ML^2\left(\frac{1}{2}-\frac{\pi}{64}\right)$

7. **The mass of each sphere are M and the mass of the rod is M. The moment of inertia of the given figure about the axis shown in figure is**



(a) $\frac{301MR^2}{15}$ (b) $\frac{302MR^2}{15}$ (c) $\frac{303MR^2}{15}$ (d) $\frac{304MR^2}{15}$

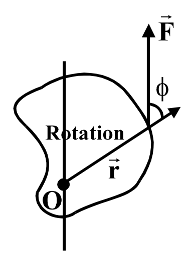
8. Two loops P and Q are made from a uniform wire. The radii of P and Q are r_1 and r_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $I_2/I_1 = 4$ then $\frac{r_2}{r_1}$ equals

(a) $4^{2/3}$ (b) $4^{1/3}$ (c) $4^{-2/3}$ (d) $4^{-1/3}$

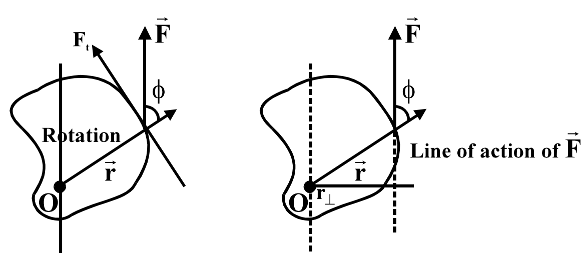
[Answers : (1) a (2) a (3) d (4) a (5) c (6) a (7) b (8) b]

C3 MOMENT OF FORCE OR TORQUE

Torque is a turning or twisting action on a body about a rotation axis due to a force $\vec{F}$. It has the same role in rotational motion as that of force in linear motion. Consider a force $\vec{F}$ is exerted at a point given by the position vector $\vec{r}$ relative to the axis, as shown in figure. Its torque about O is given by $\vec{\tau} = \vec{r} \times \vec{F}$.



Note that torque is always defined with reference to a specific point, often (but not always) the origin of a co-ordinate system.



The magnitude of the torque is $\tau = rF_t = r_\perp F = rF\sin\phi$ where F_t is the component of $\vec{F}$ perpendicular to $\vec{r}$ and ϕ is the angle between $\vec{r}$ and $\vec{F}$.

The quantity $r_\perp$ is the perpendicular distance between the rotation axis and extended line runnning through the $\vec{F}$ vector. This line is called the line of action of $\vec{F}$ and $r_\perp$ is called the moment arm of $\vec{F}$. Similarly r is the moment arm of F_t.

Newton's Second Law for Rotation

The rotational analog of newton's second law is $\tau_{net} = I\alpha$

where τ_{net} is the net torque acting on a particle or rigid body, I is the rotational inertia of the particle or body about the rotation axis, and α is the resulting angular acceleration about that axis. Here τ_{net} and I are taken with respect to the same rotation axis.

Note the following points for $\tau_{net} = I\,\alpha$

1. This equation is valid only for rigid bodies.
2. Here we consider the torque of the external forces.
3. The axis of rotation should be stationary. But, infact, this equation is valid even when the axis of rotation moves if the following two conditions are met :
   (a) The moving axis of rotation must be an axis of symmetry.
   (b) The axis must not change direction.

Equilibrium : A rigid body is said to be in equilibrium if

(a) Net external force equal to zero. This is the condition of transiational equilibrium $\sum \vec{F} = 0$.

(b) Net external torque equal to zero. This is the condition of rotational equilibrium $\sum \vec{\tau} = 0$.

<u>Practice Problems :</u>

1. A wheel of radius 10 cm and mass 12.5 kg rotates freely about an axis passing through the center and perpendicular to the plane of the wheel by applying a constant force F and it is found that its angular speed increases from zero to 2 rad/s in 1s. The force F acting on the wheel to do so

   (a) 1.25 N (b) 2.5 N (c) 4.5 N (d) 6.25 N

2. A cubical block of mass m and edge length *l* slides down the rough inclined plane of inclination α with a uniform velocity. (a) Draw the force body diagram of cubical block showin all the forces and its point of application. (b) What is the torque of the normal force acting on the block about its centre.
3. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis through its centre. A horizontal force of variable magnitude F equal to αt acts on the rod at a distance of L/4 from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.
4. A square plate of mass m and edge length *l* rotates about one of the diagonal of plate with uniform angular acceleration α by an external agent. What is torque applied by the external agent ?
5. A flywheel of moment of inertia 5.0 kg-m^2 is rotated at a speed of 60 rad/s. Because of the friction at the axle, it comes to rest in 5.0 minutes., Find (a), the average torque of the friction, (b) the total work done by the friction and (c) the angular momentum of the wheel 1 minute before it stops rotating.
6. A light rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from the ends. Find the initial angular acceleration of the rod assuming that it was horizontal in the beginning.
7. Two blocks of masses m and M connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley has a radius r and moment of inertia I about its axis and it can freely rotate about this axis. Find the acceleration of the mass M assuming that the string does not slip on the pulley.
8. A uniform metre stick of mass 200 g is suspended from the ceiling through two vertical strings of equal lengths fixed at the ends. A small object of mass 20 g is placed on the stick at a distance of 70 cm from the left end. Find the tensions in the two strings.

9. A uniform ladder of length 10.0 m and mass 16.0 kg is resting against a vertical wall making an angle of 37^0 with it. The vertical wall is frictionless but the ground is rough. An electricial weighing 60.0 kg climbs up the ladder. If he stays on the ladder at a point 8.00 m from the lower end, what will be the normal force and the froce of friction on the ladder by the ground ? What should be the minimum coefficient of friction for the electricial to work safely ?

10. Suppose the friction coefficient between the ground and the ladder of the previous problem is 0.540. Find the maximum weight of a mechanic who could go up and do the work from the same position of the ladder.

11. A 6.5 m long ladder rests against a vertical wall reaching a height of 6.0 m. A 60 kg man stands half way up the ladder. (a) Find the torque of the force exerted by the man on the ladder about the upper end of the ladder. (b) Assuming the weight of the ladder to be negligible as compared to the man and assuming the wall to be smooth, find the force exerted by the ground on the ladder.

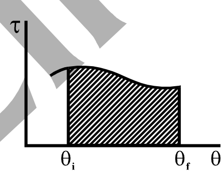
12. The door of an almirah is 6 ft high, 1.5 ft wide and weighs 8 kg. The door is supported by two hinges situated at a distance of 1 ft from the ends. If the magnitudes of the forces exerted by the hinges on the door are equal, find this magnitude.

[Answers : (1) a (2) ½mg$l$ sinα (5) (a) 1.0 N-m (b) 9.0 kJ (c) 60 kg-m²/s (6) 8.4 rad/s² (7) $\frac{Mg}{M + m + I/r^2}$ (8) 1.04 N in the left string and 1.12 N in the right (9) 745 N, 412 N, 0.553 (10) 44.0 kg (11) (a) 740 N-m (b) 590 N vertical and 120 N horizontal (12) 43 N]

C4 WORK IN ROTATIONAL MOTION

When a torque τ acts on a rigid body that undergoes an angular displacement from θ_i to θ_f then work W done by the torque is $W = \int_{\theta_i}^{\theta_f} \tau d\theta$. If the torque is constant, then $W = \tau(\theta_f - \theta_i) = \tau\Delta\theta$.

Graphical interpreration of rotational work done is shown in figure.



Work - Energy Theorem for Rotational Motion

Work energy theorem for rotational motion of a rigid body is $W = \Delta K = K_f - K_i = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$.

Practice Problems :

1. A uniform cylinder of radius R and mass M is spinned about its axis to the angular velocity ω_0 and then placed into a corner. The coefficient of friction between the walls and the cylinder is equal to μ. The total work done is

(a) $-\frac{1}{2}MR^2\omega_0^2$ (b) $\frac{1}{2}MR^2\omega_0^2$ (c) $\frac{1}{4}MR^2\omega_0^2$ (d) $-\frac{1}{4}MR^2\omega_0^2$

[Answers : (1) d]

C5 POWER IN ROTATIONAL MOTION

When the body rotates with angular velocity ω, the power P (rate at which the torque does work) is

$$P = \frac{dW}{dt} = \tau\omega .$$

Practice Problems :

1. An electric motor exerts a constant torque of τ = 10 N-m on a gridstone mounted on it shaft. The moment of inertia of the grindstone is 2 kg-m^2. If the system starts from rest, the kinetic energy at the end of 8s is

(a) 400 J (b) 800 J (c) 1600 J (d) 2000 J

2. In the above problem, the instant power at t = 8s delivered by the motor is

(a) 100 W (b) 200 W (c) 400 W (d) 800 W

[Answers : (1) c (2) c]

C6 ANGULAR MOMENTUM

1. **Angular momentum of a particle**

The angular momentum $\vec{L}$ of a particle, with linear momentum $\vec{p}$, mass m and linear velocity $\vec{v}$ is a vector quantity defined relative to a fixed point (usually an origin). It is $\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$.

The magnitude of $\vec{L}$ is given by

$|\vec{L}| = mvr\sin\phi = rp_\perp = rmv_\perp = r_\perp p = r_\perp mv$.

where ϕ is the angle between $\vec{r}$ and $\vec{p}$, $p_\perp$ and $v_\perp$ are the components of $\vec{p}$ and $\vec{v}$ respectively, perpendicular to $\vec{r}$ and $r_\perp$ is the perpendicular distance between the fixed point and the line of extension of $\vec{p}$. The direction of $\vec{L}$ is given by the right hand rule for cross products.

