WORK, ENERGY, POWER AND CURVELINEAR MOTION

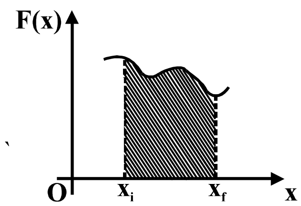
C1A Work

When a force acts on a body of particle that moves, the force can do work on the body. Mathematically, the work W_{ba} done by a force F on the particle as the particle moves from a to b is defined as $\mathbf{W_{ba}} = \int_a^b \vec{\mathbf{F}}.\mathbf{d}\vec{\mathbf{r}}$.

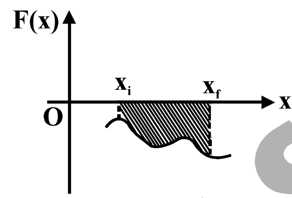
Hence the work done by a force is defined as the dot product of the force and the distanplacement of the point of application of force. The work done by a force can be positive, negative or zero. Work depends on frame of reference.

C1B Graphical Interpretation Of Work Done

Graphically, the work done by a variable force F(x) from an initial position x_i to final position x_f is interpreted as the area under the force-displacement curve, shown in figure



(A) Positive Work



(B) Negative Work

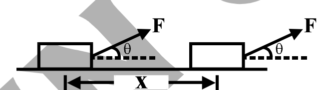
C1C Work : For Motion Along Straight Line

Case I Constant Force

Consider a body that moves along the x-axis under a constant force F then work W done by the force W =

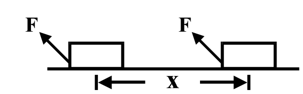
$\mathbf{W} = \vec{\mathbf{F}}.\vec{\mathbf{x}}$ = Fx cos θ

→ If $\theta < \frac{\pi}{2}$, W is positive



→ If $\theta = \frac{\pi}{2}$, W = 0

→ If $\theta > \frac{\pi}{2}$, W is negative



→ If force is in direction of displacement i.e. θ = 0, W = Fx

→ If force is in opposite to displacement i.e. θ = π, W = –Fx

Work Done By Several Constant Forces

Choose the initial and final positions of the body, and draw a free body diagram showing all the forces that act on the body. List the forces and calculate the work done by each force. Add the amounts of work done by the separate forces to find the total work done.

Case II Work Done by Variable Force

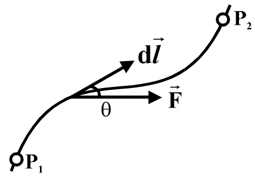
Consider a body moves along the x-axis from position x_1 to x_2 under a variable force F. Then work done by this force is $\mathbf{W} = \int_{x_1}^{x_2} \vec{\mathbf{F}}.\mathbf{d}\vec{\mathbf{x}}$.

Work done by Spring Force

The spring force is a variable force and work done by spring force to extend or compress the spring from x_1 and x_2 is given by $\mathbf{W = -\frac{1}{2}k(x_2^2 - x_1^2)}$. It does not mean that work done by spring force is always negative.

C1D Work : for Motion along Curved Path

We can generalize our definition of work further to include a force that varies in direction as well as in magnitude and a displacement that lies along a curved path. Suppose a particle moves from point P_1 to P_2 along a curve, as shown in figure :



We divide the portion of the curve between these points into many infinitesimal vector displacements $\mathbf{d\vec{l}}$. Each $\mathbf{d\vec{l}}$ is tangent to the path at its position. Let $\vec{\mathbf{F}}$ be the force at a typical point along the path, and let θ be the angle between $\vec{\mathbf{F}}$ and $\mathbf{d\vec{l}}$ at this point.

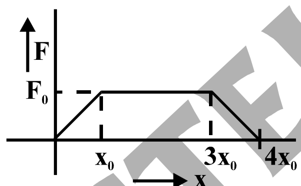
Then $\quad \mathbf{dW = \vec{F}.d\vec{l} = F\cos\theta\, dl}$

$$\Rightarrow \qquad \mathbf{W = \int_{P_1}^{P_2} \vec{F}.d\vec{l}}$$

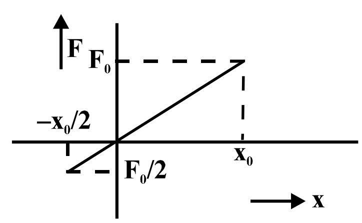
<u>Practice Problems :</u>

1. Find the work done in the following cases :

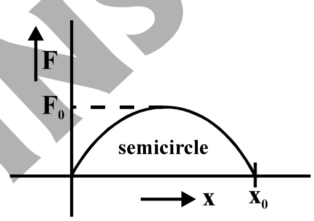
(a)



(b)



(c)



2. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he moves a distance of 40 m with an accleration of 0.5 m/s^2. Calculate work done by the man on the block during motion.

3. A man weighing 55 kg supports a body of 20 kg on his head. Calculate work done by him if he moves a distance of 20 m (i) on a horizontal road, (ii) upon incline of 1 in 5. Take g = 10 ms^{-2}.

4. An elastic string of unstretched length L and force constant k is stretched by a small length x. It is further stretched by another small length y. The work done in the second stretching is

(a) $\frac{1}{2}ky^2$ **(b)** $\frac{1}{2}k(x^2+y^2)$ **(c)** $\frac{1}{2}k(x+y)^2$ **(d)** $\frac{1}{2}ky(2x+y)$

5. A cord is used to lower vertically a block of mass M a distance d at a constant downward acceleration of g/4. Then the work done by the cord on the block is

(a) $\frac{Mgd}{4}$ **(b)** $\frac{-Mgd}{4}$ **(c)** $\frac{3Mgd}{4}$ **(d)** $\frac{-3Mgd}{4}$

6. A tracter pulls a block, of weight 14, 700 N, a distance of 20 m along the level road. The tractor exerts a constant 5000 N force at an angle of 37^0 above the horizontal. There is a 3500 N friction force opposing the motion. The total work done by all the forces

(a) **10 kJ** **(b)** **20 kJ** **(c)** **30 kJ** **(d)** **40 kJ**

[Answers : (2) 40 J (3) Zero; 3000 J (4) d (5) d (6) a]

C2 Conservative Forces

A force is conservative force if the net work it does on a particle moving around every closed path, from an initial point and then back to that point, is zero, Equivalently, it is conservative if the net work it does on a particle moving between two points does not depend on the path taken by the particle. The gravitational force and the spring force are conservatives; the kinetic frictional force is a non-conservative force. Any dissipative force is non- conservative in nature.

