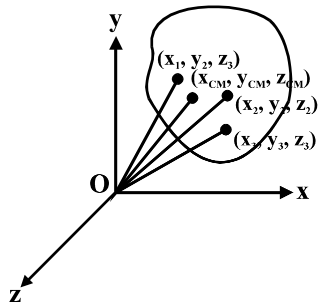
SYSTEM OF PARTICLES AND CENTRE OF MASS, COLLISION

C1 Centre of Mass

A special point of a baseball that is flipped into the air moves in a simple parabolic path, but all other points of the bat follow more complicated curved paths. In fact, that special point moves as though (1) the bat's total mass were concentrated that and (2) the gravitational force on the bat acted only there. That special point is said to be the centre of mass of the bat. In general ; the centre of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there, and all external forces were applied there.

For a system of particles, that the distributed in three dimensions shown in the figure



The center of mass of the system of particles is given by

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

The position vector of centre of mass of the system of particles is given by $\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{v} + z_{CM}\hat{k}$

i.e., $$\vec{R}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Centre of Mass of Solid Bodies

Solid bodies are treated as continuous distribution of matter and the centre of mass for these bodies is given by

$$x_{CM} = \frac{\int xdm}{\int dm} \qquad y_{CM} = \frac{\int ydm}{\int dm} \qquad z_{CM} = \frac{\int zdm}{\int dm}$$

Here x, y, z are the centre of mass of the differential elements of the solid bodies and dm is the mass of the differential elements.

<u>**Practice Problems :**</u>

1. **Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body ?**

2. **Find the centre of mass of (i) quarter circular ring of radius r (ii) half circular ring of radius r (iii) a sector of ring of radius r and forming angle α at the centre (iv) half circular disc of radius r.**

3. **Find the centre of mass (i) masses m, 2m, 3m, 4m are placed at the corners of a square of side length L (ii) masses m, 2m, 3m, 4m, 5m and 6m are placed at the corners of a regular hexagon of side length L.**

4. **Four particles of masses m, 2m, 4m, 4m are placed at (*l*, *l*), (–*l*, *l*), (–*l*, –*l*) and (*l*, –*l*) respectively. The centre of mass will lie in**

   (a) First quadrant **(b) Second quadrant**

   (c) Third quadrant **(d) Fourth quadrant**

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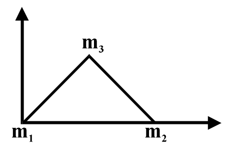
5. **Three particles of masses $m_1 = 1.2$ kg, $m_2 = 2.5$ kg and $m_3 = 3.4$ kg form an equilateral triangle of edge length $l = 140$ cm as shown in figure.**



The center of mass of this three particle system is

(a) (0.83 m, 0.58 m) **(b)** (0.78 m, 0.58 m)

(c) (0.83 m, 0.48 m) **(d)** (0.78 m, 0.48 m)

[Answers : (4) c (5) a]

C2 Newton's Second Law For System of Particles

For a system of n particles, $\mathbf{M\vec{R}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}$

Differentiating with respect to time, $\mathbf{M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}$

where $\mathbf{\vec{V}_{CM}}$ is the velocity of centre of mass of the system of particles.

Differentiating with respect to time, $\mathbf{M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n}$

where $\mathbf{\vec{a}_{CM}}$ is the acceleration of the centre of mass of the system of particles.

From Newton's second law

$$\mathbf{M\vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n}$$

Among the forces that contribute to the right side of the above equation will be forces that the particles of the system exert on each other (internal forces) and forces exerted on the particles from outside the system (external forces). By Newton's third law, the internal forces cancel out in the sum that appears on the right side of the above equation, what remains is the vector sum of all the external forces that act on the system.

Hence $\mathbf{\vec{F}_{net} = M\vec{a}_{CM}}$.

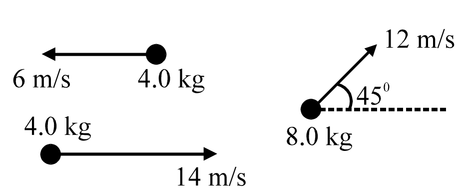
Practice Problems :

1. **If a bomb is exploded during the flight in parabolic path then what is the path of the centre of mass of the fragments after the explosion ?**

2. **Two spheres of masses M and 2M are initially at rest at a distance R apart. Due to mutual force of attraction they approach each other.When they are at separation R/2, the acceleration of their centre of mass would be**

(a) 0 **(b)** g m/s^2 **(c)** 3g m/s^2 **(d)** 12g m/s^2

3. **The three particles are moving with constant velocity in the horizontal plane as shown.**



The velocity of cm is

(a) $\{(3\sqrt{2} - 2)\hat{i} + 3\sqrt{2}\hat{j}\}$ m / s. **(b)** $\{(3\sqrt{2} + 2)\hat{i} + 3\sqrt{2}\hat{j}\}$ m / s.

(c) $\{(2\sqrt{2} + 2)\hat{i} + 3\sqrt{2}\hat{j}\}$ m / s. **(d)** $\{(2\sqrt{2} - 2)\hat{i} + 3\sqrt{2}\hat{j}\}$ m / s.

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4. **Two bodies of masses 1.0 kg and 3.0 kg are connected by a spring and rest on a frictionless surface. They are given velocities towards each other such that the 1.0 kg block travels initially at 1.7 m/s towards right. Choose the correct statement from the following :**

(a) **The velocity of centre of mass is always zero.**

(b) **The velocity of second block at the initial moment is $\frac{1.7}{3}$ m/sec towards left.**

(c) **both (a) and (b) are correct**

(d) **none**

[Answers : (2) a (3) b (4) c]

C3 Linear Momentum

For a single particle, we define a quantity $\vec{P}$ called its linear momentum as $\vec{P} = m\vec{v}$ and can write Newton's second law in terms of this momentum $\vec{F}_{external} = \frac{d\vec{P}}{dt}$

For a system of particles these relations become

$$\vec{P} = M\vec{v}_{CM} \quad \text{and} \quad \vec{F}_{external} = \frac{d\vec{P}}{dt}$$

Conservation of Linear Momentum

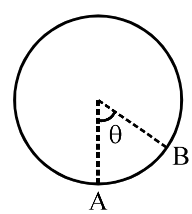
If a system is isolated so that no net external force acts on the system, the linear momentum $\vec{P}$ of the system remains constant i.e., $\vec{P} = \text{constant}$.

This can be written as $\vec{P}_i = \vec{P}_f$

where the subscripts refer to the value of $\vec{P}$ at some initial time and at a later time.

Practice Problems :

1. **A particle of mass m is moving in a circle of radius r center at O with constant speed v. The magnitude of change in linear momentum in moving from A to B is**



(a) 0 (b) $2mv\sin\frac{\theta}{2}$ (c) $2mv\cos\frac{\theta}{2}$ (d) $2mv\tan\frac{\theta}{2}$

2. **A 20.0 kg body is moving in the positive x-direction with a speed of 200 m/s when, owing to an internal explosion, if breaks into three parts. One part, with a mass of 10.0 kg moves away from the point of explosion with a speed of 100 m/s in positive y direction. A second fragment, with a mass of 4.0 kg moves in the negative x-direction with a speed of 500 m/s. The energy released in the explosion is**

(a) **1.25 MJ** (b) **2.5 MJ** (c) **3.23 MJ** (d) **4.75 MJ**

[Answers : (1) b (2) c]

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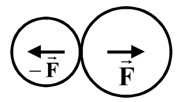
C4 Collision :

In a collision, two bodies exert strong forces on each other for a relatively short time. That's why these forces are known as impulsive. These forces are internal to the forces and direction of these forces along the normal to body system and are significantly larger than any external forces during the collision.