2. **Angular momentum of a system of particles**

The angular momentum $\vec{L}$ of a system of particles is the vector sum of the angular momentum of individual particles, $\vec{L} = \sum_{i=1}^{n} \vec{L}_i$.

3. **Angular Momentum of a Rigid Body**

When a symmetric rigid body with moment of inertia I rotates with angular velocity $\vec{\omega}$ about a stationary axis of symmetry, its angular momentum is given by $\vec{L} = I\vec{\omega}$. If the body is not symmetric or the rotation axis is not an axis of symmetry, the component of angular momentum along the axis of rotation is equal to Iω.

Practice Problems :

1. A particle of mass m is projected with speed u at an angle of θ with the horizontal in the vertical plane. Find the angular momentum of the particle about the point of projection (a) when the particle at at the highest point (b) just before the landing.

2. Two spheres each of radius r and mass m are connected by a rod of mass m and length 2r. The system is rotated with angular speed ω about an axis passing through the mid-point of the rod and perpendicular to the length of the rod. Find the angular momentum about this axis of this system.

3. A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin

(a) is zero (b) remains constant

(c) goes on increasing (d) goes on decreasing

4. When a mass is rotated in a plane about a fixed point, its angular momentum is directed along

(a) the radius

(b) the tangent to the orbit

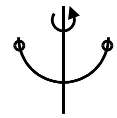
(c) a line at an angle of 45^0 to the plane of rotation

(d) the axis of rotation.

5. If the radius of earth contarcts to half of its present day value, the mass remaining unchanged, the duration of the day will be

(a) 48 hrs (b) 24 hrs (c) 12 hrs (d) 6 hrs

6. Two beads (each of mass m) can move freely in a frictionless semicircular wire of mass m and radius is r. The angular velocity ω_0 when the beads are at distance r from the axis as shown in figure. The angular velocity of the system when the beads are at a distance r/2 from the axis is



(a) $\frac{5}{4}\omega_0$ (b) $\frac{3}{2}\omega_0$ (c) $\frac{5}{2}\omega_0$ (d) $2\,\omega_0$

[Answers : (3) b (4) d (5) d (6) c]

C7 RELATION BETWEEN TORQUE AND ANGULAR MOMENTUM

The rate of change of angular momentum of a rigid body equals the net torque acting on it i.e., $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$.

Practice Problems :

1. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 seconds. The magnitude of this torque is

(a) $\frac{3A_0}{4}$ (b) A_0 (c) $4\,A_0$ (d) $12\,A_0$

[Answers : (1) a]

C8 CONSERVATION OF ANGULAR MOMENTUM

The angular momentum $\vec{L}$ of a system remains constant if the net external torque acting on the system is zero i.e. $\vec{\tau} = \frac{d\vec{L}}{dt} = 0 = \vec{L} = \text{constant}$.

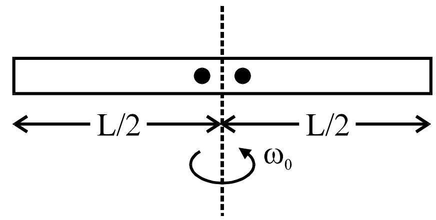
This is a law of conservation of angular momentum. It is one of the fundamental conservation laws of nature, having been verified even in situation (involving high speed particles or subatomic dimension) in which newton's laws are not applicable.

Practice Problems :

1. A thin circular ring of mass M is rotating about its axis with a constant angular velocity ω. Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity.

(a) $\frac{\omega M}{M+m}$ (b) $\frac{\omega(M-2m)}{M+2m}$ (c) $\frac{\omega M}{M+2m}$ (d) $\frac{\omega(M+2m)}{M}$

2. A smooth uniform rod of length L and mass M has two identical beads of negligible size, each of mass m, which can slide freely along the rod. Initially the two beads are at the centre of the rod and the system is rotating with an angular velocity ω_0 about an axis perpendicular to the rod and passing through the mid-point of the rod. There are no external forces. When the beads reach the ends of the rod, the angular velocity of the system is



(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+4m}$ (c) $\frac{M\omega_0}{M+5m}$ (d) $\frac{M\omega_0}{M+6m}$

[Answers : (1) c (2) d]

C9 ANGULAR IMPULSE

The angular impulse of a torque τ in a time interval dt is defined as $d\vec{J} = \vec{\tau}dt \Rightarrow \Delta\vec{J} = \int_{t_1}^{t_2} \vec{\tau}dt$.

$\therefore \quad \vec{\tau} = \frac{d\vec{L}}{dt}$

$\therefore \quad \Delta\vec{J} = \Delta\vec{L} = \vec{\tau}\Delta t$

Thus, the change in angular momentum is equal to the angular impulse of the resultant torque. This theorem is known as angular impulse - momentum theorem.

Practice Problems :

1. A uniform rod of mass M and length L is placed on a frictionless horizontal surface with one of the end of the rod is tapped. A sharp linear impulse J, perpendicular to length of the rod, is provided at the mid-point of the rod. The time taken by the rod to complete one revolution is

(a) $\frac{4ML\pi}{3J}$ (b) $\frac{\pi ML}{12J}$ (c) $\frac{2ML\pi}{3J}$ (d) $\frac{\pi ML}{6J}$

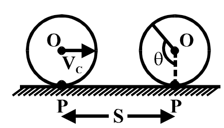
[Answers : (1) a]

C10 ROLLING MOTION

Rigid body rotation about a moving axis i.e., the motion of the body as combined translation and rotation is defined as rolling motion. The key to understanding such situation is this : Every possible motion of a rigid body can be represented as a combination of translational motion of the centre of mass and rotation about an axis through the centre of mass. Note that the axis passing through the centre of mass is a symmetry axis that does not change direction as the body moves.

Rolling without slipping (Pure Rolling)

In case of pure rolling the point P at which the body makes contact with the surface over which the whell rolls move same distance as the centre of mass O of the rolling body during the same time i.e. s = θ R



where R is the radius of rolling body.

Differentiating the above equation w.r.t. time, we get $v_c = R\omega$.

Further differentiation w.r.t. to time gives $a_c = R\alpha$.

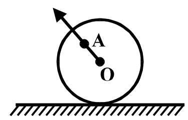
Note that $v_c = \omega R$ is not the condition of pure rolling. Condition of pure rolling; The instantaneous velocity of the point of contact.

C11 <u>DISPLACEMENT, VELOCITY AND ACCELERATION OF A POINT ON THE ROLING BODY</u>

Consider a point A on the rolling body. Here O is centre of mass of the body.

Displacement of A

$\Delta\vec{r}_A = \Delta\vec{r}_0 + \Delta\vec{r}_{AO}$, **here $\Delta\vec{r}_{AO}$ is the displacement of point A w.r.t. O.**



Velocity of A

$\vec{v}_A = \vec{v}_0 + \vec{v}_{AO}$, **here $\vec{v}_{AO}$ is the velocity of point A w.r.t. O which equals to $\vec{v}_{AO} = \vec{\omega}\times\vec{r} = \vec{\omega}\times\overrightarrow{OA}$ where $\vec{\omega}$ is the angular velocity vector normal to the plane of the motion in the sense determined by the right hand rule.**

Acceleration of A

$\vec{a}_A = \vec{a}_0 + \vec{a}_{AO}$, **here $\vec{a}_{AO}$, acceleration of A w.r.t. O, has two components $\vec{a}_{AO} = (\vec{a}_{AO})_n + (\vec{a}_{AO})_t$.**

Here $(\vec{a}_{AO})_n$ = normal component of $\vec{a}_{AO} = \vec{\omega}\times(\vec{\omega}\times\vec{r}) = \vec{\omega}\times(\vec{\omega}\times\overrightarrow{OA}) = \vec{\omega}\times\vec{v}_{AO}$.

The magnitude of $(\vec{a}_{AO})_n$

$$|(\vec{a}_{AO})_n| = \omega^2 r = \frac{v^0_{AO}}{r}$$

And $(\vec{a}_{AO})$ = tangential component of $\vec{a}_{AO} = \vec{\alpha}\times\vec{r} = \vec{\alpha}\times\overrightarrow{OA}$

Here $\vec{\alpha}$ is the angular acceleration of the body.

C12 <u>ROLLING AS PURE ROTATION</u>

For a body in pure rolling, the point of contact P has the zero instantaneous velocity. Hence an axis passing through the point P and perpendicular to the plane of body is an axis of rotation or the rotation axis and about point P we can consider the pure rotation of the pure rolling body. This point P is called the instantaneous centre of rotation or instantaneous centre of zero velocity. This point may lie within or outside the body. Note that the angular velocity of the body about this point is the same as that about its centre of mass.

C13 <u>THE DYNAMICS OF ROLLING MOTION</u>

(a) For translation of centre of mass $\sum\vec{F}_{ext} = M\vec{a}_{CM}$

For rotation about the centre of mass $\sum\tau = I_{CM}\alpha$

(b) The work energy theorem for rolling

$$W_T + W_R = \Delta K = \frac{1}{2}MV^2_{2CM} + \frac{1}{2}I_{cm}\omega_2^2 - \frac{1}{2}MV^2_{1CM} - \frac{1}{2}I_{cm}\omega_2^2$$

C14 <u>ANGULAR MOMENTUM FOR ROLLING BODY</u>

$$\vec{L} = \vec{L}_{CM} + M\vec{r}\times\vec{v}_{CM}$$

The first term $\vec{L}_{CM}$ represents the angular momentum of the body about the centre of mass frame.