C3 Kinetic Energy

Kinetic energy is a scalar quantity associated with the motion of the particle. Mathematically kinetic energy of a body or particle of mass m having speed v is given by $\frac{1}{2}mv^2$. Kinetic energy depends on frame of reference.

C4 Work Energy Theorem

Work is associated with kinetic energy. In any displacement of a particle, the change in its kinetic energy equals the total work done by all the forces acting on the particle.

$$W = K_f - K_i = \Delta K$$

This result is the work - energy theorem.

→ Work energy theorem is valid even when force varies during the displacement and for all types of motions.

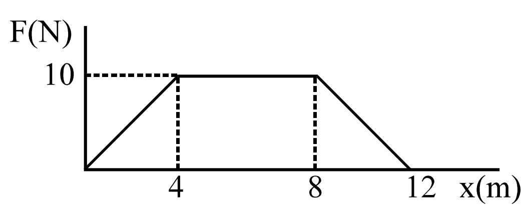
→ Work energy theorem is valid for all types of force acting on the body i.e. for conservative and non-conservative forces.

Practice Problems :

1. The displacement x of a particle of mass m kg moving in one dimension, under the action of a force, is related to the time t by the equation $t = \sqrt{x} + 3$ where x is in metres and t is in seconds. The work done by the force in the first six seconds in joules is

(a) **0** **(b)** **3 m** **(c)** **6 m** **(d)** **9 m**

2. A particle of mass 0.1 kg (moving along x-axis) is subjected to a force, which varies with distance (x) as shown in figure. It starts journey from rest at x = 0, its speed at x = 12 m is



(a) **20 m/s** **(b)** **40 m/s** **(c)** **60 m/s** **(d)** **80 m/s**

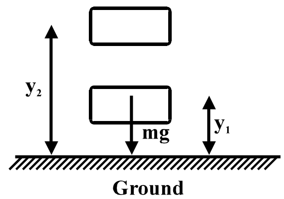
[Answers : (1) a (2) b]

C5 Potential Energy

Potential energy is energy associated with the position of a system rather than its motion. Potential energy function is defined only for conservative forces. The work done on a body by conservative forces can be expressed as a change in potential energy i.e. $dW_c = -dU$.

Gravitational Potential Energy

Work done by gravitational force during the vertical motion of a body from an initial position y_1 to final position y_2 is given $\mathbf{W_c = -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)}$



As $dW_c = -dU \Rightarrow U_f - U_i = mgy_2 - mgy_1$.

Above expression shows that we can express work done by gravitational force in terms of the values of the quantity mgy at the beginning and end of the displacement. This quantity, the product of the weight mg and the position y above the reference level, is called the gravitational potential energy i.e. U = mgy.

Remember that to define the gravitational potential energy we need a reference level. Above the reference level the gravitational potential energy is positive, on the reference level the gravitational potential energy is zero whereas below the reference level the gravitational potential energy is taken as negative.

Spring Potential Energy

When a spring is elongated (or compressed) work is done against the spring force. This work done is stored in the spring as spring or elastic potential energy.

Work done in stretching or compression a spring from x_1 to x_2 is given by $-\frac{1}{2}k(x_2^2 - x_1^2)$

Hence, change in spring potential energy = $\frac{1}{2}k(x_2^2 - x_1^2)$.

The spring potential energy for a spring extended by amount 'x' or compressed by amount 'x' is $\frac{1}{2}kx^2$.

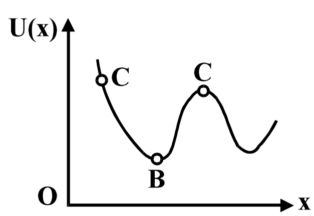
C6 Conservative Force And Potential Energy Function

A conservative force can be derived from a scalar potential energy function. The slope of the potential energy function is a measure of the magnitude of conservative force.

In one dimension, it is defined as the negative derivative of potential energy with respect to distance i.e.,

$$F_c = -\frac{dU}{dx}$$

The figure shows a graph of potential energy function U(x) for one dimensional motion. Three specific points A, B and C are chosen.



At point A, a very strong force is acting because the slope is very large.

At B and C, no force acts as the slope is zero, these two are the equilibrium positions. At equilibrium, $\frac{dU}{dx} = 0$.

The point B is the position of stable equilibrium, as U is minimum and $\frac{d^2U}{dx^2} > 0$

The point C is the position of unstable equilibrium, as U is maximum and $\frac{d^2U}{dx^2} < 0$

C7 Mechanical Energy

The mechanical energy E_{mec} of a system is the sum of its kinetic K and its potential energy U :

$$E_{mec} = K + U$$

As isolated system is one in which no external force causes energy changes. If only conservatives force do work within an isolated system, then the mechanical energy E_{mec} of the system cannot change. This **principle of conservation of mechanical energy** is written as

$$K_2 + U_2 = K_1 + U_1,$$

in which the subscripts refer to different instants during an energy transfer process. This conservation principle can also be written as

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

<u>**Practice Problems :**</u>

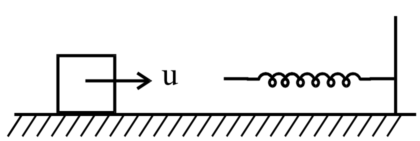
1. **The potential energy of a 2 kg particle free to move along the x-axis is given by $U(x) = \left(\frac{x}{b}\right)^4 - 5\left(\frac{x}{b}\right)^2$ J where b = 1 m. Plot this potential, identifying the extremum points. Identify the regions where particle may be found and its maximum speed given that the total mechanical energy is (i) 36 J; (ii) –4 J. Also find the force acting on the particle as a function of x.**

2. **A block of mass 5.0 kg is suspended from the end of a vertical spring which is stretched by 10 cm under the load of the block. The block is given a sharp impulse from below so that it acquires an upward speed of 2.0 m/s. How high will it rise ? Take g = 10 m/s².**

3. **A uniform chain of length L and mass M is lying on a smooth table and 1/n of its length is hanging vertically down over the edge of the table. The work required to pull the hanging part on the table is**

   (a) MgL (b) $\frac{MgL}{n}$ (c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

4. **An ideal spring of force constant k is attached to a vertical wall as shown in figure. A block of mass m is projected with speed u towards the spring. The horizontal surface is smooth. The maximum compression in the spring is**



   (a) $2u\sqrt{\frac{m}{k}}$ (b) $u\sqrt{\frac{m}{k}}$ (c) $\frac{u}{2}\sqrt{\frac{m}{k}}$ (d) $\frac{u}{4}\sqrt{\frac{m}{k}}$