These forces are third law force pair. Their magnitude vary with time during the collision, but at any given instant those magnitudes are equal. Figure shows the third law force pair, $\vec{F}$ **and** $-\vec{F}$, that acts during the collision.



C5 Impulse and Linear Momentum

The forces during the collision will change the linear momentum of both bodies; the amount of change will depend not only on the forces, but also on the time dt during which they act and they are related by

$$d\vec{p} = \vec{F}dt \quad \Rightarrow \quad \int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} \vec{F}dt \qquad (1)$$

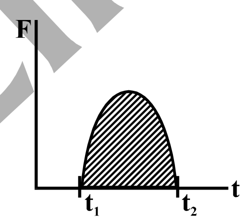
Here left side represent change in linear momentum and the right side, which is a measure of both the strength and the duration of the collision force, this is called the impulse $\vec{\tau}$ of the collision thus, $\vec{\tau} = \int_{t_i}^{t_f} \vec{F}dt$

Equation (1) represents the **Impulse – Linear Momentum Theorem**

In component form $\quad p_{fx} - p_{ix} = \Delta p_x = \tau_x, p_{fy} - p_{iy} = \Delta p_y = \tau_y, p_{fz} - p_{iz} = \Delta p_z = \tau_z$

If F_{avg} is the average magnitude of $\vec{F}$ during the collision and Δt is the duration of the collision then for one dimensional motion the equation (1) becomes $\Delta p = \tau = F_{avg}\Delta t$

Equation (1) tells us that of the magnitude of the impulse is equal to the area under as shown in figure.



C6 Coefficient of Restitution and types of collision

Collisions can be classified according to the relation of velocities of the two bodies. When two bodies collide, their relative velocity after impact is in a constant ratio to their relative velocity before impact. This constant ratio is known as coefficient of restitution of two bodies, given by

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The velocities are taken along the direction of impulsive force. The value of e is varying from zero to one. On the basis of the value of e, we have three types of collision :

(a) Elastic Collision

For elastic collision, e = 1 i.e, $v_1 - v_2 = u_2 - u_1$. In an elastic collision between two bodies, the initial and final kinetic energies are equal.

(b) Perfectly Inelastic Collision

For perfectly inelastic collision , e = 0 i.e, the two bodies have the same final velocity after collision i.e., the two bodies will stick together and then move. In an inelastic two bodies collision the final total kinetic energy is less than the initial total kinetic energy.

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(c) Inelastic Collision

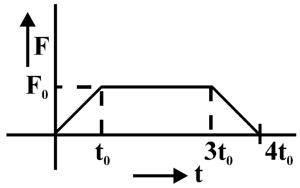
For inelastic collision, the value of e is given by $0 < e < 1$.

In this case the two bodies do not have the same final velocity after collision. In an inelastic two body collision the final total kinetic energy is less than the initial kinetic energy.

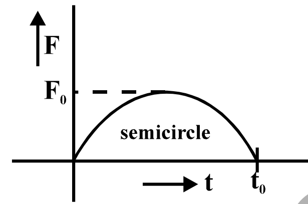
Practice Problems :

1. Find the kinetic energy of the particle of mass 'm' at the end time if initial speed is zero :

(a)



(b)



2. An impulse is supplied to a moving object with the force at an angle of θ with the velocity vector. The angle between the impulse vector and the change in momentum vector is

(a) θ **(b)** 0 **(c)** $\theta/2$ **(d)** 2θ

3. A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. The marble strike the disc normally and rebound downward with the same speed. If the mass of each marble is 5 g, the velocity with which the marble are striking the disc is ($g = 9.8\ m/s^2$)

(a) 0.98 m/s **(b)** 9.8 m/s **(c)** 1.96 m/s **(d)** 19.6 m/s

4. A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that if is turned back with velocity of 20 m/s. The force of blow acts for 0.01 s. The average force exerted by the bat on the ball is

(a) 120 N **(b)** 240 N **(c)** 480 N **(d)** 960 N

5. A ball A, moving with a speed u, collides directly with another similar ball B moving with a speed v in the opposite direction. A comes to rest after the collision. If the coefficient of restitution is e then u/v is

(a) $\frac{1+e}{1-e}$ **(b)** $\frac{1-e}{1+e}$ **(c)** $\frac{e}{1-e}$ **(d)** $\frac{e}{1+e}$

6. A ball is dropped from a height of 1 m. If the coefficient of restitution between the surface and the ball is 0.6, the ball rebounds to a height of

(a) 0.6 m **(b)** 0.4 m **(c)** 0.16 m **(d)** 0.36 m

7. A ball of mass m collides head-on and elastically with a ball of mass nm, initially at rest. The fraction of the incident energy transferred to the heavier ball is

(a) $\frac{n}{n+1}$ **(b)** $\frac{n}{(n+1)^2}$ **(c)** $\frac{2n}{(n+1)^2}$ **(d)** $\frac{4n}{(n+1)^2}$

8. A body of mass m_1, moving with a velocity u_1, collides head-on with a body of mass m_2, moving with a velocity u_2. The two bodies stick together after the collision. The loss of kinetic energy during the collision is

(a) $\frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$ **(b)** $\frac{2m_2 m_1 (u_2 - u_1)^2}{(m_1 + m_2)}$

(c) $\frac{m_2 m_1 (u_2 + u_1)^2}{2(m_1 - m_2)}$ **(d)** $\frac{2m_2 m_1 (u_2 - u_1)^2}{(m_1 - m_2)}$

[Answers : (2) b (3) a (4) c (5) a (6) d (7) d (8) a]

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SINGLE CORRECT CHOICE TYPE

1. A canon ball is fired with a velocity of 200 m/s at an angle of 60^0 with the horizontal. At the highest point it explodes into three equal fragments. One goes vertically upwards with a velocity of 100 m/s, the second one falls vertically downwards with a velocity of 100 m/s. The third one moves with a velocity of

(a) 100 m/s horizontally

(b) 300 m/s horizontally

(c) 200 m/s at 60^0 with the horizontal

(d) 300 m/s at 60^0 with the horizontal

2. A body of mass 2.9 kg is suspended from a string of length 2.5 m and is at rest. A bullet of mass 100 g, moving horizontally with a speed of 150 m/s, strikes and sticks to it. The maximum angle made by the string with the vertical after the impact is

(a) 30^0 (b) 45^0

(c) 60^0 (d) 90^0

3. A 1 kg ball, moving at 12 m/s, collides head-on with a 2 kg ball moving in the opposite direction at 24 m/s. If the coefficient of restitution is 2/3, then the energy lost in the collision is

(a) 60 J (b) 120 J

(c) 240 J (d) 480 J

4. A loaded spring gun, initially at rest on a horizontal frictionless surface, fires a marble at angle of elevation θ. The mass of the gun is M, the velocity of the marble is v_0 with respect to gun. The velocity of the gun after the fire is