The second term $M\vec{r}\times\vec{v}_{CM}$ equals the angular momentum of the body about the particular point if the body is assumed to be concentrated at the centre of mass translating with the velocity $\vec{v}_{cm}$.

Practice Problems :

1. **A solid cylinder of mass M and radius R rolls down an inclined plane from height h without slipping. The speed of its centre of mass when it reaches the bottom is**

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{4}{3}gh}$ (c) $\sqrt{\frac{3}{4}gh}$ (d) $\sqrt{\frac{4g}{h}}$

2. **A thin, uniform, circular disc is rolling down an inclined plane of inclination 30^0 without slipping. Its linear acceleration along the plane is**

(a) g/4 (b) g/3 (c) g/2 (d) 2g/3

3. **A solid sphere, a hollow sphere and a solid cylinder, all of the same radius, roll down an inclined plane from the same height, starting from rest. Which of them takes the least time in reaching the bottom of the plane ?**

(a) Solid sphere (b) Hollow sphere

(c) Solid cylinder (d) All will take the same time

4. **A ring is rolling without slipping on a horizontal surface. The velocity of centre of mass of the ring is v. The fraction of rotational kinetic energy of the total kinetic energy is**

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

5. **Find the angular momentum of the rolling disc having centre of mass speed v about any point on the horizontal surface ? Also find the kinetic energy of this body. What fraction of kinetic energy associated with the rotational part ? Do this problem for any rigid body which can roll on the horizontal surface. This body has the moment of inertia I.**

6. **A rigid body is rolling with slipping along the horizontal frictional surface. Which of the following quantity is conserved ?**

(a) Linear momentum

(b) Angular momentum about the centre of mass

(c) Angular momentum about any point on the surface

(d) Kinetic energy

7. **A sphere is rolling without slipping on a rough horizotnal surface. It collides with a smooth vertical wall. Which of the following statement is correct ?**

(a) After the collision the sphere doesnot roll with slipping.

(b) After the collision the sphere has same angular speed before the collision.

(c) The linear velocity of the sphere after the collision doesnot change

(d) all are correct

[Answers : (1) b (2) b (3) a (4) a (6) c (7) b]

SINGLE CORRECT CHOICE TYPE

1. A cube is resting on an inclined plane. The value of coefficient of friction between cube and plane so that cube topples before sliding

(a) $\mu = 1/2$ (b) $\mu < 1$
(c) $\mu > 1$ (d) $\mu > 1/2$

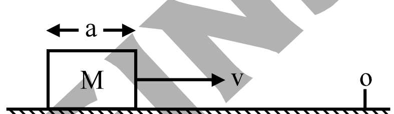
2. What must be the relation between length and radius of a cylinder of given mass and density so that its moment of inertia about the axis through its centre of mass and perpendicular to its length may be minimum ?

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$
(c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{2}{3}}$

3. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts

(a) in the backward direction on the front wheel and in the forward direction on the rear wheel.
(b) in the forward direction on the front wheel and in the backward direction on the rear wheel.
(c) in the backward direction on both the front and the rear wheels.
(d) both (a) and (c) are correct

4. A cubical block of side a is moving with velocity v on a horozintal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is



(a) 3v/(4a) (b) 3v/(2a)
(c) √3v/(√2a) (d) zero

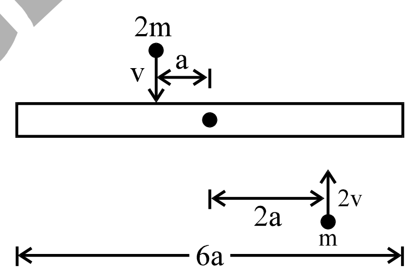
5. A hollow sphere and a solid sphere, having the same mass, are released from rest simultaneously from the top of a smooth inclined plane. Which of the two will reach the bottom first ?

(a) solid sphere
(b) hollow sphere
(c) the one which has the greater density
(d) both will reach the bottom simultaneously

6. A particle of mass m is projected with a velocity v making an angle of 45^0 with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is

(a) zero (b) $\frac{mv^3}{4\sqrt{2}g}$
(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{gh^3}$

7. A uniform bar of length 6a and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane with speeds 2v and v, respectively, strike the bar as shown in figure and stick to it after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and V_c respectively, we have after collision. choose the incorrect option ?



(a) $V_C = 0$ (b) $\omega = \frac{3v}{5a}$
(c) $\omega = \frac{v}{5a}$ (d) $E = \frac{3mv^2}{5}$

8. Four spheres, each of mass M and diameter 2r, are placed with their centres on the four corners of a square of side a (> 2r). The moment of inertia of the system about one side of the square is

(a) $\frac{2}{5}M(5r^2 + 4a^2)$
(b) $\frac{2}{5}M(5r^2 + 2a^2)$
(c) $\frac{2}{5}M(2r^2 + 5a^2)$
(d) $\frac{2}{5}M(4r^2 + 5a^2)$

9. A cord is wound round the circumference of a wheel of radius r. The axis of the wheel is horizontal and its moment of inertia about this axis is I. A weight mg is attached to the end of the cord and allowed to fall from rest. The angular velocity of the wheel, when the weight has fallen through a distance h, is

(a) $\left[\frac{2gh}{I+mr}\right]^{1/2}$ (b) $\left[\frac{2mgh}{I+mr^2}\right]^{1/2}$

(c) $\left[\frac{2mgh}{I+2mr^2}\right]^{1/2}$ (d) $(2gh)^{1/2}$

10. A body of mass M and radius r, rolling on a smooth horizontal floor with velocity v, rolls up an irregular inclined plane up to a vertical height $\frac{3v^2}{4g}$. The body may be

(a) sphere (b) solid cylinder

(c) disc (d) both (b) and (c)

11. Two points masses m_1 and m_2 are joined by a massless rod of length r. The moment of inertia of the system about an axis passing through the center of mass and perpendicular to the rod is

(a) $(m_1+m_2)\frac{r^2}{4}$

(b) $(m_1-m_2)\frac{r^2}{4}$

(c) $\frac{m_1m_2}{m_1+m_2}r^2$

(d) $\frac{m_1m_2}{m_1-m_2}\frac{r^2}{4}$

12. A thin rod of length L and mass M is held vertically with one end on the floor and is allowed to fall. The velocity of the other end when it hits the floor, assuming that the end on the floor does not slip

(a) $\sqrt{3gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{gL}$ (d) $2\sqrt{gL}$

13. Three uniform rods each of mass m and length L, is used to form an equilateral triangle. The moment of inertia of this frame about an axis through the centroid and perpendicular to the plane of triangle is

(a) mL^2 (b) $\frac{mL^2}{2}$

(c) $\frac{mL^2}{3}$ (d) $\frac{mL^2}{4}$

14. Three uniform rods each of mass m and length L, is used to form an equilateral triangle. The moment of inertia of this frame about one of the side

(a) mL^2 (b) $\frac{mL^2}{2}$

(c) $\frac{mL^2}{3}$ (d) $\frac{mL^2}{4}$

15. A disc starts from rest with angular acceleration $(9-12t)$ rad/s^2 in anticlockwise direction where t is the time. The number of revolutions that the disc makes before it starts to move in clockwise direction is

(a) $\frac{3.375}{2\pi}$ (b) $\frac{3.375}{3\pi}$

(c) $\frac{3.375}{4\pi}$ (d) none of these

16. A diver makes 2.5 revolution on the way from a 10 m high platform to the water. Assuming zero initial vertical velocity, the diver's average angular velocity during a dive is

(a) 8 rad/s (b) 9 rad/s

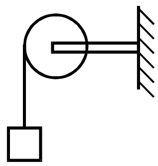
(c) 10 rad/s (d) 11 rad/s

17. A wheel has a constant angular acceleration of 3 rad/s^2. During a certain 4s interval, it turns through an angle of 120 rad. Assuming that the wheel starts from rest, how long is it in motion at the start of this 4s interval ?

(a) 4s (b) 8s

(c) 12s (d) 16s

18. A uniform disk, with mass M and radius R mounted on a fixed horizontal axle. A block with mass M hangs from a massless cord that is wrapped around the rim of the disk.