5. **In the above problem, the speed of the block when it compress the spring by an amount half of the maximum compression is**

   (a) u/2 (b) u/4 (c) u/√2 (d) none of these

6. An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstreched. Then the maximum extension in the spring is

(a) $\frac{4Mg}{k}$ **(b)** $\frac{2Mg}{k}$ **(c)** $\frac{Mg}{k}$ **(d)** $\frac{Mg}{2k}$

[Answers : (1) (a) –3m < x < 3m, v_{max} = 5.45 m/s (b) –2m < x < –1m, 1m < x < 2m, v_{max} = 1.5 m/s (2) 20 cm (3) d (4) b (5) d (6) b]

C8 Power

The time rate of doing work is defined as power and is given by

$$(\text{Power})_{\text{instant}} = \frac{dW}{dt} \text{ and } (\text{Power})_{\text{av.}} = \frac{\Delta W}{\Delta t}$$

From $P = \frac{dW}{dt}$, we can write P = $\vec{F}.\vec{v}$

<u>**Practice Problems :**</u>

1. The human heart discharges 75 ml of blood at each beat against a pressure of 0.1 m of Hg. Calculate the power of the heart assuming that the pulse frequency is 80 beats per minute. Given, density of mercury = 13.6 × 10^3 kg/m^3.

2. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain 'n' times water from the same pipe in the same time by what amount (a) the force and (b) power of the motor should be increased.

3. A particle is projected with speed u at an angle θ with the horizontal in a vertical plane. The instantaneous power of the particle at the instant when it at the highest point of its trajectory is

(a) 0 **(b)** mgu cosθ **(c)** mgu sinθ **(d)** mgu tanθ

4. In the above problem, the average power during the time from point of projection to the instant when the particle is at the highest point of its trajectory is

(a) 0 **(b)** –½mgu cosθ **(c)** –½mgu sinθ **(d)** –½mgu tanθ

5. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(a) $t^{1/2}$ **(b)** $t^{3/4}$ **(c)** $t^{3/2}$ **(d)** t^2

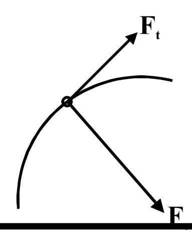
6. A pump can take out 36000 kg of water per hour from a 100 m deep well. It has efficiency of 50%, its power is (g = 10 m/s^2).

(a) 5 kW **(b)** 10 kW **(c)** 15 kW **(d)** 20 kW

[Answers : (1) 1.33 watt (2) (a) n^2 (b) n^3 (3) a (4) c (5) c (6) d]

C9A Curvelinear Motion

The velocity of a particle traversing curved path, can change both in mgnitude as well as in direction. Remember that the velocity vectos is always tangential to the path. The force which is responsible to change the magnitude of velocity vector i.e. speed is known as tangential force (F_t) where the force which is responsible to change the direction of velocity vector is known as centripetal force (F_c).Centripetal force is also known as normal force or radial force. Tangential force acts along the tangent whereas the centripetal force directed towards the centre as shown in figure.



According to Newton's second law,

the net force along the tangent $\sum F_t = ma_t = m\frac{dv}{dt}$

the net force along the normal $\sum F_c = ma_c = \frac{mv^2}{R}$

Here a_t : tangential acceleration

a_c : centripetal acceleration

V : speed of the particle

R : radius of curvature of the path

Hence the net force acting on the particle

$F = \sqrt{F_t^2 + F_c^2} = m\sqrt{a_t^2 + a_c^2} = ma$ where $a = \sqrt{a_t^2 + a_c^2}$ is the net acceleration. Circular motion and projectile motion are the examples of curvelinear motion.

C9B Uniform Circular Motion

When a particle moves in a circle with constant speed, the motion is called uniform circular motion. There is no tangential acceleration i.e.,

$$a_t = \frac{dv}{dt} = 0 \text{ but } a_c = \frac{v^2}{R}$$

In uniform circular motion the acceleration is perpendicular to the velocity at each instant; as the direction of the velocity changes, the direction of this acceleration also changes. The centripetal acceleration (a_c) at each point in the circular path is directed toward the centre of the circle. We can also express the magnitude of the acceleration in uniform circular motion in terms of the period T of the motion, the time for one revolution

$$T = \frac{2\pi R}{v} \Rightarrow a_c = \frac{4\pi^2 R}{T^2}$$

If the speed varies, we call the motion is non-uniform circular motion. In this case

$$a_t = \frac{dv}{dt} \neq 0 \text{ and } a_c = \frac{v^2}{R}$$

Cetrifugal force is a pseudo force, which is experienced by a non-inertial observer moving in a circular path with constant speed. Its magnitude is equal to that of the centripetal force but its direction is exactly opposite to that.

<u>**Practice Problems :**</u>

1. Find the angle through which a cyclist bends when he covers a circular path 34.3 m long in √22 sec. Given g = 9.8 ms^{-2}.

2. A body of mass 0.5 kg is whirled in a vertical circle by a string 1 m long. Calculate (i) minimum speed it must have at the bottom of the circle so that the string may not slack when the body reaches the top. (ii) In that case, will the velocity at the top of the circle be zero ?

3. A stone of mass 0.3 kg tied to the end of a string in a horizontal plane is whirled around in a circle of radius 1 m with a speed of 40 rev./min. What is the tension in the string ? What is the maximum speed with which the stone can be whirled around if the string can withstand the maximum tension of 200 N ?

4. A boy is sitting on the horizontal platform of a joy wheel at a distance of 5 m from the centre. The joy wheel begins to rotate and when the angular speed exceeds 10 r.p.m., the boy just slips. What is the coefficient of friction between the boy and the platform ?

5. A bucket containing water is tied to one end of a rope 2.45 m long and rotated about the other end in a vertical circle. Find the minimum velocity at the highest and lowest points in order that water in the bucket may not spil.

6. **A motor cyclist loops the loop whose diameter is 8 m. From what minimum height must be start in order to roll down and go around the loop ? If the cyclist has the mass of 60 kg then find the speed, force exerted by the loop, and net force acting on the cyclist at the following points :**

   (i) at the lowermost point

   (ii) at the highest points

7. **A simple pendulum of length l has a bob of mass m. It is displaced through an angle of θ from the vertical and then released. Choose the incorrect option**

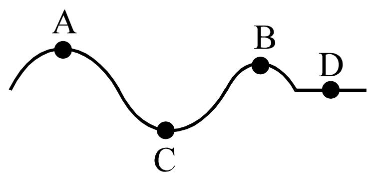
   (a) The speed of the bob at the lowest most point is $\sqrt{2gl(1-\cos\theta)}$

   (b) Tension in the string at the lowest most point is $(3 - 2\cos\theta)mg$

   (c) Tension in the string at the point where it is released is 0

   (d) The centripetal force at the point where it is released is zero.