(a) $\frac{mv_0\cos\theta}{m+M}$ (b) $\frac{mv_0\cos\theta}{M}$

(c) $\frac{Mv_0\cos\theta}{m}$ (d) $\frac{mv_0\sin\theta}{M}$

5. In the above problem, let the angle of elevation as seen from the ground is α then tan α equals to

(a) $\left(\frac{m+M}{m}\right)\tan\theta$

(b) $\left(\frac{m+M}{M}\right)\tan\theta$

(c) $\tan\theta$

(d) $\left(\frac{m+M}{M}\right)\cos\theta$

6. Two blocks m_1 and m_2, having masses 10 kg and 5 kg respectively, are placed on a frictionless horizontal surface and are connected by a light spring of force constant 5 N/m. m_1 is in contact with a rigid wall. m_2 is pushed through a distance of 4 cm towards m_1 and then released. The velocity of the centre of mass of the system when m_1 breaks off the wall is

(a) 2/3 cm/s (b) 4/3 cm/s

(c) 2 cm/s (d) 4 cm/s

7. A ball collides with a fixed plane with initial speed u at an angle α with the normal The coefficient of restitution for the collision is e. The sphere rebounds with speed v at an angle β with normal. Then tan β equals to

(a) $\frac{\sin\alpha}{e}$ (b) $\frac{\cos\alpha}{e}$

(c) $\frac{\cot\alpha}{e}$ (d) $\frac{\tan\alpha}{e}$

8. Two equal balls of mass *m* are in contact on a smooth horizontal table. A third identical ball with velocity u collides symmetrically on them and comes to rest. The coefficient of restitution for the collision is

(a) 1/3 (b) 2/3

(c) 2/5 (d) 1/5

9. Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of diameter. A is projected along the groove and at the end of time t impinges on B. If e is the coefficient of restitution then the time after which the second impact will occur

(a) t/e (b) 2t/e

(c) 3t/e (d) 4t/e

10. A ball is dropped from a height h onto a floor. If in each collision its speed becomes e times of its striking value. The time taken by the ball to stop rebounding

(a) $\sqrt{\frac{2h}{g}}\left(\frac{1-e}{1+e}\right)$ (b) $\sqrt{\frac{h}{g}}\left(\frac{1-e}{1+e}\right)$

(c) $\sqrt{\frac{h}{g}}\left(\frac{1+e}{1-e}\right)$ (d) $\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$

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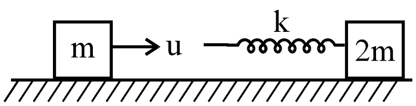
11. A perfectly elastic oblique collision takes place between a moving particle and a stationary particle of the same mass. After the collision the angle between their directions of motion is

(a) $\pi/3$ (b) $\pi/4$

(c) $\pi/2$

(d) cannot be determined

12. A block of mass m is projected with speed u as shown in figure towards the block of mass 2m. A spring of force constant k is connected to the block of mass 2m. Assume all the surfaces are frictionless. At the maximum compression of the spring the velocity of each block are given by



(a) u/2, u/2 (b) u/3, u/3

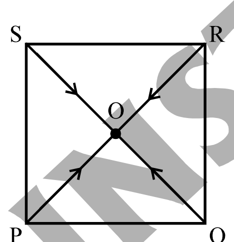
(c) u/2, u/3 (d) u/3, u/2

13. In the above problem the maximum compression of the spring is

(a) $2u\sqrt{\frac{m}{k}}$ (b) $u\sqrt{\frac{m}{k}}$

(c) $\frac{u}{2}\sqrt{\frac{m}{k}}$ (d) none of these

14. Four particles P, Q, R and S of equal mass move with equal speed v along the diagonals of square as shown in figure.



They collide at the center O of the square. After the collision, P comes to rest and S retraces its path with speed v. Then it is possible that

(a) R comes to rest whereas Q retraces its path with speed v.

(b) Both Q and R comes to rest.

(c) Q comes to rest whereas R retrices its path with speed v.

(d) Both Q and R retraces its path with speed v.

15. Two skaters A and B, having masses 50 kg and 70 kg respectively, stand facing each other 6 m apart on a horizontal smooth surface. They pull on a rope stretched between them. How far does each move before they meet ?

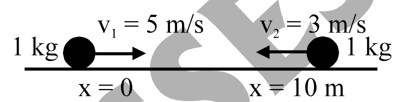
(a) both move 3 m

(b) A moves 2.5 m and B moves 3.5 m

(c) A moves 3.5 m and B moves 2.5 m

(d) none of the above

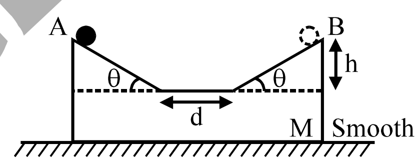
16. At t = 0, the position and velocities of two particles are as shown in the figure. They are kept on a smooth surface and being mutually attracted by gravitational force. The position of centre of mass at t = 2s is



(a) x = 5 m (b) x = 7 m

(c) x = 3 m (d) x = 2 m

17. A ball of mass m is allowed to roll down the wedge of mass M as shown in the figure. The displacement of wedge when the ball reaches from A to B



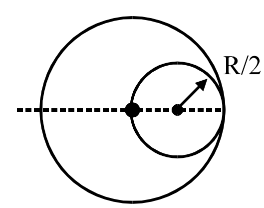
(a) $\left(\frac{m}{M+m}\right)(d+2h\cot\theta)$ towards left

(b) $\left(\frac{m}{2M-m}\right)(d+2h\cot\theta)$ towards left

(c) $\left(\frac{m}{M}\right)(d+2h\cot\theta)$ towards right

(d) $\left(\frac{m+M}{2M}\right)(d+2h\cot\theta)$ towards left

18. A circular hole of radius R/2 is cut from a homogenous circular disc of a radius R. The centre of mass of the remaining disc is



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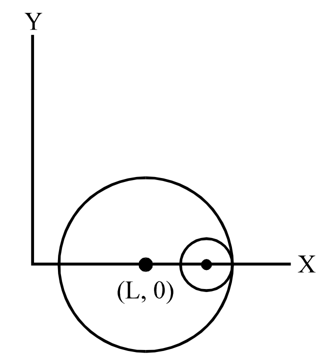
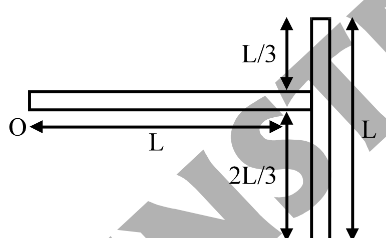
(a) R/6 towards left
(b) R/6 towards right
(c) R/3 towards left
(d) R/3 towards right

19. A small sphere of radius R is held against the inner surface of a larger sphere of radius 6R. The masses of large and small sphere are 4M and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surface of contact. The small sphere is now released. The coordinates of the center of the larger sphere when the smaller sphere reaches the other extreme position is