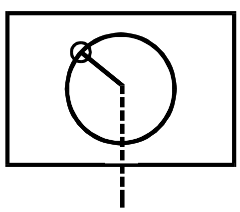


The cord does not slip, and there is no friction at the axle. The acceleration of the falling block is

(a) g (b) g/2

(c) 2/3 g (d) g/3

19. Figure shows a mass m placed on a frictionless horizontal table and attached to a string passing through a small hole in the surface. Initially, the mass moves in a circle of radius r_0 with a speed v_0 and a person holds the free end of the string. The person pulls on the string slowly to decrease the radius of the circle to r. Let the tension in the string when the mass moves depends on radius r as r^n. The value of n is



(a) –1 (b) –2

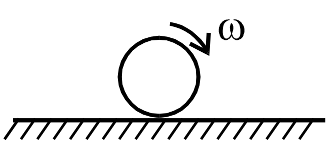
(c) –3 (d) –4

20. A cylinder of mass m and radius R is placed with pure linear speed v_0 on the frictional ground having coefficient of friction μ. The time after which it starts pure rolling is

(a) $\frac{2v_0}{3\mu g}$ (b) $\frac{v_0}{3\mu g}$

(c) $\frac{2v_0}{5\mu g}$ (d) $\frac{5v_0}{7\mu g}$

21. A sphere of mass m and radius R is rolling without slipping with angular speed ω on a horizontal plane as shown.



The angular momentum of the sphere about any point lying on the surface is

(a) $2/5\ mR^2\omega$ (b) $3/5\ mR^2\omega$

(c) $7/5\ mR^2\omega$ (d) $8/5\ mR^2\omega$

22. A cylinder is rolling with slipping on an inclined plane of inclination θ then the coefficient of static friction between plane and cylinder may be

(a) $\frac{1}{3}\tan\theta$ (b) $\frac{1}{2}\tan\theta$

(c) $\frac{1}{4}\tan\theta$ (d) $\frac{2}{3}\tan\theta$

23. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground may not be

(a) $\sqrt{2}u$ (b) $2\sqrt{2}u$

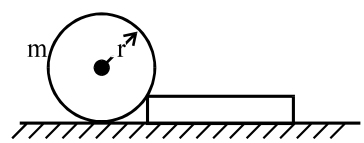
(c) 2u (d) 0

24. A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is held a distance $\frac{3R}{5}$ above the center line. The ball leaves the cue with a speed v_0 and angular speed ω_0. Then the relation between v_0 and ω_0 is

(a) $3v_0 = 2\omega_0 R$ (b) $v_0 = \omega_0 R$

(c) $2v_0 = 3\omega_0 R$ (d) $5v_0 = 3\omega_0 R$

25. A wheel of radius r and mass m stands in contact with step of height h.



The least horizontal force F which should be applied to the axle of the wheel to force it climb onto the step is

(a) $\frac{mg\sqrt{[h(2r-h)]}}{r-h}$

(b) $\frac{mgh(2r-h)}{r-h}$

(c) $\frac{mgh}{r-h}$

(d) None of these

26. **A thin, flat, uniform disk, has mass M and radius R. A circular hole of radius $\frac{R}{4}$, centered at a point $\frac{R}{2}$ from the disk's center, is then punched in the disk. The moment of inertia of the disk with the hole about an axis through the original center of the disk, perpendicular to the plane of the disk.**

(a) $\frac{247}{512}MR^2$ (b) $\frac{212}{513}MR^2$

(c) $\frac{145}{573}MR^2$ (d) $\frac{245}{673}MR^2$

27. **A non uniform rod of length L and the mass per unit length (a + bx), where a and b are constants and x is measured from the left end of the rod. The moment of inertia of the rod about an axis passing through one of the end (left) and perpendicular to the length of the rod is**

(a) $a\frac{L^3}{2}+b\frac{L^4}{4}$ (b) $a\frac{L^3}{3}+b\frac{L^4}{4}$

(c) $a\frac{L^3}{3}+b\frac{L^4}{5}$ (d) $a\frac{L^3}{4}+b\frac{L^4}{5}$

28. **A particle of mass m is projected with speed u at an angle α with the horizontal. The angular momentum of the particle about the point of projection at the instant when the velocity vector will become perpendicular to the initial velocity of projection**

(a) $\frac{mu^3\sin^2\alpha\cos\alpha}{2g}$

(b) $\frac{mu^3\cot\alpha}{2g\sin\alpha}$

(c) $\frac{mu^3\cot^2\alpha}{2g\sin\alpha}$

(d) $\frac{mu^3\cot^2\alpha}{2g\sin^2\alpha}$

29. **A thin rod hangs from a ceiling by means of two inextensible cords. Mass of the rod is M and length 2L. Rod is held at an angle θ with the horizontal. If the string at the right end breaks, the instantaneous angular acceleration of rod is immediately after the string breaks**

(a) $\frac{3g\cos\theta}{L(1+3\cos^2\theta)}$

(b) $\frac{4g\cos\theta}{L(1+3\cos^2\theta)}$

(c) $\frac{5g\cos\theta}{L(1+3\cos^2\theta)}$

(d) $\frac{6g\cos\theta}{L(1+3\cos^2\theta)}$

30. **A thin uniform rod of length L is initially at rest with respect to an inertial frame of reference. The rod is tapped at one end perpendicular to its length. Neglect gravitation effect. The distance does the centre of mass translate while the rod completes one revolution about its centre of mass**

(a) $\frac{\pi L}{3}$ (b) $\frac{2\pi L}{3}$

(c) πL (d) $\frac{4\pi L}{3}$

31. **A uniform disc of radius R is spinned about its axis to an angular velocity ω_0 and then carefully placed on a rough horizontal surface of coefficient of friction μ. The time taken by the disc to stop rotation if it is placed with its plane on the table is**

(a) $\frac{3}{4}\frac{R\omega_0}{\mu g}$ (b) $\frac{1}{4}\frac{R\omega_0}{\mu g}$

(c) $\frac{2}{3}\frac{R\omega_0}{\mu g}$ (d) $\frac{1}{2}\frac{R\omega_0}{\mu g}$

32. **A rigid body with moment of inertia I spins one enery T seconds. The rotation is slowing down, so $\frac{dT}{dt}>0$. The rate of change of kinetic energy is directly proportinal to T^n. The value of n is**

(a) zero (b) –1

(c) –2 (d) –3

33. **Consider a non-uniform rod of total mass M and length L. The mass per unit length of the rod increasing from the left end in the proportion of its length. The moment of inertia of the rod about an axis passing through the leftend and perpendicular to the rod is**

(a) $\frac{ML^2}{3}$ (b) $\frac{ML^2}{4}$

(c) $\frac{ML^2}{2}$ (d) $\frac{ML^2}{5}$

34. In the above problem the moment of inertia of the rod about an axis passing through the centre of mass of the rod and perpendicular to the legnth of the rod is

(a) $\frac{5}{25}ML^2$ (b) $\frac{3}{12}ML^2$

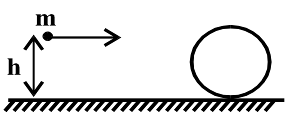
(c) $\frac{5}{14}ML^2$ (d) $\frac{5}{18}ML^2$

EXCERCISE BASED ON NEW PATTERN

COMPREHENSION TYPE

Comprehension-1

A hollow sphere shown in figure lies on a rough plane and a particle of mass m travelling at a speed v_0 collides and sticks to it. Assume that the mass M of the sphre is large compared to the mass of the particle so that the center of mass of the combined system is not appreciably shifted from the centre of the sphere. The line of motion of the particle is at a distance h above the plane.



1. The linear speed of the combined system just after the collision is

(a) $\frac{mv_0}{M+m}$ (b) $\frac{mv_0}{M}$

(c) $\frac{Mv_0}{M+m}$ (d) v_0

2. The angular speed of the system about the center of the sphere just after the collision is

(a) $\frac{mv_0h}{\left(\frac{2}{3}M+m\right)R^2}$ (b) $\frac{mv_0(h-R)}{\left(\frac{2}{3}M+m\right)R^2}$

(c) $\frac{mv_0(h+R)}{\left(\frac{2}{3}M+m\right)R^2}$ (d) $\frac{mv_0(h+R)}{\left(\frac{8}{5}M+m\right)R^2}$

3. The value of h for which the sphere starts pure rolling on the plane immediately after the collision is

(a) $\frac{5}{4}R$ (b) R

(c) $\frac{7}{5}R$ (d) $\frac{5}{3}R$

4. If h = R then immediately after the collision

(a) sphere will have pure translatory motion

(b) sphere will have pure rotational motion

(c) sphere will have rolling with slipping

(d) sphere will have pure rolling motion

5. If h is greater than $\frac{5}{3}R$ then immediately after the collision

(a) sphere will have pure translatory motion

(b) sphere will have pure rotational motion

(c) sphere will have rolling with slipping

(d) sphere will have pure rolling motion

6. If R < h < $\frac{5}{3}R$ then immediately after the collision

(a) the friction force is zero

(b) the friction force is in forward direction

(c) the friction force is in backward direction

(d) cannot be decided as the data is insufficient

7. If $\frac{5}{3}R$ < h < 2R then immediately after the collision

(a) the friction force is zero

(b) the friction force is in forward direction

(c) the friction force is in backward direction

(d) cannot be decided as the data is insufficient

8. If h = $\frac{5}{3}R$ then immediately after the collision

(a) the friction force is zero

(b) the friction force is in forward direction

(c) the friction force is in backward direction

(d) cannot be decided as the data is insufficient

Comprehension-2

A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the point of the rod with a velocity v_0 in a direction perpendicular to AB at the distance d from the centre of the rod. The collision is completely elastic. After the collision the particle comes to rest.