8. **A car moves at a constant speed on a road as shown in figure. The normal force by the road on the car is N_A, N_B, N_C and N_D when it is at the points A, B, C and D respectively.**



   (a) $N_A = N_B = N_C = N_D$ (b) $N_A > N_B > N_C > N_D$

   (c) $N_A < N_B < N_C < N_D$ (d) $N_C > N_D > N_A > N_B$

9. **A simple pendulum has a string of length l and bob of mass m. When the bob is at its lowest position, it is given the minimum horizontal speed necessary for it to move in a circular path about the point of suspension. The tension in the string at the lowest position of the bob is**

   (a) 3 mg (b) 4 mg (c) 5 mg (d) 6 mg

10. **The speed at which a car can run round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4, is**

   (a) 5 m/s (b) 10 m/s (c) 20 m/s (d) both (a) and (b)

11. **Keeping the banking angle same, to increase the maximum speed with which a vehicle can travel on a circular road by 10%, the radius of curvature of the road has to be changed from 20 m to**

   (a) 16 m (b) 18 m (c) 24.2 m (d) 30.5 m

12. **A particle is moving along a circular path with a power, directly proportional to time. The tangential force acting on the particle is directly proportional to t^n. Then the value of n is**

   (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

13. **A particle is projected with velocity u at an angle θ with the horizontal in a gravitational field of intensity E (this is also called the acceleration due to gravity). The field intensity is vertical and acting downward. The time after which the tangential acceleration of the particle becomes zero is**

   (a) $\frac{2u\sin\theta}{E}$ (b) $\frac{3u\sin\theta}{2E}$ (c) $\frac{u\sin\theta}{3E}$ (d) $\frac{u\sin\theta}{E}$

14. **A particle is projected with speed u at an angle θ with the horizontal in vertical plane. The radius of curvature of the path traversed by the particle at the highest point of its trajectory is**

   (a) $\frac{u^2\cos^2\theta}{g}$ (b) $\frac{u^2\cos^2\theta}{2g}$ (c) $\frac{u^2\cos^2\theta}{3g}$ (d) $\frac{u^2\cos^2\theta}{4g}$

[Answers : (1) 45^0 (2) (i) 7 ms^{-1} (ii) 3.1 ms^{-1} (3) 200 N, 25.8 ms^{-1} (4) 0.56 (5) 10.78 ms^{-1} (6) 10 m (7) c (8) d (9) d (10) d (11) c (12) a (13) d (14) a]

SINGLE CORRECT CHOICE TYPE

1. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical.

(a) $\theta = \cos^{-1}(1/3)$ (b) $\theta = 60^0$

(c) $\theta = \cos^{-1}(2/3)$ (d) $\theta = 0$

2. A particle of mass m is fixed to one end of a light rigid rod of length l and rotated in a vertical circular path about its other end. The minimum speed of the particle at its lowest point must be

(a) zero (b) $\sqrt{gl}$

(b) $\sqrt{5gl}$ (d) $2\sqrt{gl}$

3. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a is varrying with time as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(a) $2\pi mk^2r^2t$ (b) mk^2r^2t

(c) $\frac{1}{3}mk^4r^2t^5$ (d) 0

4. Under the action of a force, a 2 kg body moves such that the position x as a function of time t is given by $x = 2t^3$, where x is in metres and t is in seconds. The work done by the force in the first three seconds is

(a) 144 J (b) 720 J

(c) 1660 J (d) none of these

5. A block of mass m is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. The friction coefficient between the block and the surface is μ. If the block travels at a uniform speed, the work done by this applied force during a displacement d of the block is

(a) $\frac{\mu Mgd\cos\theta}{\cos\theta + \mu\sin\theta}$

(b) $\frac{\mu Mgd\sin\theta}{\cos\theta + \mu\sin\theta}$

(c) $\frac{\mu Mgd\sin\theta}{\sin\theta + \mu\cos\theta}$

(d) $\frac{\mu Mgd\cos\theta}{\sin\theta + \mu\cos\theta}$

6. A force $\vec{F} = -k(y\hat{i} + x\hat{j})$, where k is a positive constant, acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0), and then parallel to the y-axis to the point (a, a). The total work done by the force on the particle is

(a) 0 (b) ka^2

(c) $-ka^2$ (d) $-2ka^2$

7. The speed v reached by a car of mass m that is driven with constant power P is given by

(a) $\left(\frac{3xP}{m}\right)^{1/3}$ (b) $\left(\frac{2xP}{m}\right)^{1/3}$

(c) $\left(\frac{2xP}{m}\right)^{1/2}$ (d) $\left(\frac{3xP}{m}\right)^{1/2}$

where x is the distance traveled by car from rest.

8. A small particle of mass m attached to a string of length l is rotated about a vertical axis at constant angular speed ω in the horizontal plane. The string of the pendulum makes an angle of θ with the vertical axis. The value of ω is

(a) $\sqrt{\frac{g}{l\sin\theta}}$ (b) $\sqrt{\frac{g}{l\tan\theta}}$

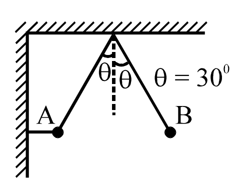
(c) $\sqrt{\frac{g}{l\cot\theta}}$ (d) $\sqrt{\frac{g}{l\cos\theta}}$

9. A small block of mass m moving on the inside of a smooth fixed hollow hemisphere of radius r, describes a horizontal circle at a distance of r/2 below the centre of the sphere. The force with which the block pushes against the hemisphere is

(a) mg (b) 2mg

(c) 3mg (d) $\sqrt{3}$mg/2

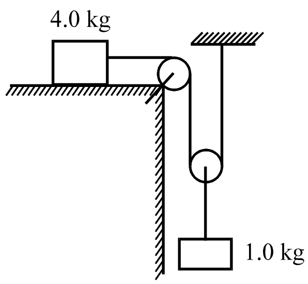
10. A ball is held at a rest in position A by two light cords. The horizontal cord is now cut and the ball swings to the position B. What is the ratio of the tension in the cord in position B to that in position A.