(a) {(L + 2R), 0} (b) {(L – 2R), 0}
(c) {(L – R), 0} (d) {(L + R), 0}

20. Two homogenous rod of same mass and equal length L is joint as shown.



The cm of this 'T' shape at the distance from O is

(a) $\frac{L}{2}$ (b) $\frac{L}{\sqrt{3}}$
(c) $\frac{L}{\sqrt{2}}$ (d) none

21. A block of mass m is released from the top of a smooth wedge of mass M, which rests on a smooth horizontal floor. The distance moved by the wedge till the mass m touches the floor is

(a) L (b) $\frac{mL}{m+M}$
(c) $\frac{mL}{M}$ (d) $\frac{ML}{m+M}$

22. A 4.5 kg dog stands on an 18 kg boat and is 6.1 m from the shore. He walks 2.4 m along the boat toward shore and then stops. Assuming there is no friction between the boat and water, the distance of the dog from the shore is

(a) 3.6 m (b) 3.8 m
(c) 4.0 m (d) 4.2 m

23. A bullet of mass m moving with a horizontal velocity v strikes a stationary block of mass M suspended by a string of length L. The bullet gets embeded in the block. The maximum angle made by the string after impact is

(a) $\cos^{-1}\left[1-\frac{1}{2gL}\left(\frac{mv}{m+M}\right)^2\right]$

(b) $\sin^{-1}\left[1-\frac{1}{2gL}\left(\frac{mv}{m+M}\right)^2\right]$

(c) $\tan^{-1}\left[1-\frac{1}{2gL}\left(\frac{mv}{m+M}\right)^2\right]$

(d) $\sec^{-1}\left[1-\frac{1}{2gL}\left(\frac{mv}{m+M}\right)^2\right]$

24. Two blocks of masses m_1 and m_2 are connected by a light inextensible string passing over a smooth fixed pulley of negligible mass. The acceleration of the centre of mass of the system when blocks move under gravity is

(a) g

(b) $\left(\frac{m_1-m_2}{m_1+m_2}\right)g$

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(c) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$

(d) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^3 g$

25. **A projective is fired at a speed of 100 m/s at an angle of 37^0 above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the smaller coming to rest. The distance from the launching point to the point where the heavier piece lands is**

(a)	920 m	(b)	1020 m
(c)	1120 m	(d)	1220 m

26. **Two 30 kg children, each with the speed of 4 m/s, are sliding on a frictionless frozen pond when they collide and stick together because they have velcro straps on their jackets. The two children then collide and stick to a 75 kg man who was sliding at 2.0 m/s. After this collision, the three person composite is stationary. The angle between the initial velocity vectors of the two children is**

(a)	zero	(b)	30^0
(c)	60^0	(d)	none

27. **A rail road freight car of mass 3.18×10^4 kg collides with a stationary caboose car. They couple together, and 27.0 % of the initial K.E. is transferred to thermal energy, sound, vibrations, and so on. The mass of the caboose is**

(a)	1.18×10^4 kg	(b)	2.18×10^4 kg
(c)	3.18×10^4 kg	(d)	4.18×10^4 kg

28. **Two particles A and B of mass 1 kg and 2 kg respectively are projected in the opposite directions with speed u_A = 200m/s and u_B = 50 m/s. Initially they were 90 m apart. The maximum height attained by the centre of mass of the particles is**

(a)	110.55 m	(b)	115.55 m
(c)	125.55 m	(d)	135.55 m

ANSWERS (SINGLE CORRECT CHOICE TYPE)

1.	b	11.	c	21.	b
2.	c	12.	b	22.	d
3.	c	13.	d	23.	a
4.	a	14.	a	24.	c
5.	b	15.	c	25.	c
6.	b	16.	b	26.	d
7.	d	17.	a	27.	a
8.	b	18.	a	28.	b
9.	b	19.	a		
10.	d	20.	d		

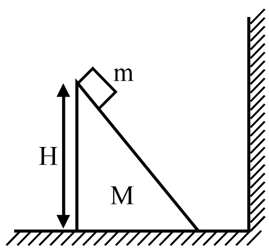
EINSTEIN CLASSES

EXCERCISE BASED ON NEW PATTERN

COMPREHENSION TYPE

Comprehension-1

In the figure, the block of mass m starts from rest at the top of a wedge of mass M. All the surfaces are frictionless. The block slides down on to the ground, moves along it with a speed v, has an elastic collision with the wall, and climbs back onto the wedge.



1. The velocity of cm of the system before the collision is

(a) zero (b) $\frac{mv}{M}$

(c) $\frac{mv}{M+m}$ (d) $\frac{2mv}{M+m}$

2. The velocity of cm of the system after the collision is

(a) zero (b) $\frac{mv}{M}$

(c) $\frac{mv}{M+m}$ (d) $\frac{2mv}{M+m}$

3. If the time of contact is Δt then the force exerted by the block on the wall is

(a) $\frac{mv}{\Delta t}$ (b) $\frac{2mv}{\Delta t}$

(c) infinite (d) zero

4. The height reached by the block onto the wedge after the collision of block with wall is

(a) H

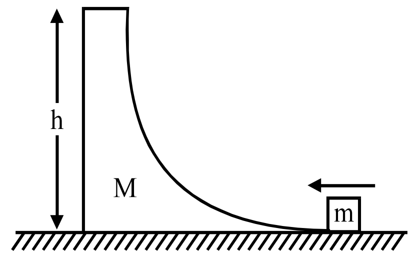
(b) less than H

(c) independent of mass of the block

(d) independent of mass of the wedge

Comprehension-2

Figure shows a small body of mass m placed over a larger mass M whose surface is horizontal near the smaller mass and gradually curves to become vertical. The smaller mass is pushed on the longer one at a speed v and the system is left to itself. Assume that all the surfaces are frictionless.



5. The speed of the larger block when the smaller block is sliding on the vertical part is

(a) $\frac{mv}{M+m}$ (b) $\frac{Mv}{M+m}$

(c) $\frac{mv}{M}$ (d) $\frac{Mv}{m}$

6. The speed of the smaller mass when it breaks off the larger mass at a height h.

(a) $\left(\frac{M^2+Mm+m^2}{(M+m)^2}v^2+2gh\right)^{1/2}$

(b) $\left(\frac{M^2+Mm+m^2}{(M+m)^2}v^2-2gh\right)^{1/2}$

(c) $\left(\frac{M^2+Mm+m^2}{2(M+m)^2}v^2-2gh\right)^{1/2}$

(d) none

7. The maximum height (from the ground) that the smaller mass ascends

(a) $\frac{mv^2}{2g(M+m)}$ (b) $\frac{Mv^2}{g(M+m)}$

(c) $\frac{Mv^2}{2g(M+m)}$ (d) $\frac{mv^2}{g(M+m)}$

8. The path of the small body as observed from the ground when it is in air is

(a) vertical straight line

(b) horizontal straight line

(c) parabolic path

(d) none

EINSTEIN CLASSES

9. The distance transversed by a bigger block during the time when the smaller block was in its flight under gravity is

(a) $\dfrac{2mv[Mv^2 + 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

(b) $\dfrac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

(c) $\dfrac{2Mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

(d) $\dfrac{2Mv[Mv^2 + 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

Comprehension-3

A particle of mass 6m is projected with speed 'u' in the vertical plane at an angle of θ with the horizontal. At the highest point of the trajectory the particle is exploded into three fragments in the mass ratio 1 : 2 : 3. The heaviest fragment retraces the original path and comes to initial point of projection whereas lightest fragment immediately comes to rest after the explosion.