9. The value of $\frac{m}{M}$ is

(a) $\frac{L^2}{L^2+6d^2}$ (b) $\frac{L^2}{L^2+8d^2}$

(c) $\frac{L^2}{L^2+10d^2}$ (d) $\frac{L^2}{L^2+12d^2}$

10. A point P on the rod is at rest immediately after the collision. The distance of this point from the mid point of the rod is

(a) $\frac{L^2}{6d}$ (b) $\frac{L^2}{8d}$

(c) $\frac{L^2}{10d}$ (d) $\frac{L^2}{12d}$

11. The value of d must be such that immediately after the collision one of the point in the rod is at rest

(a) $\geq \frac{L}{6}$ (b) $\geq \frac{L}{8}$

(c) $\geq \frac{L}{10}$ (d) $\geq \frac{L}{10}$

for the following problems consider $d = \frac{L}{2}$

12. If $d = \frac{L}{2}$ then the linear speed of the point, which is stationary immediately after the collision, at the time $\frac{\pi L}{3v_0}$ is

(a) $\frac{v_0}{2\sqrt{2}}$ (b) v_0

(c) $\frac{v_0}{2}$ (d) $\sqrt{2}v_0$

13. The distance travelled by the rod during the above time is

(a) $\frac{\pi L}{12}$ (b) $\frac{\pi L}{10}$

(c) $\frac{\pi L}{8}$ (d) $\frac{\pi L}{6}$

14. The time after which the rod will strike the particle is

(a) $\frac{\pi L}{3v_0}$ (b) $\frac{2\pi L}{3v_0}$

(c) $\frac{\pi L}{v_0}$ (d) $\frac{4\pi L}{3v_0}$

15. The kinetic energy of the rod immediately after the collision

(a) $\frac{1}{2}Mv_0^2$ (b) $\frac{19}{32}Mv_0^2$

(c) $\frac{1}{8}Mv_0^2$ (d) none

16. The linear impulse suffered by the rod immediately after the collision is

(a) Mv_0 (b) $\frac{Mv_0}{2}$

(c) $\frac{Mv_0}{3}$ (d) $\frac{Mv_0}{4}$

17. The angular impulse suffered by the rod immediately after the collision is

(a) Mv_0L (b) $\frac{Mv_0}{2}L$

(c) $\frac{Mv_0}{3}L$ (d) $\frac{Mv_0}{8}L$

18. If the time of collision is Δt then the impulsive force acted on the rod during the collision is

(a) $\frac{Mv_0}{\Delta t}$ (b) $\frac{Mv_0}{2\Delta t}$

(c) $\frac{Mv_0}{3\Delta t}$ (d) $\frac{Mv_0}{4\Delta t}$

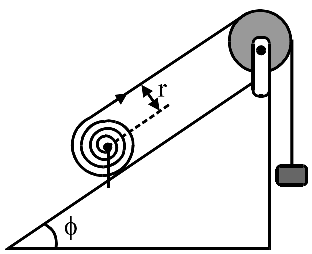
19. The force with which one half of the rod will act on the other half in the process of motion after the collision is

(a) $\frac{Mv_0^2}{L}$ (b) $\frac{9Mv_0^2}{32L}$

(c) $\frac{7Mv_0^2}{39L}$ (d) $\frac{Mv_0^2}{4L}$

Comprehension-3

A tape is wrapped around a cylinder of mass M and radius r. The tape is pulled as shown in figure to prevent the centre of mass from falling as the cylinder unwinds the tape. Assume that the inclined plane is frictionless. The pulley is massless and frictionless.



20. The angular acceleration of the cylinder is

(a) $\frac{2g\sin\phi}{r}$ (b) $\frac{3g\sin\phi}{r}$

(c) $\frac{3g\sin\phi}{2r}$ (d) $\frac{5g\sin\phi}{2r}$

21. The mass of the block is

(a) $\frac{M}{1-2\sin\phi}$ (b) $\frac{M\sin\phi}{1-2\sin\phi}$

(c) $\frac{2M\sin\phi}{1-2\sin\phi}$ (d) $\frac{4M}{1-2\sin\phi}$

22. The work has been done on the cylinder when it has reached an angular velocity ω is

(a) $\frac{1}{4}(M\omega^2r^2)$ (b) $\frac{1}{3}(M\omega^2r^2)$

(c) $\frac{1}{2}(M\omega^2r^2)$ (d) $M\omega^2r^2$

23. The length of the tape unwound in this time is

(a) $\frac{\omega^2r^2}{4g\sin\phi}$ (b) $\frac{\omega^2r^2}{3g\sin\phi}$

(c) $\frac{\omega^2r^2}{2g\sin\phi}$ (d) $\frac{\omega^2r^2}{g\sin\phi}$

Comprehension-4

A man of mass 100 kg stands at the rim of a turntable of radius 2m, moment of inertia 4000 kg.m^2 mounted on a vertical frictional shaft at its centre. The whole system is initially at rest. The man now walks along the outer edge of the turntable with a velocity of 1 m/s relative to the earth :

24. The angular velocity of the turntable is

(a) 0.02 rad/s (b) 0.03 rad/s

(c) 0.04 rad/s (d) 0.05 rad/s

25. The angle through which it will be rotated when the man reaches his initial position on the turnable

(a) π/11 rad (b) 2π/11 rad

(c) 3π/11 rad (d) 4π/11 rad

26. The angle through which it will be rotated when the man reaches his initial position relative to earth

(a) 18^0 (b) 27^0

(c) 36^0 (d) 45^0

Comprehension-5

A uniform rod of length a is freely pivoted at one end. It is initially held horizontally and then released from rest. When the rod is vertical it breaks at its midpoint. Assume that no impulsive forces are generated when the rod breaks.

27. The angular velocity just before the rod breaks is

(a) $\sqrt{\frac{g}{a}}$ (b) $\sqrt{\frac{2g}{a}}$

(c) $\sqrt{\frac{3g}{a}}$ (d) $2\sqrt{\frac{g}{a}}$

28. The largest angle from the vertical reached by the upper part of the rod in its subsequent motion is

(a) 45^0 (b) 60^0

(c) 75^0 (d) 90^0

29. The motion of the lower part of the rod after the break is

(a) vertical straight line

(b) parabolic path

(c) circular path

(d) elliptical path

Comprehension-6

A uniform rod of length *l* and mass 2m rests on a smooth horizontal table. A point mass m moving horizontally at right angles to the rod with an initial velocity V collides with one end of the rod and sticks to it.

30. The angular velocity of the system after the collision is

(a) $\frac{V}{l}$ (b) $\frac{V}{2l}$

(c) $\frac{2V}{3l}$ (d) $\frac{V}{4l}$

31. The position of the instant axis of rotation immediately after the collision is

(a) $\frac{l}{3}$ from the end to which the mass sticks

(b) $\frac{2l}{3}$ from the end to which the mass sticks

(c) $\frac{l}{4}$ from the end to which the mass sticks

(d) $\frac{l}{2}$ from the end to which the mass sticks

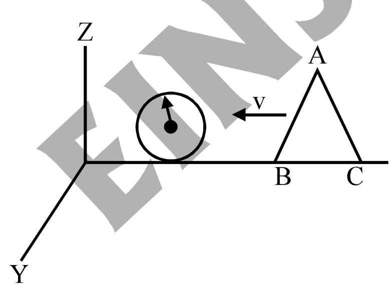
32. The change in kinetic energy of the system as a whole as a result of the collision is

(a) $\frac{1}{2}mV^2$ (b) $\frac{1}{4}mV^2$

(c) $\frac{1}{3}mV^2$ (d) none

Comprehension-7

A wedge of mass m and triangular cross section (AB = BC = CA = 2R) is moving with a constant velocity $-v\hat{i}$ towards sphere of radius R fixed on smooth horizontal table as shown in figure.



The wedge makes an elastic collison with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt, during which the sphere exerts a constant force F on the wedge.

33. The magnitude of force F is

(a) $\frac{2mV}{\Delta t}$ (b) $\frac{2mV}{\sqrt{3}\Delta t}$

(c) $\frac{4mV}{\sqrt{3}\Delta t}$ (d) $\frac{mV}{\sqrt{3}\Delta t}$

34. The normal force N exerted by the table on the wedge during the time Δt is

(a) $mg + \frac{2mV}{\Delta t}$ (b) $mg + \frac{2mV}{\sqrt{3}\Delta t}$

(c) $mg + \frac{4mV}{\sqrt{3}\Delta t}$ (d) $mg + \frac{mV}{\sqrt{3}\Delta t}$

35. Let h denotes the perpendicular distance between the centre of mass of the wedge and the line of action of F. The magnitude of the torque due to the normal force N about the centre of the wedge during the interval Δt

(a) $\frac{2mVh}{\Delta t}$ (b) $\frac{2mVh}{\sqrt{3}\Delta t}$

(c) $\frac{4mVh}{\sqrt{3}\Delta t}$ (d) $\frac{mVh}{\sqrt{3}\Delta t}$

36. Let the normal reaction shift by distance 'x' during the collision. The value of 'x' is

(a) zero

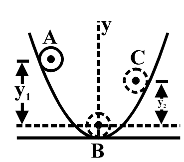
(b) $\frac{4vh}{\sqrt{3}g\Delta t + 2v}$ towards left

(c) $\frac{4vh}{\sqrt{3}g\Delta t + 2v}$ towards right

(d) can't be decided as the data is insufficient.

Comprehension-8

A uniform cylinder rolls from rest down the side of a trough whose vertical dimension y is given by equation $y = k \cdot x^2$.



From A to B on the trough there is sufficient friction such that the cylinder has rolling without slipping but the surface of the trough is frictionless from B to C.