(a) 3 (b) 3/4

(c) 1/2 (d) 1

11. Consider the situation shown in figure. The system is released from rest and the block of mass 1.0 kg is found to have a speed 0.3 m/s after it has descended through a distance of 1 m.



The coefficient of kinetic friction between the block and the table is

(a) 0.1 (b) 0.2

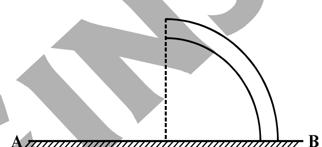
(c) 0.3 (d) none

12. A stone with weight w is thrown vertically upward into the air with initial speed v_0. If a constant force f due to air drag acts on the stone throughout its flight then the maximum height reached by the stone is

(a) $\dfrac{v_0^2}{2g\left(1+\dfrac{f}{w}\right)}$ (b) $\dfrac{v_0^2}{4g\left(1+\dfrac{f}{w}\right)}$

(c) $\dfrac{v_0^2}{6g\left(1+\dfrac{f}{w}\right)}$ (d) $\dfrac{v_0^2}{8g\left(1+\dfrac{f}{w}\right)}$

13. A rod of mass per unit length λ and length L is bent into a quarter circular ring and placed vertically on the ground as shown in figure.



The potential energy of the quarter circular ring with respect to the reference level AB is

(a) $6\lambda gr^2$ (b) $4\lambda gr^2$

(c) $2\lambda gr^2$ (d) λgr^2

14. A block of mass 2.0 kg is pulled up on a smooth inclined of angle 30^0 with the horizontal. If the block moves with an acceleration of 1.0 m/s², the power delivered by the pulling force at time 4.0 s after the motion starts is

(a) 24 W (b) 47 W

(c) 42 W (d) 74 W

15. A particle of mass m moving in a circular path of radius R has kinetic energy k given by $k = \alpha t^2$. Here α is a constant quantity. The net force acting on the particle is

(a) $\sqrt{5\alpha m+\dfrac{4\alpha^2t^4}{R^2}}$ (b) $\sqrt{4\alpha m+\dfrac{4\alpha^2t^4}{R^2}}$

(c) $\sqrt{3\alpha m+\dfrac{4\alpha^2t^4}{R^2}}$ (d) $\sqrt{2\alpha m+\dfrac{4\alpha^2t^4}{R^2}}$

16. A particle of mass m slides down a smooth inclined surface, which ends into a vertical loop of radius R. The minimum height from which the particle be released so that it does not fall at the uppermost point of the loop

(a) $\dfrac{3R}{2}$ (b) 2R

(c) $\dfrac{5}{2}R$ (d) 3R

17. A small ball of mass 1 g is placed at the bottom of a watch glass of radius 1 m. It is displaced vertically by 1 cm along the glass surface and released. The total distance described by it before it comes to rest at the bottom, if the coefficient of kinetic friction between the ball and the watch glass is 0.1

(a) 10 cm (b) 12 cm

(c) 14 cm (d) 16 cm

18. A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to (K/r^2), where K is constant. The total energy of the particle is

(a) $-\dfrac{K}{r}$ (b) $-\dfrac{K}{2r}$

(c) $-\dfrac{2K}{r}$ (d) $\dfrac{K}{r}$

19. Two identical cylindrical vessels with their bases at the same level, each contain a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other vessel is h_2. The area of either base is A. What is the work done by gravity in equalizing the levels when the vessels are interconnected ?

(a) $A\rho g\left(\dfrac{h_1+h_2}{2}\right)^2$ (b) $A\rho g\left(\dfrac{h_1-h_2}{2}\right)^2$

(c) $A\rho g\left(\dfrac{h_1+h_2}{4}\right)^2$ (d) $A\rho g\left(\dfrac{h_1-h_2}{4}\right)^2$

20. A shied, fixed vertically, is made of two plates of wood and iron, the wood being 4 cm thick and the iron 2 cm thick. A bullet fired horizontally goes through the iron first and then penetrates 2 cm into the wood. A similar bullet fired from the opposite direction goes through the wood first and then penetrates 1 cm into the iron. The ratio of resistive forces offered by the iron and the wood is

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) $\frac{3}{2}$

21. A particle slides along a track with elevated ends and a flat central part. The flat part has a length of 2m. The curved portions of the track are frictionless. For the flat part the coefficient of kinetic friction is 0.2. The particle is released from point A which is at a height 1 m above the flat part of the track. The total distance covered by the particle finally come to rest

(a) 2 m (b) 3 m

(c) 4 m (d) 5 m

22. A stone tied to a string of length L is whirled in a vertical circle, with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

23. The kinetic energy K of a particle moving along a circle of radius R depends on the distance covered s as $K = as^2$, where a is a constant. The force acting on the particle as a function of s is

(a) $2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$ (b) $4as\left[1+\frac{s^2}{R^2}\right]^{1/2}$

(c) $6as\left[1+\frac{s^2}{R^2}\right]^{1/2}$ (d) $8as\left[1+\frac{s^2}{R^2}\right]^{1/2}$

24. A uniform rod of mass m and length l rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is

(a) independent of x

(b) increasing linearly with x

(c) decreasing linearly with x

(d) none of these

25. A block of mass m is moved along the horizontal frictional surface with coefficient of friction μ such that the minimum force is required to pull the block. If the block is displaced d then work done by this force is

(a) $\frac{\mu mgd}{1+\mu^2}$ (b) μmgd

(c) $\frac{\mu mgd}{\sqrt{1+\mu^2}}$ (d) 2μmgd

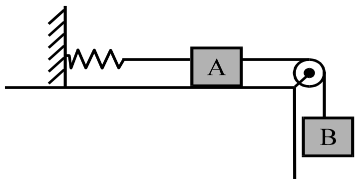
ANSWERS (SINGLE CORRECT CHOICE TYPE)

1.	c	14.	b
2.	d	15.	d
3.	b	16.	c
4.	d	17.	a
5.	a	18.	b
6.	c	19.	b
7.	a	20.	b
8.	d	21.	d
9.	b	22.	d
10.	b	23.	a
11.	d	24.	d
12.	a	25.	a
13.	d		

COMPREHENSION TYPE

Comprehension-1

Consider the situation shown in figure. Mass of block A is m and that of block B is 2m. The force constant of spring is K. Friction is absent everywhere. System is released from rest with the spring unstretched.