10. The trajectory of centre of mass of three fragments will follow

(a) original parabolic path

(b) new parabolic path

(c) first parabolic path and then straight line

(d) straight line path

11. The time after which heaviest fragments land on the ground is

(a) $\dfrac{u\sin\theta}{2g}$ (b) $\dfrac{u\sin\theta}{g}$

(c) $\dfrac{3u\sin\theta}{2g}$ (d) $\dfrac{2u\sin\theta}{g}$

12. The time after which lightest fragments land on the ground is

(a) $\dfrac{u\sin\theta}{2g}$ (b) $\dfrac{u\sin\theta}{g}$

(c) $\dfrac{3u\sin\theta}{2g}$ (d) $\dfrac{2u\sin\theta}{g}$

13. The time after which second fragments land on the ground is

(a) $\dfrac{u\sin\theta}{2g}$ (b) $\dfrac{u\sin\theta}{g}$

(c) $\dfrac{3u\sin\theta}{2g}$ (d) $\dfrac{2u\sin\theta}{g}$

14. Let the lightest fragment and second fragment lands on the ground at the distance x_1 and x_2 from the point of projection then $x_1 : x_2$ is

(a) 1 : 5 (b) 2 : 11

(c) 3 : 7 (d) 4 : 13

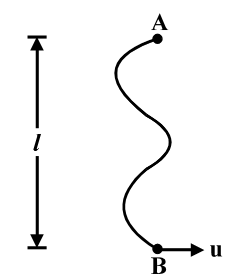
15. The speed of the middle fragment just after the the explosion is

(a) 3ucos θ (b) 3.5ucos θ

(c) 4ucos θ (d) 4.5ucos θ

Comprehension-4

Two particles A and B of equal mass m each are attached by a string of length 2*l* and initially placed over a smooth horizontal table in the position shown in figure. Particle B is projected across the table with speed u perpendicular to AB as shown in figure.



16. The speed of the particle A after the string becomes taut

(a) $\dfrac{\sqrt{3}}{4}u$ (b) $\dfrac{\sqrt{5}}{4}u$

(c) $\dfrac{\sqrt{7}}{4}u$ (d) $\dfrac{\sqrt{11}}{4}u$

17. The speed of the particle B after the string becomes taut

(a) $\dfrac{\sqrt{3}}{4}u$ (b) $\dfrac{\sqrt{5}}{4}u$

(c) $\dfrac{\sqrt{7}}{4}u$ (d) $\dfrac{\sqrt{11}}{4}u$

EINSTEIN CLASSES

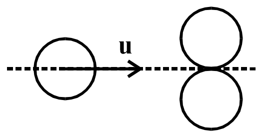
18. The magnitude of impulsive tension is

(a) $\frac{\sqrt{3}}{4}mu$ (b) $\frac{\sqrt{5}}{4}mu$

(c) $\frac{\sqrt{7}}{4}mu$ (d) $\frac{\sqrt{11}}{4}mu$

Comprehension-5

Three identical discs each of mass 'm' are placed on the smooth horizontal table. The first disc has given a velocity u as shown in figure which collides with another discs symetrically and comes to rest after the collision.



19. The impulse suffered by the stationary disc is

(a) mu (b) $\frac{mu}{2}$

(c) $\frac{mu}{\sqrt{3}}$ (d) $\frac{2mu}{\sqrt{3}}$

20. If the time of collision of Δt then the impulsive force acted on the moving disc during the collision is

(a) $\frac{mu}{\Delta t}$ (b) $\frac{mu}{2\Delta t}$

(c) $\frac{mu}{\sqrt{3}\Delta t}$ (d) $\frac{2mu}{\sqrt{3}\Delta t}$

21. The coefficient of restitution of the collision is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Comprehension-6

A sphere collides with a fixed horizontal plane with initial speed u at an angle α with the vertical. The coefficient of restitution for the collision is e. The sphere rebounds with speed v at an angle β with the vertical.

22. The value of 'v' is

(a) $u\sqrt{e^2\cos^2\alpha+\sin^2\alpha}$

(b) $u\sqrt{e^2\sin^2\alpha+\cos^2\alpha}$

(c) $u\sqrt{e^2\tan^2\alpha+\cos^2\alpha}$

(d) $u\sqrt{e^2\tan^2\alpha-\cos^2\alpha}$

23. The value of 'β' is

(a) α (b) $\tan^{-1}\left(\frac{\tan\alpha}{e}\right)$

(c) $\tan^{-1}(e\tan\alpha)$ (d) $\tan^{-1}(e^2\tan\alpha)$

24. The time after which it collides with the plane once again is

(a) $\frac{2u\sin\alpha}{g}$ (b) $\frac{2eu\sin\alpha}{g}$

(c) $\frac{2u\cos\alpha}{g}$ (d) $\frac{2eu\cos\alpha}{g}$

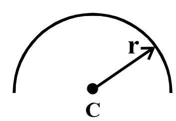
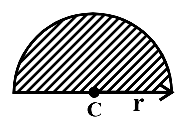
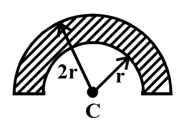
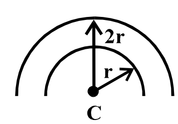
25. The total horizontal distance covered by the ball before coming to rest is

(a) $\frac{eu^2\sin2\alpha}{(1-e)g}$ (b) $\frac{u^2\sin2\alpha}{(1-e)g}$

(c) $\frac{e^2u^2\sin2\alpha}{(1-e^2)g}$ (d) $\frac{e^2u^2\sin2\alpha}{(1+e^2)g}$

MATRIX-MATCH TYPE

Matching-1

Configuration		Centre of mass of the configuration	
(A)		(p)	$\frac{10r}{3\pi}$ above C
(B)		(q)	$\frac{2r}{\pi}$ above C
(C)		(r)	$\frac{4r}{3\pi}$ above C
(D)		(s)	$\frac{28r}{9\pi}$ above C

Matching-2

Two smooth spheres, A and B, of equal radius, lie on a horizontal table. A is of mass 1 kg and B is of mass 3kg. The spheres are projected towards each other with velocity vectors $5\vec{i}+2\vec{j}$ and $2\vec{i}-\vec{j}$ respectively and when they collide the line joining their centres is parallel to the vector $\vec{i}$.

If the coefficient of restitution between A and B is $\frac{1}{3}$.