37. The speed of the cylinder when it just reaches the point B for the first time

(a) $\sqrt{2gy_1}$ (b) $\sqrt{\frac{4gy_1}{3}}$

(c) $\sqrt{\frac{2gy_1}{3}}$ (d) $\sqrt{gy_1}$

38. The ratio $y_1 : y_2$ is

(a) 4 : 3 (b) 2 : 1

(c) 3 : 2 (d) 1 : 1

39. The speed of the cylinder when it reaches the point B second time

(a) $\sqrt{2gy_1}$ (b) $\sqrt{\frac{4gy_1}{3}}$

(c) $\sqrt{\frac{2gy_1}{3}}$ (d) $\sqrt{gy_1}$

40. The frictional force acting on the cylinder during the rolling motion in part AB is

(a) constant
(b) increases
(c) decreasing
(d) first increases and then decreases

41. The acceleration of centre of mass during the rolling motion in part AB is

(a) constant
(b) increases
(c) decreasing
(d) first increases and then decreases

42. The motion of the cylinder in part BC is

(a) pure translational
(b) pure rotational
(c) pure rolling
(d) rolling with slipping

Comprehension-9

A cylinder of mass m and radius r is attached with a spring of spring constant k. One end of the spring is attached to the wall and another end with the centre of the cylinder. Initially the spring is extended by an amount x_0 and released from rest. The horizontal surface is frictional. Immediately after the release the cylinder will have pure rolling motion.

43. The velocity of cylinder when the spring is relaxed

(a) $x_0\sqrt{\frac{2k}{3m}}$ (b) $x_0\sqrt{\frac{k}{m}}$

(c) $x_0\sqrt{\frac{2k}{m}}$ (d) $x_0\sqrt{\frac{3k}{2m}}$

44. The acceleration of the cylinder

(a) remains constant
(b) changes continuously in magnitude only
(c) changes continuoulsy in magnitude and also direction will change
(d) changes only direction.

45. Choose the incorrect option

(a) work done by the frictional force acting on the cylinder is zero.
(b) frictional force acting on the cylinder remains constant.
(c) frictional force acting on the cylinder changes in magnitude and also direction will change
(d) mechanical energy of the system remains constant.

46. The maximum acceleration of the cylinder is

(a) $\frac{kx_0}{m}$ (b) $\frac{2kx_0}{3m}$

(c) $\frac{2kx_0}{m}$ (d) $\frac{kx_0}{3m}$

47. If the coefficient of static frinction is μ then the maximum value of the x_0 such that the cylinder will have pure rolling motion

(a) $\frac{\mu mg}{k}$ (b) $\frac{2\mu mg}{k}$

(c) $\frac{3\mu mg}{k}$ (d) $\frac{3\mu mg}{2k}$

Consider the surface is frictionless :

48. After the release the cylinder will have

(a) pure translational motion
(b) pure rotational motion
(c) pure rolling motion
(d) rolling with slipping motion

49. The maximum speed of the cylinder is

(a) $x_0\sqrt{\frac{2k}{3m}}$ (b) $x_0\sqrt{\frac{k}{m}}$

(c) $x_0\sqrt{\frac{2k}{m}}$ (d) $x_0\sqrt{\frac{3k}{2m}}$

If one end of the spring is attached at the top most point of the cylinder and there is a sufficient friction such that the cylinder will have pure rolling motion on the surface after the release.

50. The initial acceleration of the cylinder just after the release is

(a) $\frac{4kx_0}{m}$ (b) $\frac{2kx_0}{3m}$

(c) $\frac{2kx_0}{m}$ (d) $\frac{4kx_0}{3m}$

51. The initial frictional force acting on the cylinder is

(a) $\frac{kx_0}{3}$ towards the wall

(b) $\frac{kx_0}{3}$ away from the wall

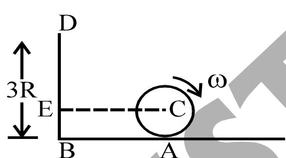
(c) $\frac{2kx_0}{3}$ towards the wall

(d) $\frac{2kx_0}{3}$ away from th wall

MATRIX-MATCH TYPE

Matching-1

A sphere of mass m and radius R is rolling without slipping with angular speed ω on a horizontal plane as shown.



	Column - A		Column - B
(A)	The angular momentum about point A is	(p)	$\frac{7}{5}mR^2\omega$
(B)	The angular momentum about point B is	(q)	$\frac{2}{5}mR^2\omega$
(C)	The angular momentum about point D is	(r)	$\frac{8}{5}mR^2\omega$
(D)	The angular momentum about point E is	(s)	$\frac{3}{5}mR^2\omega$

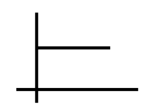
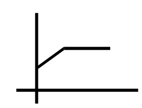
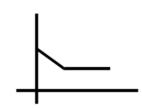
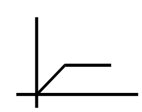
Matching-2

Consider a square frame consists of four uniform rods each of mass m and length *l*.

	Column - A		Column - B
(A)	Moment of inertia of the frame about an axis passing through the centre and perpendicular to the plane of the figure	(p)	$4/3\ ml^2$
(B)	Moment of inertia about an axis passing through one of the corner and perpendicular to the plane of the figure	(q)	$10/3\ ml^2$
(C)	Moment of inertia about a diagonal	(r)	$2/3\ ml^2$
(D)	Moment of inertia about any side	(s)	$5/3\ ml^2$

Matching-3

A ring has given pure angular speed and gently placed on a frictional horizontal surface.

	Column - A		Column - B
(A)	Variation of angular momentum of the ring about any point of the surface with time	(p)	
(B)	Variation of angular speed of the ring	(q)	
(C)	Variation of linear speed of the ring	(r)	
(D)	Variation of angular momentum of the ring about the centre with time	(s)	

MULTIPLE CORRECT CHOICE TYPE

1. An object rotates about a fixed axis, and the angular position of a reference line on the object is given by $\theta = 0.4\ e^{2t}$, where θ is in radians and t is in second. Consider a point on the object that is 4cm from the axis of rotation. At t = 0

(a) The magnitude of tangential acceleration of the point is 0.064 m/s^2

(b) The magnitude of radial acceleration of the point is 0.0256 m/s^2

(c) The net acceleration of the point is 0.075 m/s^2

(d) All the above

2. A string is wrapped around a cylinder of mass M and radius r. The string is pulled vertically upward to prevent the center of mass to fall as the cylinder unwinds the string. Choose the correct options :

(a) The tension in the string is Mg.

(b) The work has been done on the cylinder once it has reached an angular speed ω is $\frac{1}{4}M\omega^2 r^2$

(c) The length of string unwound in the time it acquires angular speed ω is $\frac{1}{4}\frac{r^2\omega^2}{g}$

(d) none of these are correct

3. Two discs with moment of inertia I_1 and I_2 initially they are rotating with angular velocities ω_1 and ω_2 respectively in anticlockwise direction, are pushed with forces acting along the axis. The disks rub against each other and eventually reach a common final angular velocity ω. Choose the correct statements

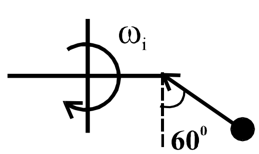
(a) The final angular speed acheived is $\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$ in the anticlockwise sense.

(b) There is a loss of kinetic energy

(c) The angular momentum of the two discs system will be conserved about the axis.

(d) Work done by the frictional force is $\frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}(\omega_1 - \omega_2)^2$

4. Four thin, uniform rods, each of mass M and length d = 0.5 m, are rigidly connected to a vertical axle to form a turnstile. The turnstile rotates clockwise about the axle, which is attached to a floor, with initial angular velocity ω_i = – 2 rad/s.



A mud ball of mass $m = \frac{1}{3}M$ and initial speed v_i = 12 m/s is thrown along the path shown and strikes to the end of one rod. Choose the correct statements

(a) The direction of final rotation is clockwise

(b) The direction of final rotation is anticlockwise

(c) The final angular speed is 0.8 rad/s

(d) The kinetic energy and angular momentum about the axis will be conserved in the collision process.

5. A particle of mass 3 kg is moving under the action of a central force whose potential is given by $U = 10\,r^3$ J in a circular orbit of radius 10 m. Let the total energy of the particle, angular momentum about the centre and time period are E, L, T respectively. Then

(a) $E = 2.5 \times 10^4$ J (b) $L = 3 \times 10^3$ J–s

(c) $T = \frac{\pi}{5}$ s (d) none

6. Two skaters, each of mass 50 kg, approach each other along parallel paths separated by 3m. They have equal and opposite velocities of 10 m/s. The first skater carries a long light pole, 3 m long, and the second skater grabs the end of it as he passes. Assume that the surface is frictionless. Then

(a) After the skaters are connected they have eventually rotational motion of angular speed 20/3 rad/s

(b) If they will pull each other then angular speed will increase

(c) If they will pull each other then their rotational kinetic energy will increase

(d) If they will pull each other there angular momentum will remain constant.

7. A rigid body of moment of inertia I (about the cm) mass m and radius R is placed with pure linear speed v_0 on the frictional ground having coefficient of friction μ. Then

(a) The time after which body starts pure rolling is $\frac{Iv_0}{\mu g(I + mR^2)}$

(b) The angular velocity at the moment the body starts pure rolling is $\frac{mRv_0}{I + mR^2}$

(c) The angular momentum at any time of the body about any point on the ground is Mv_0R.

(d) The kinetic energy of the body at any time after the pure rolling starts remains constant

8. A solid sphere of radius R rolling on a rough horizontal surface with a linear speed v collides elastically with a fixed, smooth vertical wall. Then

(a) Immediately after the collision, the angular speed of the sphere is $\frac{v}{R}$ in the same sense as before the collision.

(b) The angular momentum of the sphere just before the collision and just after the collision remains constant about any point on the horizontal surface.