1. The maximum extension of the spring x_m is

(a) $\frac{mg}{K}$ (b) $2\frac{mg}{K}$

(c) $3\frac{mg}{K}$ (d) $4\frac{mg}{K}$

2. The speed of block A when the extension in the spring is $x = \frac{x_m}{2}$

(a) $g\sqrt{\frac{m}{3K}}$ (b) $2g\sqrt{\frac{m}{3K}}$

(c) $3g\sqrt{\frac{m}{3K}}$ (d) $4g\sqrt{\frac{m}{3K}}$

3. The acceleration of block B when extension in the spring is $x = \frac{x_m}{4}$

(a) $\frac{g}{2}$ (b) $\frac{g}{3}$

(c) $\frac{g}{4}$ (d) $\frac{g}{5}$

Comprehsion-2

A single conservative force F(x) acts on a 1.0 kg particle that moves along the x-axis. The potential energy U(x) is given by :

$$U(x) = 20 + (x - 2)^2$$

where x is in metres. At x = 5.0 m the particle has a kinetic energy of 20 J.

4. The mechanical energy of the system is

(a) 49 J (b) 39 J

(c) 29 J (d) 19 J

5. The maximum kinetic energy of the particle is

(a) 49 J (b) 39 J

(c) 29 J (d) 19 J

6. Choose the incorrect statement for the acceleration of the particle

(a) does not depend on 'x'

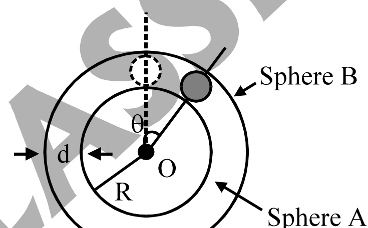
(b) depends on 'x' linearly

(c) is zero at x = 2 m

(d) always directed towards some fixed point

Comprehension-3

A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure).



The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in figure).

7. Let the ball loose the contact with the inner sphere when the angle θ is

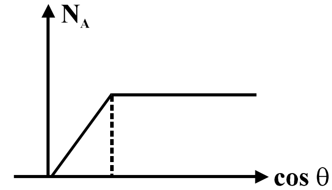
(a) $\cos^{-1}\frac{1}{3}$ (b) $\cos^{-1}\frac{2}{3}$

(c) $\cos^{-1}\frac{1}{\sqrt{3}}$ (d) $\cos^{-1}\frac{2}{\sqrt{3}}$

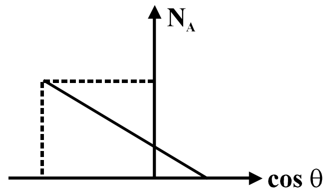
Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively.

8. The variations of N_A as functions of cos θ is given by

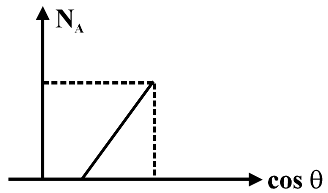
(a)



(b)



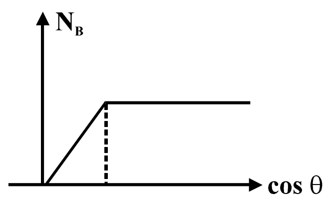
(c)



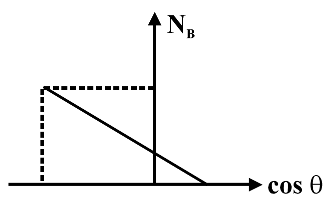
(d) none

9. The variations of N_B as functions of cos θ is given by

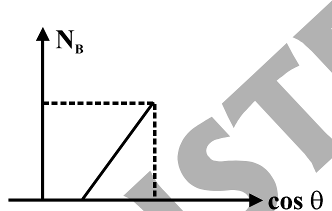
(a)



(b)



(c)



(d) none

Comprehension-4

A smooth semicircular wire track of radius R is fixed in a vertical plane. One end of a massless spring of natural length (3R/4) is attached to the lowest point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60^0 with the vertical. The spring constant K = mg/R.

10. The tangential acceleration of the ring when it is released

(a) $\frac{\sqrt{3}}{8}g$ (b) $3\frac{\sqrt{3}}{8}g$

(c) $5\frac{\sqrt{3}}{8}g$ (d) $7\frac{\sqrt{3}}{8}g$

11. The normal reaction on the ring by the track is when it is released

(a) $\frac{mg}{8}$ (b) $\frac{3mg}{8}$

(c) $\frac{5mg}{8}$ (d) $\frac{7mg}{8}$

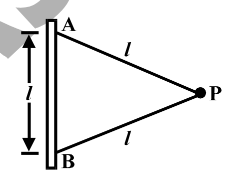
12. The net acceleration of the ring is when it is released

(a) $\frac{\sqrt{3}}{8}g$ (b) $3\frac{\sqrt{3}}{8}g$

(c) $5\frac{\sqrt{3}}{8}g$ (d) $7\frac{\sqrt{3}}{8}g$

MATRIX-MATCH TYPE

Matching-1



A particle P of mass m is attached to a vertical axis by two strings AP and BP. P rotates around the axis with an angular velocity ω.

	Column - A		Column - B
(A)	Tension in the string AP	(p)	zero
(B)	Tension in the string BP	(q)	$\frac{m\omega^2 l + 2mg}{2}$
(C)	Centripetal force on P	(r)	$\frac{m\omega^2 l - 2mg}{2}$
(D)	Tangential force on P	(s)	$\frac{\sqrt{3}}{2}m\omega^2 l$

Matching-2

A bob of mass m is suspended from a thread of length l and the other end is attached to a point in the vertical plane. The bob performs a circular motion in the vertical plane if it has given a speed of $\sqrt{6gl}$ at the lowermost point.

	Column - A		Column - B
(A)	Tension at the lowermost point	(p)	3/5 mg
(B)	Tension at the uppermost point	(q)	mg

(C)	Net force on the bob when the string will become horizontal	(r)	7 mg
(D)	Tangential force on the bob at the point when the angle made by the string with the vertical is 37^0	(s)	$\sqrt{17}$ mg

MULTIPLE CORRECT CHOICE TYPE

1. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

(d) it moves in a circular path

2. No work is done by a force on an object if

(a) the force is always perpendicular to its velocity

(b) the force is always perpendicular to its acceleration

(c) the object is stationary but the point of application of the forces moves on it

(d) the object moves in such a way that the point of application of the force remains stationary

3. One end of a light spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement x, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are

(a) the spring was initially compressed by a distance x and was finally in its natural length

(b) it was initially stretched by a distance x and finally was in its natural length

(c) it was initially in its natural length and finally in a compressed position

(d) it was initially in its natural length and finally in a stretched position

4. A ball of mass is attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, and comes to rest again after descending through a distance x. Then

(a) x = mg/k

(b) x = 2mg/k

(c) The ball will have no acceleration at the position where it has descended through x/2.