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	Column - A	Column - B	
(A)	speed of A after the impact in m/sec	(p)	2
(B)	speed of B after the impact in m/sec	(q)	3
(C)	loss of kinetic energy in the impact in Joule	(r)	4
(D)	magnitude of impulse during the collision in SI unit	(s)	5

MULTIPLE CORRECT CHOICE TYPE

1. **Two bodies A and B of masses m and 2m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A. At a certain instant of time after collision it is found that the instantaneous velocities of A and B are same. Further at this instant the compression of the spring is x_0.**

(a) **The common velocity of A and B are $\frac{v_0}{3}$**

(b) **The spring constant of the spring is $\frac{2mv_0^2}{3x_0^2}$**

(c) **Initial velocity of B is v_0**

(d) **The velocity of centre of mass of the system A + B + C is always constant**

2. **A uniform thin rod of mass M and length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at time t = 0 causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling.**

(a) **The path of the centre of mass of the rod during the fall of rod is vertical straight line.**

(b) **The path of any point on the rod other than centre of mass at the distance r from the lower end is ellipse.**

(c) **The centre of mass of a point at the distance $L/4$ from the lower end is circular.**

(d) **The horizontal force on the rod at any moment is zero.**

3. **In a gravity free space, a car is standing at the height 'h' above the floor. The driver of the car throws a ball of mass m with certain speed u. The combined mass of the driver and car is M. Choose the correct statement from the following :**

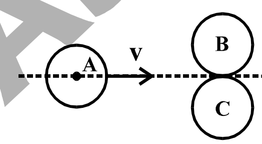
(a) **If the driver throw the ball downward then the car will move upward.**

(b) **The ball reach the ground in time h/u if the ball is thrown downward.**

(c) **The total distance travelled by the car is $\frac{hm}{M}$ upward during the time when the ball reach the ground if ball is projected downward.**

(d) **The centre of mass of the car + ball system does not move.**

4. **Three identical discs A, B and C rests on a smooth horizontal plane. The disc A is set in motion with velocity v along the perpendicular bisector of the line BC joining the cetres of the stationary discs. The distance BC between the centres of the stationary discs B and C is n times the diameter of each disc. The collision is elastic**



Choose the correct statement from the following :

(a) **for n = √2, the disc A will stop after the collision.**

(b) **for n > √2, the disc A will move on in the same direction after the collision.**

(c) **for n < √2, the disc A will recoil after the collision.**

(d) **cannot be predicted as the data is insufficient.**

5. **A ball of mass m approaches a moving wall of infinite mass with speed v along the normal to the wall. If the speed of the wall is u towards the ball then the speed of the ball after the elastic collision with the wall is v_1. If the speed of the wall is u away from the ball then the speed of the ball after the elastic collision with the wall is v_2. Then**

(a) **$v_1 = 2u + v$ away from the wall**

(b) **$v_1 = 2u - v$ away from the wall**

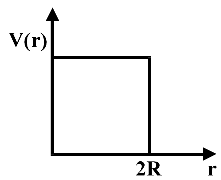
(c) **$v_2 = v - 2u$ away from the wall**

(d) **$v_2 = v + 2u$ away from the wall**

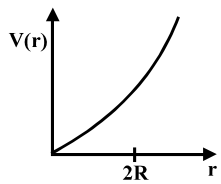
EINSTEIN CLASSES

6. Which of the following potential energy curves cannot possibly describe the elastic collision of two billiards balls ? Here r is the distance between centres of the balls and R is the radius of the ball.

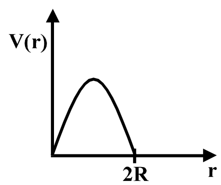
(a)



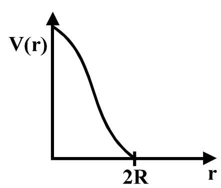
(b)



(c)



(d)



7. A ball hits the floor and rebounds after an inelastic collision. In this case choose the correct alternative

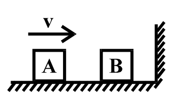
(a) The momentum of the ball just after the collision is the same as that just before the collision.

(b) The kinetic energy of the ball will decrease just after the collision.

(c) The total energy of the ball and the earth is conserved.

(d) The momentum of the ball + earth is conserved.

8. Two blocks as shown in figure are identical, the first moving with speed v towards right and the second staying at rest. The wall at the extreme right is fixed and smooth. Assuming all collisions to be elastic, which of the following statements are correct ?



(a) There are only three collisions

(b) There is no loss of kinetic energy of any block

(c) The speed of balls remain unchanged after allcollisions have taken place

(d) The velocities of balls remain unchanged after allcollisions have taken place

9. When a canon ball is fired from the canon then

(a) Kinetic energy of canon ball is greater than the kinetic energy of canon.

(b) Momentum of canon ball + canon is conserved along the horizontal direction.

(c) Momentum of canon ball will be conserved with respect to canon.

(d) Recoiled speed of canon is less than the canon ball.

10. Choose the correct statement for the centre of mass

(a) The centre of mass depends on the distribution of masses.

(b) The centre of mass depends on the frame of reference.

(c) Internal force does not affect the motion of centre of mass

(d) Incentre of mass frame the momentum of a system is always zero.

Assertion-Reason Type

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

1. STATEMENT-1 : In an elastic collision of two billiard balls, the total kinetic energy is conserved during the short time of collision of the balls when there are in contact.

STATEMENT-2 : Part of the kinetic energy will be converted into potential energy during the collision.

2. STATEMENT-1 : Total linear momentum will be conserved during the short time of an elastic collision of the two balls or inelastic collision of the two balls.

STATEMENT-2 : Linear momentum will be conserved when the net external force acted on the system is zero.

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3. STATEMENT-1 : A car and truck are moving with the same kinetic energy. The truck has the greater momentum.

STATEMENT-2 : A car and truck are moving with the same momentum. The car has the greater kinetic energy.

4. STATEMENT-1 : Kinetic energy of a system can be changed without changing its momentum.

STATEMENT-2 : Momentum of the system can be changed without changing its kinetic energy.

5. STATEMENT-1 : A meteorite burns in the atmosphere before it reaches earth surface. The momentum of the meteorite is transfered to air molecules.

STATEMENT-2 : The momentum of the meteorite + air molecules always remains same.

6. STATEMENT-1 : Three particles A, B and C of equal masses move with equal speed v along the medians of an equilateral triangle. They collide at the centroid G of the triangle. After the collision A comes to rest, while B retraces its path with the speed v and the particle C after the collision will move with the vleocity of B before collision.

STATEMENT-2 : The momentum of the three particle system is always conserved.

7. STATEMENT-1 : Centre of mass and centre of gravity of a body are at the same point.

STATEMENT-2 : Centre of gravity is the point at which the total weight of the body is concentrated.

8. STATEMENT-1 : Consider a very high rise building with uniform mass distribution along the height. The centre of gravity will be in the lower half of the building whereas the centre of mass will be at the mid-point of the building.

STATEMENT-2 : The centre of gravity and centre of mass of the body will coincide in uniform gravitational field.

9. STATEMENT-1 : Two particles of masses 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2 m/s, their centres of mass has a velocity of 0.5 m/s. If the relative velocity approach will change then the velocity of centre of mass will also change.

STATEMENT-2 : In the absence of external force on the system, the velocity of centre of mass remains constant.

10. STATEMENT-1 : When a ball is thrown up the magnitude of its momentum decreases and then increases.

STATEMENT-2 : This is the violation of conservation of linear momentum.