(c) After the collision the sphere will have rolling with slipping and at a particular moment pure translational motion and finally the sphere will acheive pure rolling motion.

(d) The final linear speed of a top most point of the sphere is $\frac{6v}{7}$

9. A uniform ladder of mass 10 kg rests against a smooth vertical wall making an angle of 53^0 with it. The other end rests on a rough horizontal floor with coefficient of friction $\frac{\sqrt{3}}{2}$.

(a) The normal reaction by the horizontal floor is 98 N

(b) The frictional force is 65 N

(c) The normal reaction by the wall is 65 N

(d) The maximum angle made by the ladder with the wall is 60^0.

10. A force F acts tangentially at the highest point of a sphere of mass m kept on a rough horizontal plane. If the sphere starts rolling without slipping then

(a) The acceleration of the sphere is $\frac{10F}{7m}$.

(b) The frictional force is $\frac{3F}{7}$ in forward direction.

(c) The frictional force is $\frac{3F}{7}$ in backward direction.

(d) The angular acceleration about the instant point of contact is $\frac{10F}{7mR}$.

<u>Assertion-Reason Type</u>

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

1. STATEMENT-1 : Moment of inertia depends on distribution of masses.

STATEMENT-2 : Moment of inertia of a uniform ring or non-uniform ring having same masses will be the same about an axis passing through the centre and perpendicular to the plane.

2. STATEMENT-1 : The condition of pure rolling is $v = \omega R$ where v is the velocity of the centre of mass and ω is the angular velocity of the centre of mass for a rigid body in rolling motion on the horizontal fixed ground.

STATEMENT-2 : The kinetic energy of a pure rolling body is always constant.

3. STATEMENT-1 : A cylinder cannot have pure rolling motion on a smooth inclined plane.

STATEMENT-2 : In case of pure rolling motion, the instant point of contact will be stationary with respect to the surface on which the body will roll.

4. STATEMENT-1 : A cubical block slides down a rough inclined plane with uniform speed. The normal reaction will pass through the centre of the cube.

STATEMENT-2 : The torque of the normal reaction acting on a block about the centre is same as the torque due to frictional force about the centre.

5. STATEMENT-1 : A solid sphere, a hollow sphere, a cylinder, a ring and a disc with different masses and different radius are placed at the top of a smooth inclined and released. All of them will reach the bottom at the same time.

STATEMENT-2 : All of them will have pure rolling motion on the inclined.

6. STATEMENT-1 : Work done by the friction force in pure rolling motion is always zero.

STATEMENT-2 : Work done depends on the frame of reference.

7. STATEMENT-1 : A solid sphere, a hollow sphere, a cylinder, a ring and a disc with same masses and same radius are placed at the top of a frictional inclined and released. The friction coefficient

between the objects and the inclined are same and not sufficient to allow pure rolling. All of them will reach the bottom at the same time.

STATEMENT-2 : The smallest kinetic energy at the bottom of the inclined will be achieved by the ring.

8. STATEMENT-1 : A solid sphere, a hollow sphere, a cylinder, a ring and a disc with same masses and same radius are placed at the top of a frictional inclined and released. The friction coefficient between the objects and the inclined are same and sufficient to allow pure rolling. All of them will reach the bottom at different time.

STATEMENT-2 : The kinetic energy at the bottom of the inclined will be the same for each.

9. STATEMENT-1 : Two uniform cylinders having a different masses and different radius are released from rest from the same height on the rough inclined, such that they have pure rolling motion. Both will reach the bottom at the same time.

STATEMENT-2 : The minimum coefficient of friction for the pure rolling motion for both will be the same.

10. STATEMENT-1 : A particle is projected from the ground at certain angle with the horizontal. The angular momentum of the particle about the point of projection changes with time.

STATEMENT-2 : The torque acting on the particle about the point of projection is non-zero but constant.

11. STATEMENT-1 : A ring, disc, cylinder and sphere have different masses and same radius are given the same kinetic energy and released on the flat horizontal surface such that they begin to roll as soon as released towards a wall which is at the same distance from each. The rolling friction is negligible in each case. All of them will reach the wall at different time.

STATEMENT-2 : All of them will have the same linear speed immediately after the release.

12. STATEMENT-1 : The centre of gravity and centre of mass of a very high rise building with uiform mass distribution will coincide.

STATEMENT-2 : The centre of gravity is a point where the total weight of the body is concentrated.

13. STATEMENT-1 : A ring is placed on a horizontal surface with its plane perpendicular to the surface. A bullet moving at the height equal to the diameter of the ring touches the ring and passed on. The ring will have pure rolling motion.

STATEMENT-2 : The speed of the bullet will decrease after the contact.

(Answers) EXCERCISE BASED ON NEW PATTERN

COMPREHENSION TYPE

1.	a	2.	b	3.	d	4.	a	5.	c	6.	c
7.	b	8.	a	9.	d	10.	d	11.	a	12.	a
13.	a	14.	b	15.	c	16.	d	17.	d	18.	d
19.	b	20.	a	21.	b	22.	a	23.	a	24.	d
25.	b	26.	c	27.	c	28.	b	29.	b	30.	a
31.	b	32.	d	33.	c	34.	b	35.	c	36.	b
37.	b	38.	c	39.	b	40.	c	41.	c	42.	d
43.	a	44.	c	45.	b	46.	b	47.	c	48.	a
49.	b	50.	d	51.	a						

MATRIX-MATCH TYPE

1. [A-p, B-p, C-r, D-q] 2. [A-p; B-q; C-r; D-s] 3. [A-p; B-r; C-s; D-r]

MULTIPLE CORRECT CHOICE TYPE

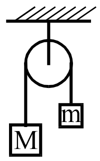
1. a, b 2. a, b, c 3. a, b, c, d 4. b, c 5. a, b, c
6. a, b, c, d 7. a, b, c, d 8. a, b, c, d 9. a, b, c, d
10. a, b, d

ASSERTION-REASON TYPE

1.	B	2.	C	3.	A	4.	D	5.	C	6.	D
7.	B	8.	B	9.	B	10.	C	11.	C	12.	D
13.	B										

INITIAL STEP EXERCISE (SUBJECTIVE)

1. The pulley shown in figure has a moment of inertia I about its axis and its radius is R. Find the magnitude of the acceleration of the two blocks.



Assume that the string is light and doesnot slip on the pulley.

2. A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is held horizontally a distance h above the centre line. The ball leaves the cue with a speed v_0 and, because of its forward english eventually acquires a final speed of $\frac{9v_0}{7}$, show that $h = \frac{4R}{5}$, where R is the radius of the ball.

3. A cylinder of mass m is suspended through two strings wrapped around it. Find (a) the tenstion T in the string (b) the speed of the cylinder as it falls through a distance h.

4. A cylinder of radius R and mass M has density that increases linearly with distance r from the cylinder axis, $\rho = \alpha r$. Find the moment of inertia of the cylinder about a longitudinal axis through its center in terms of M and R.

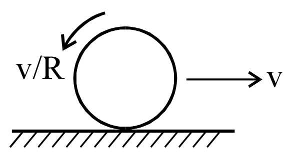
5. A uniform cylinder of height h and radius r is placed with its circular face on a rough inclined. The inclination of the plane to the horizontal is gradually increased. If μ is the coefficient of friction, then under what condition the cylinder will (a) slide before toppling (b) topple before sliding ?

6. A uniform cylinder of radius R and mass M is spinned about its axis to the angular velocity ω_0 and then placed into a corner. The coefficient of friction between the walls and the cylinder is equal to μ. (a) What are the normal reactions acting on the cylinder by the wall ? (b) Find the work done by the frictional forces acting on the cylinder ? Also find the total work done ?

7. A particle is projected at time t = 0 from a point P with a speed v_0 at an angle of $\frac{\pi}{4}$ to the horizontal. Find the magnitude and the direction of the angular momentum of the particle about the point P at $t = \frac{v_0}{g}$.

8. The cylinder has a fixed axes and is initially at rest. A block of mass M is initially moving to the right with speed V_1 passes over the cylinder. When it first makes contact with the cylinder its slips on the cylinder but the friction is large enough so that slipping ceases before M lose contact with the cylinder. Find the speed V_2 in terms of V_1, M, I (moment of interia) and radius R.

9. A solid sphere is set into motion on a rough horizontal surface with a linear speed v in the forward direction and an angular speed $\frac{v}{R}$ in the anticlockwise direction.

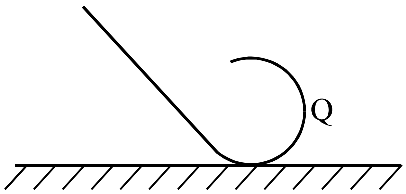


Find the linear speed of the sphere (a) when it stops rotating and (b) when slipping finally ceases and pure rolling starts.

10. A solid sphere of mass 0.5 kg is kept on a horizontal surface. The coefficient of static friction between the surfaces in contact is $\frac{2}{7}$. What maximum force can be applied at the highest point in the horizontal direction so that the sphere does not slip on the surface ?

11. A small sphere is released from rest on a fixed bigger sphere from the top most point. There is sufficient friction such that the small sphere starts pure rolling motion immediately after the release. At what angle from the vertical does the small sphere leave the contact of bigger sphere ?