(d) The ball will have an upward acceleration equal to g at its lowermost position.

5. A simple pendulum has a bob of mass m and swings with an angular amplitude ϕ. The tension in the tread is T. At a certain time, the string makes an angle θ with the vertical ($\theta \le \phi$).

(a) $T = mg\cos\theta$, for all values of θ

(b) $T = mg\cos\theta$, only for $\theta = \phi$

(c) $T = mg$, for $\theta = \cos^{-1}\left[\frac{1}{3}(2\cos\phi + 1)\right]$

(d) T will be larger for smaller value of θ

6. A particle is projected with speed u at an angle θ with the horizontal in the vertical plane. Choose the correct statement at the point just after the projection

(a) The tangential acceleration at this point is $g\sin\theta$.

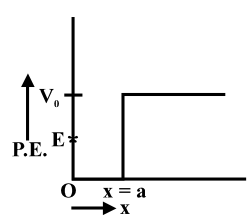
(b) The radius of curvature of the path at this point is $\frac{u^2}{g\cos\theta}$

(c) The centripetal acceleration at this point is zero

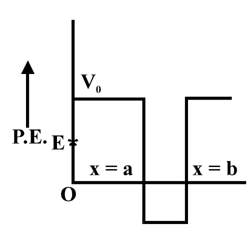
(d) The centripetal acceleration at this point is 'g'

7. The potential energy of an object varies with x as shown in figure in different cases. The total energy E of the object is indicated by the cross mark on energy axis.

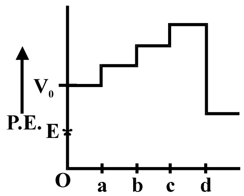
(i)



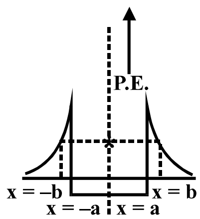
(ii)



(iii)



(iv)



Choose the correct statement from the following :

(a) The object does not exist for x > a in case (i)

(b) The object does not exist for x < a and x > b in case (ii)

(c) The object does not exist in any region in case (iii)

(d) The object does not exist in the region between a < x < b in case (iv)

8. Two springs A and B with spring constant k_A and k_B respectively and $k_A > k_B$. Choose the correct statement

(a) If they are stretched by the same force then more work is done on B.

(b) If they are stretched by the same force then more work is done on A.

(c) If they are stretched by the same amount then more work is done on B.

(d) If they are stretched by the same amount then more work is done on A.

9. A simple pendulum of length l is set in motion such that the bob, of mass m, moves along a horizontal circular path, and the string makes a constant angle θ with the vertical. The time period of rotation of the bob is t and the tension in the thread is T.

(a) $t = 2\pi\sqrt{l/g}$

(b) $t = 2\pi\sqrt{l\cos\theta/g}$

(c) $T = \frac{4\pi^2 m l}{t^2}$

(d) The bob is in equilibrium

10. A body moves on a horizontal circular road of radius r, with a tangential acceleration a_t. The coefficient of friction between the body and the road surface is μ. It begins to slip when its speed is v.

(a) $v^2 = \mu r g$

(b) $\mu g = \frac{v^2}{r} + a_t$

(c) $\mu^2 g^2 = \frac{v^4}{r^2} + a_t^2$

(d) The force of friction makes an angle $\tan^{-1}(v^2/a_t r)$ with the direction of motion at the point of slipping

Assertion-Reason Type

Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

1. STATEMENT-1 : Work energy theorem is valid for conservative as well as non-conservative forces.

STATEMENT-2 : Conservation of mechanical energy is applied when the forces in the system are conservative.

2. STATEMENT-1 : Work energy theorem is not valid in non-inertial frame.

STATEMENT-2 : Non-inertial frames are accelerated frame with respect to ground.

3. STATEMENT-1 : Consider a tug-of-war, in which the two teams pulling on the rope are evenly matched, so that no motion takes place. Work is being done only within the bodies of the pullers. For e.g., the heart of each puller is applying forces on the blood to move it through the body.

STATEMENT-2 : There is no work done on the rope.

4. STATEMENT-1 : Kinetic energy never be negative but the change in kinetic energy may be negative.

STATEMENT-2 : Kinetic energy is a scalar quantity.

5. STATEMENT-1 : A particle is projected from the ground in the vertical plane. During the time of ascent the centripetal acceleration decreases and tangential acceleration increases.

STATEMENT-2 : The net acceleration at any moment during the motion is 'g' (neglect the air friction).

6. STATEMENT-1 : Work done by the force in uniform circular motion is always zero.

STATEMENT-2 : In uniform circular motion force and displacement are always perpendicular to each other.

7. STATEMENT-1 : A truck and a car moving with the same speed are stopped by applying the same retarding force. The truck will cover the larger distance.

STATEMENT-2 : The distance covered is directly proportional to mass.

8. STATEMENT-1 : A truck and a car moving with the same kinetic energy are stopped by applying the same retarding force. The truck will cover the larger distance.

STATEMENT-2 : The distance covered is independent of mass in this case.

9. STATEMENT-1 : Work done by the spring force is always negative.

STATEMENT-2 : Spring force is a conservative force.

10. STATEMENT-1 : Work done by the frictional force is always negative.

STATEMENT-2 : Friction force is a non conservative force.

11. STATEMENT-1 : A block is slowly moved up along a smooth inclined by the variable force. Let the work done by this force is W_1. Now the same block is displaced very slowly along the same plane by a constant force. Let the work done by this force is W_2. If the displacement of the block in both cases are same then $W_1 = W_2$.

STATEMENT-2 : If a block is moved very slowly then the change in kinetic energy of the block is zero.

(Answers) EXCERCISE BASED ON NEW PATTERN

COMPREHENSION TYPE

1.	d	2.	b	3.	b	4.	a	5.	c	6.	a
7.	b	8.	c	9.	b	10.	c	11.	b	12.	c

MATRIX-MATCH TYPE

1. [A-q, B-r, C-s, D-p]   2. [A-r, B-q, C-s, D-p]

MULTIPLE CORRECT CHOICE TYPE

1.	c, d	2.	a, c, d	3.	a, b	4.	b, c, d	5.	b, c, d	6.	a, b
7.	a, b, c, d	8.	a, d	9.	b, c	10.	c, d				

ASSERTION-REASON TYPE

1.	B	2.	D	3.	B	4.	B	5.	D	6.	A
7.	A	8.	D	9.	D	10.	D	11.	A		

INITIAL STEP EXERCISE (SUBJECTIVE)

1. A chain of length *l* and mass *m* lies on the surface of a smooth sphere of radius $R > l$ with one end tied to the top of the sphere, find the gravitational potential energy of the chain with reference level to the centre of the sphere.