EINSTEIN CLASSES

(Answers) EXCERCISE BASED ON NEW PATTERN

COMPREHENSION TYPE

1.	a	2.	d	3.	b	4.	b	5.	a	6.	b
7.	c	8.	c	9.	b	10.	a	11.	b	12.	b
13.	b	14.	b	15.	d	16.	a	17.	c	18.	a
19.	c	20.	a	21.	b	22.	a	23.	b	24.	d
25.	a										

MATRIX-MATCH TYPE

1. [A-q, B-r, C-s, D-p] 2. [A-p, B-q, C-q, D-q]

MULTIPLE CORRECT CHOICE TYPE

1.	a, b, c, d	2.	a, b, c, d	3.	a, b, c, d	4.	a, b, c
5.	a, c	6.	a, b, c	7.	b, c, d	8.	a, b, c
9.	a, b, d	10.	a, c, d				

ASSERTION-REASON TYPE

1.	D	2.	A	3.	B	4.	B	5.	A	6.	A
7.	D	8.	A	9.	D	10.	C				

INITIAL STEP EXERCISE (SUBJECTIVE)

1. A particle (m = 0.5 kg) lies at the origin and another particle m(1.5 kg) lies at the point (1, 2)m in the xy plane. At t = 0 a force $\vec{F}_1 = (2\hat{i} + 3\hat{j})N$ being to act on the smaller particle and a force $\vec{F}_2 = (-3\hat{i} - 2\hat{j})N$ begins to act on the bigger particle. In unit vector, find the position of cm of the both particle system at t = 4s ?

2. A moving particle of mass m makes a head on elastic collision with a particle of mass 2m which is initially at rest. Prove that the colliding particle losses $\frac{8}{9}$ th of its energy after collision.

3. A ball of mass m_1 moving with a speed u_1 collides directly with another ball of mass m_2 moving with speed u_2 ($u_1 > u_2$). The coefficient of restitution of collision is e. Find the velocities of the balls after impact. Discuss the following cases for elastic collision : (a) $m_1 = m_2$ and $u_2 = 0$ (b) $m_1 = m_2$ (c) $m_2 >> m_1$ and $u_2 = 0$ (d) $m_2 >> m_1$.

4. A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10.0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the friction coefficient between the block and the surface ?

5. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a perfectly elastic collision. Where will it hit the plane again ?

6. Two blocks of masses m_1 and m_2 are connected by a spring of spring constant k. The block of mass m_2 is given a sharp impulse so that it acquires a velocity v_0 towards right. Find (a) the velocity of cm (b) the maximum elongation that the spring will suffer.

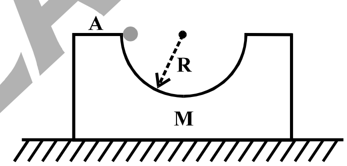
7. A stone is dropped at t = 0. A second stone, with twice the mass of the first, is dropped from the same point at t = 100 ms (a) How far below the release point is the cm of the two stones at t = 300 ms ? (b) How fast is the cm of the two stone system moving at that time ?

8. A particle of mass m_1 experienced a perfectly elastic collision with a stationary particle of mass m_2. What fraction of the K.E. does the striking particle lose, if (a) it recoils at right angles to its original motion direction (b) the collision is a head-on one ?

9. A closed system consists of two particles of masses m_1 and m_2 which move at right angles to each other with velocities v_1 and v_2. Find (a) the momentum of each particle (b) the total K.E of two particles in the reference frame fixed to the center of inertia.

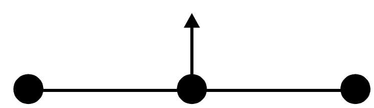
10. A block of mass m rests on a wedge with angle of inclination θ of mass M which in turn rests on a horizontal table. All the surfaces are smooth and the system is at rest initially. Find the velocity of the wedge at the instant when block after sliding on the wedge touches the table. Express your answer in terms of m, M, h and θ.

11. A block of mass M with a semicircular track of radius R rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A. The cylinder slips in the semicircular frictionless track.



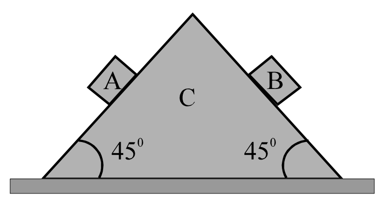
(a) How far the block moved when the cylinder reaches the bottom (point B) of the track ? (b) How fast is the block moving when the cylinder reaches the bottom of the track ?

12. Three identical balls each of mass m = 0.5 kg are connected with each other as shown in figure and rest over a smooth horizontal table. At t = 0, ball B is imparted a velocity v_0 = 9m/s.



Find the velocity of A when it collides with ball C.

13. Two blocks A and B of equal mass are released on two sides of a fixed wedge C as shown in figure.



Find the acceleration of centre of mass of blocks A and B. Neglect friction.

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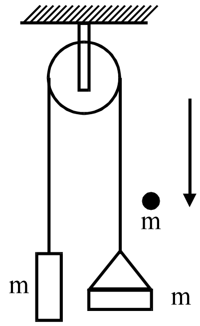
14. **A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown. Initially the system is at rest when a particle of mass m falls on the pan and stricks to it.**



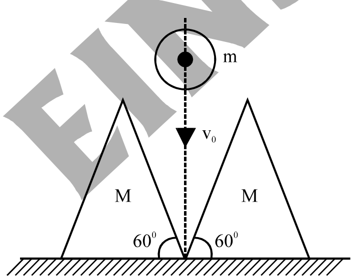
If the particle strikes the pan with speed v find the speed with which the system moves just after the collision.

15. **Two wedges, each of inclination 45^0 and mass M, lie on a smooth horizontal plane. A ball of mass m is dropped from a height. After falling through a distance H it strikes one wedge and then the other and finally bounces vertically upwards. If both the impacts are elastic then find the height to which the ball bounces.**

FINAL STEP EXERCISE (SUBJECTIVE)

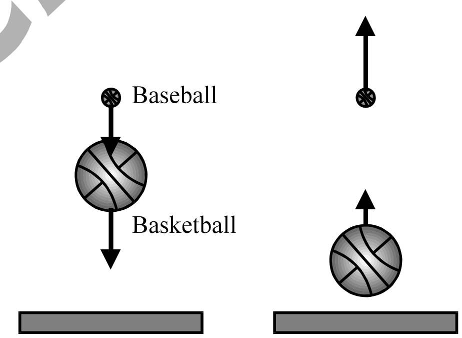
1. **A projectile is thrown on to a vertical wall at a certain angle with velocity u. Prove that in order that the projectile may return to the same point after hitting the wall, the distance of the wall must be less than $\left(\frac{e}{1+e}\frac{u^2}{g}\right)$ where e is the coefficient of restitution.**

2. **Two identical equilateral triangular wedges of mass M rest on a smooth horizontal surface. A smooth sphere of mass m moving vertically down with a velocity v_0 strikes the wedge symmetrically.**



If the coefficient of restitution is e find the velocities of the sphere and that of the wedge just after collision.

3. **A small ball of mass m is aligned above a larger, ball of mass M (with a slight separation, as in figure), and the two are dropped simultaneously from height h. (Assume radius of each ball is negligible compared to h).**



(a) If the larger ball rebounds elastically from the floor and then the small ball rebounds elastically from the larger ball, what ratio $\frac{m}{M}$ results in the larger ball stopping upon its collision with the small ball ? (b) what height does the small ball then reach ?

4. **Two blocks of mass 2kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m . The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block m after the collision moves 0.5 m up and comes to rest. Find the coefficient of restitution between the blocks and the mass of the block M. [Take sin θ $\cong$ tanθ = 0.05, g = $10 m/s^2$].**

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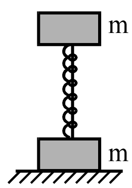
5. Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in figure (a) find the position of the cm at time t (b) if the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.