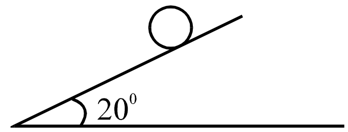
12. **A small solid marble of mass m and radius r, rolls without slipping along a loop-the-loop track.**



(a) From what minimum height above the bottom of the track must the marble be released in order that it just stay on the track at the top of the loop ? (R >> r) (b) If the marble is released from height 6R above the bottom of the track what is the horizontal component of the force acting on it at point Q ? Also calculate the maximum frictional force acting on the marble during the motion.

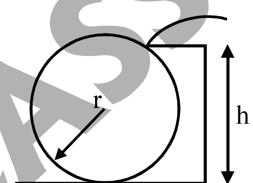
13. **A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligible push is given to it. Find the horizontal velocity of the axis of the cylinderical part of the carpet when its radius reduces to $\frac{R}{2}$.**

14. **A metal hoop with a radius r = .15 m is released from rest on the 20^0 incline. If the coefficients of the static and kinetic friction are $\mu_s = 0.15$ and $\mu_k = 0.12$.**



Determine the angular acceleration of the hoop and the time t for the loop to move a distance of 3m down the incline.

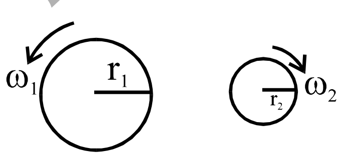
15. **Find the ratio of the height h of a cushion on a snooker table to the radius r of a ball as shown in figure, such that when the ball hits the cushion with a pure rolling motion it rebounds with a pure rolling motion.**



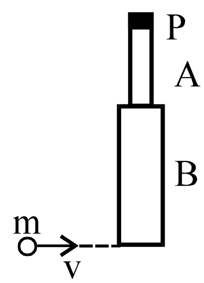
(Assume that the force exerted on the ball by the cushion is horizontal during the impact and that the ball hits the cushion normally).

FINAL STEP EXERCISE (SUBJECTIVE)

1. **Figure shows two cylinders of radii r_1 and r_2 having moments of inertia I_1 and I_2 about their respective axes with angular speeds ω_1 and ω_2 as shown in figure. The cylinders are moved closer to touch each other keeping the axes parallel the cylinder first slip over each other at the constant but the slipping finally ceases due to friction between them. If the duration for the existance of friction is Δt then (a) find the friction force (b) find the angular speeds of the cylinders after the slipping ceases.**

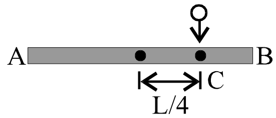


2. **Two uniform thin rodes A and B of length 0.6m each and of masses 0.01 kg and 0.02 kg respectively, are rigidly joined end to end. The combination is pivoted at the lighter end P as shown, such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg moving horizontally hits the lower end of the combination and sticks to it.**



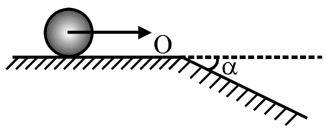
What should be the velocity of the object so that the system could just be raised to the horizontal position ?

3. **A homogenous rod AB of length L = 1.8 m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane as shown. The rod is in the horizontal position. An insect of the same mass M falls vertically with speed v on the point C. Immediately after falling, the insect moves towards the end B such that the rod rotates with constant angular velocity ω in terms of V and L. (a)**



Find the velocity ω in terms of V and L (b) If the insect reaches the end B when the rod has turned through an angle of 90^0, determine V.

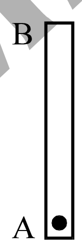
4. A uniform solid cylinder of radius R rolls over a horizontal plane and passes on to an inclined meeting the horizontal plane at an angle α with the horizontal.



Find the maximum value of velocity v_0 which still permits the cylinder to roll on the plane without a jump. Assume sliding to be absent.

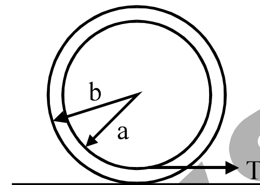
5. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius r is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine : (a) the angle θ through which the cylinder rotates before it leave contact with the edge, (b) the speed of the centre of mass of the cylinder before leaving contact with the edge and (c) the ratio of translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

6. A uniform slender bar AB of mass m and length L supported by a frictionless pivot at A, is released from rest at its near vertical position as shown in figure. Calculate the reaction at the pivot when the bar just acquires horizontal position shown dotted.



If at this instant, the bar is released from its support gently and allowed to move for t second further, estimate its angular speed and the velocity of the centre of mass at that instant.

7. A cotton reel is made up of a hub of radius a and two end caps of radius b. The mass of the complete reel is m and its moment of inertia about its longitudinal axis is I. The reel rests on a perfectly rough table (so that only rolling motion is possible) and a tension T is applied to the free end of the cotton wrapped around the hub as shown in figure. In what direction does the reel begin to move ?



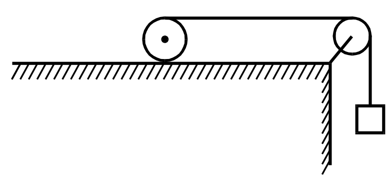
Find the frictional force exerted by the table and the direction in which it acts.

8. An elastic spherical ball of mass M and radius a moving with velocity v strikes a rigid surface at an angle θ to the normal. Assuming it skids while in contact with the surface, the tangential frictional force being a constant fraction μ of the normal reaction force, show that :

(a) The ball is reflected at an angle ϕ to the normal where $(\tan\theta - \tan\phi) = 2\mu$.

(b) The angular velocity of the rebounding ball changes by an amount $\frac{5\mu v}{a}\cos\theta$.

9. In the arrangement shown the mass of the cylinder is 1 kg and radius is 0.25 m. There is no slipping between the cylinder and the string. The pulley and the strings are massless and there is no friction between the pulley and the string. There is a friction between the table and the cylinder for which $\mu_s = 0.5$ and $\mu_c = 0.4$.



Find the acceleration of the cylinder and the friction force between the cylinder and the table for the following cases :

(a) The mass of the block is 0.25 kg

(b) The mass of the block is 2 kg

[Take $g = 10\ m/s^2$]

10. A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of 60^0 and then released. Find the magnitude of the force acting on a particle of mass dm at the tip of the rod when the rod makes an angle of 37^0 with the vertical.

ANSWERS SUBJECTIVE (INITIAL STEP EXERCISE)

1. $\dfrac{(M-m)gR^2}{I+(M+m)R^2}$

3. (a) $\dfrac{mg}{6}$ (b) $\sqrt{\dfrac{4gh}{3}}$

4. $\dfrac{3}{5}MR^2$

5. (a) $\mu < \dfrac{2r}{h}$ (b) $\mu > \dfrac{2r}{h}$

6. (a) $\dfrac{\mu Mg}{1+\mu^2}, \dfrac{Mg}{1+\mu^2}$ (b) $-\dfrac{1}{4}\dfrac{MR^2\omega_0^2}{1+\mu}, -\dfrac{\mu}{\mu+1}\dfrac{MR^2\omega_0^2}{4}, -\dfrac{1}{4}MR^2\omega_0^2$

7. $\dfrac{mv_0^3}{2\sqrt{2}g}$

8. $\dfrac{MR^2v_1}{MR^2+I}$

9. (a) $\dfrac{3v}{5}$ (b) $\dfrac{3v}{7}$

10. 3.3 N

11. 54^0

12. (a) 2.7 R (b) 7 mg, 5/7 mg

13. $\sqrt{\dfrac{14gR}{3}}$

14. 7.37 rad/s^2, 1.633s

15. $h = \dfrac{7r}{5}$

ANSWERS SUBJECTIVE (FINAL STEP EXERCISE)

1. (a) $\dfrac{I_1\omega_1 r_2 + I_2\omega_2 r_1}{I_2 r_1^2 + I_1 r_2^2} r_2$ (b) $\dfrac{I_1\omega_1 r_2 + I_2\omega_2 r_1}{I_2 r_1^2 + I_1 r_2^2} r_1$

2. 6.3 m/s.

3. (a) $\dfrac{12}{7}\dfrac{v}{L}$ (b) 3.5 m/s

4. $\sqrt{\dfrac{1}{3}gR(7\cos\alpha - 4)}$

5. (a) $\theta = \cos^{-1}(4/7)$ (b) $v = \sqrt{(4/7)gr}$ (c) 6

6. $v = \dfrac{1}{2}\sqrt{3gl} + gt$

7. $bF - aT = \dfrac{IT(b-a)}{I+mb^2}$ which are both positive as we have assumed.

8. (a) $|\tan\theta - \tan\phi| = 2\mu$ (b) $\dfrac{5\mu v\cos\theta}{a}$

10. $0.9\sqrt{2}$ dm g

ANSWERS (SINGLE CORRECT CHOICE TYPE)

1.	**c**	**7.**	**b**	**13.**	**b**	**19.**	**c**	**24.**	**a**	**29.**	**a**	**34.**	**d**
2.	**c**	**8.**	**d**	**14.**	**b**	**20.**	**b**	**25.**	**a**	**30.**	**a**		
3.	**d**	**9.**	**b**	**15.**	**a**	**21.**	**c**	**26.**	**a**	**31.**	**a**		
4.	**a**	**10.**	**d**	**16.**	**d**	**22.**	**c**	**27.**	**b**	**32.**	**d**		
5.	**d**	**11.**	**c**	**17.**	**b**	**23.**	**b**	**28.**	**b**	**33.**	**c**		
6.	**b**	**12.**	**a**	**18.**	**c**								