6. A uniform chain of mass M and length L is held vertically in such a way that its lower end just touches the horizontal floor. The chain is released from rest in this position. Any portion that strikes the floor comes to rest. Assume that the chain does not form a heap on the floor, Find the force exerted by it on the floor when a length x has reached the floor.

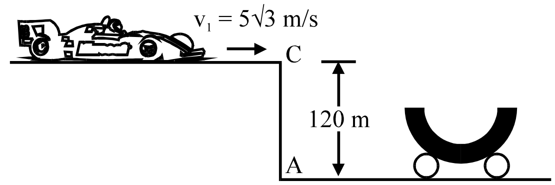
7. A system consists of two identical cubes, each of mass m, linked together by the compressed massless spring of stiffness k. The cubes are also connected by a thread which is burned through at a certain moment.



Find (a) at what values of Δl, the initial compression of the spring, the lower cube will bounce up after the thread has been burnt. (b) to what height h the c.g. of this system will rise if the initial compression of the spring $\Delta l = \dfrac{7mg}{k}$.

8. Two identical buggies 1 and 2 with one man in each move without friction due to inertia along the parallel rails toward each other. When the buggies get opposite each other, the men exchange their places by jumping in the direction perpendicular to the motion direction. As a consequence, buggy 1 stops and buggy 2 keeps moving in the same direction, with its velocity becoming equal to v. Find the initial velocity of the buggies $\vec{v}_1$ and $\vec{v}_2$ if the mass of each buggy (without a man) equals M and the mass of each man m.

9. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30^0 with the horizontal. The first cannot ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannot ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second cannot ball also hits and sticks to the carriage.

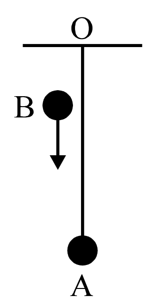


What will be the horizontal velocity of the carriage just after the second impact ? Take $g = 10\ m/s^2$.

10. A cylindrical solid of mass 10^{-2} kg and cross-sectional area $10^{-4}\ m^2$ is moving parallel to its axis (the X-axis) with a uniform speed of 10^3 m/s in the positive direction. At t = 0, its front face passes the plane x = 0. The region to the right of this plane is filled with stationary dust particles of uniform density $10^{-3}\ kg/m^3$. When a dust particle collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remains practically unchanged, and that the dust sticks only to the front face of the cylinder, find the x-coordinate of the front of the cylinder at t = 150 s.

11. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position and released. The ball hits the wall, the coefficient of restitution being $\dfrac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillation becomes less than 60 degress ?

12. A small steel ball A is suspended by an inextensible thread of length l = 1.5 m from O. Another identical ball is thrown vertically downwards such that its surface remains just in contact with thread during downward motion and collides elastically with the suspended ball. If the suspended ball just completes vertical circle after collision,



calculate the velocity of the falling ball just before collison. ($g = 10\ ms^{-2}$).

13. A cart is moving along +x direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity

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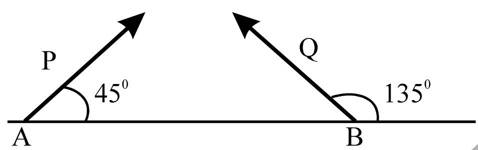
of 6m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30^0 with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine

(i) the speed of the combined mass immediately after the collision with respect to an observer on the ground.

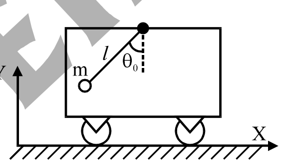
(ii) the length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

14. Particles P and Q of mass 20 gm and 40 gm respectively are simultaneously projected from point A and B on the ground. The initial velocities of P and Q make 45^0 and 135^0 angles respectively with the horizontal AB as shown in the fig. Each particle has an initial speed of 49 m. The separation AB is 249 m. Both particles travel in the same vertical plane and undergo a collision. After collision P retraces its path.



Determine the position of Q when it hits the ground. How much time after the collision does the particle Q take to reach the ground ? (Take g = 9.8 m/s²)

15. A wagon with a mass M can move without friction along horizontal rails. A mathematical pendulum (a sphere with mass m suspended from a long string of length *l*) is fastened on the wagon. At the initial moment the wagon and the pendulum are at rest and the string was deflected through an angle θ_0 from the verticle.



What will the velocity of the wagon be when the pendulum string forms an angle θ ($\theta < \theta_0$) with the vertical.

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ANSWERS SUBJECTIVE (INITIAL STEP EXERCISE)

1. $(-3.25\,\hat{i} + 5.5\,\hat{j})$

3. $v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1+m_2}u_2, v_2 = \frac{(1+e)m_1}{m_1+m_2}u_1 + \frac{m_2 - em_1}{m_1+m_2}u_2$

   (a) $v_1 = 0, v_2 = u_1$ **(b)** $v_1 = u_2, v_2 = u_1$ **(c)** $v_1 = -u_1, v_2 = 0$

   (d) $v_1 = -u_1 + 2u_2, v_2 = -u_2$

4. 0.16

5. $8h \sin\theta$

6. (a) $\frac{m_2 v_0}{m_1 + m_2}$ (b) $v_0\sqrt{\frac{m_1 m_2}{(m_1+m_2)k}}$

7. (a) 28 cm (b) 2.3 m/s.

8. (a) $\frac{2m_1}{m_1+m_2}$ (b) $\frac{4m_1m_2}{(m_1+m_2)^2}$

9. (a) $\mu\sqrt{v_1^2 + v_2^2}$ (b) $\frac{1}{2}\mu\left(v_1^2 + v_2^2\right); \mu = \frac{m_1m_2}{m_1+m_2}$

10. $\left[\frac{2m^2 g\cos^2\theta h}{(M+m)(M+m\sin^2\theta)}\right]^{1/2}$

11. (a) $\frac{m(R-r)}{m+M}$ (b) $m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

12. 6 m/s.

13. $\frac{1}{2}g$ downwards.

14. $\frac{v}{3}$

15. $\frac{\mathbf{M-m}}{\mathbf{M+m}}\mathbf{H}$

ANSWERS SUBJECTIVE (FINAL STEP EXERCISE)

2. $\frac{\sqrt{3}(1+e)mv_0}{2M+3m}, \frac{(2eM-3m)v_0}{2M+3m}$

3. (a) $\frac{1}{3}$ (b) 4h

4. 0.84, 15 kg

5. $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right),\ x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$

6. $\frac{3Mgx}{L}$

7. (a) $\Delta l > \frac{3mg}{k}$, (b) $\frac{8mg}{k}$

8. $\frac{-mv}{M-m}, \frac{mv}{M-m}$

9. $t_0 = 12s$

10. $x = 10^5$ m

11. 4

12. 12.5 ms^{-1}

13. (i) 2.5 m/s (ii) 0.3188 m

14. 3.536 s

15. $v^2 = \frac{2m^2gl}{(m+M)}\frac{(\cos\theta - \cos\theta_0)\cos^2\theta}{(M+m\sin^2\theta)}$

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