2. A 61 kg bungee – cord jumper is on a bridge 45 m above a river. The elastic bungee cord has a relaxed length L = 25 m with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height h of her feet above the water at her lowest point ?

3. A 12 kg block is released from rest on a 30^0 frictionless incline. Below the block there is a spring that can be compressed 2 cm by a force of 270 N. The block momentarily stops when it compressed the spring by 5.5 cm.

   (a) How far does the block move down the incline from its rest position to this stopping point ?

   (b) What is the speed of the block just as it touches the spring ?

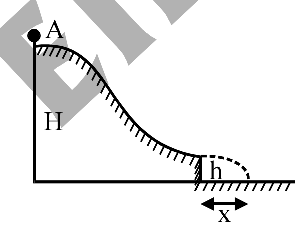
4. A particle of mass m moves along a circle of radius R with a normal acceleration varying with time as $a_n = kt^2$, where k is a constant.

   (a) Find the tangential force acting on the particle.

   (b) Find the net force acting on the particle.

   (c) Find time dependence of power developed by all the forces acting on the particle and the mean value of this power averaged over the first t second after the beginning of the motion.

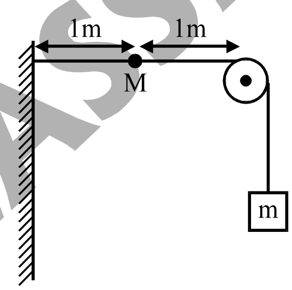
5. A small pebble A sliding down the top of a smooth hill of height *H* which has a horizontal portion at the base.



What must be the height of the horizontal portion *h* so that the pebble covers the maximum distance x along the horizontal base. What is it equal to ?

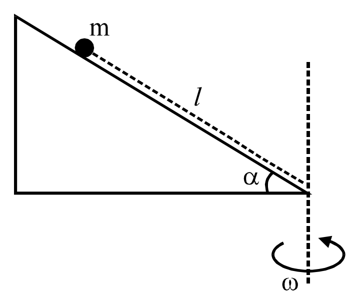
6. A system consists of two identical cubes, each of mass m, linked together by a compressed weightless spring of force constant k. The cubes are also connected by a thread which is burnt at a certain moment. At what values of initial compression x_0 of the spring, will the lower cube bounce up after the thread is burnt through ?

7. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2 m from the wall, has a point mass M = 2 kg attached to it at a distance of 1 m from the wall. A mass m = 0.5 kg attached at the free ends is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley.



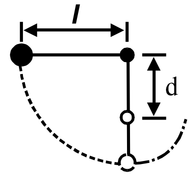
What will be the speed with which the mass M will hit the wall when the mass m is released ?

8. A ball is placed on a smooth chute inclined at an angle $\alpha = 30^0$ to the horizontal and rotating at frequency n = 30 rpm about a vertical axis passing through its lower end.



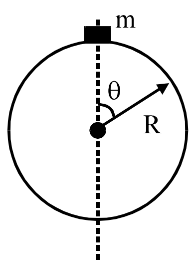
At what distance from the end of the chute does the ball rest ?

9. The nail in figure, is located at a distance d below the point of suspension.



Show that d must at least 0.6 *l* if the ball is to swing completely around in a circle centered on the nail.

10. A point mass m starts from rest and slides down the surface of a frictionless solid sphere of radius R as shown in figure. If we measured angles from the vertical and potential energy from the top then determine



(a) the change in potential energy of the mass with angle θ,

(b) the kinetic energy as a function of angle θ,

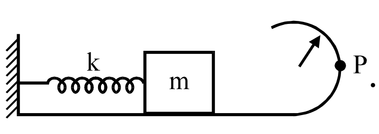
(c) the radial and tangential accelerations as a function of angle

(d) the angle at which the mass flies off the sphere

If there is friction between the mass and the sphere, does the mass fly off at a greater or lesser angle that in part (d) ?

11. A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of the ring moves with a speed v. Find the tension in the ring.

12. Figure shows a smooth track, a part of which is a circle of a radius R. A block of mass m is pushed against a spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal



13. As a particle moves in xy plane, it is acted on by a force whose components are functions of the coordinates of the particle $F_x = 8xy$ and $F_y = 6y^2$. Evaluate the work done by this force as the particle moves from the origin to the point (3, 3) along the path y = x. Also find the work using the path $y = \frac{1}{3}x^2$. Is the work done by this force independent of path ?

14. A block of mass m is pushed against a spring of spring constant k fixed at one end to a wall. The block can slide on a frictionless table. The natural length of the spring if L_0 and it is compressed to half its natural length when the block is released. Find the velocity of the block as a function of its distance x from the wall.

15. A car starts moving from rest along the circumference of a horizontal circle of radius R = 40 m with a constant tangential acceleration $a_t = 0.62\ ms^{-2}$. The coefficient of sliding firction is μ = 0.2. What distance will the car describe without sliding ?

16. A particle of mass m moves in a circle of radius R in such a way that it speed (v) varies with distance (s) as $v = a\sqrt{s}$ where a is a constant. Calculate the acceleration and force on the particle.

17. A cyclist rides along a circular path in a horizontal plane where the coefficient of friction varies with distance from the centre O of the circular path as $\mu = \mu_0\left(1 - \frac{r}{R}\right)$ where R is the maximum distance up to which the road is rough. Find the radius of the circular path along which the cylist can ride with maximum velocity. What is this maximum velocity ?

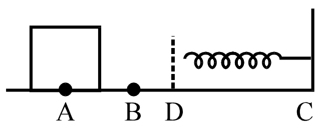
18. A very small cube of mass m is placed on the inside of a conical funnel of semivertical angle (π/2 – θ). The funnel is then set in rotation. If the coefficient of static friction between the cube and the funnel is μ and the centre of the cube is at a distance r from the axis of rotation, what are the largest and smallest angular velocities with which the funnel can be rotated so that the block will not move with respect to the funnel ?

19. A particle of mass m is suspended by a string of length *l* from a fixed rigid support. A sufficient horizontal velocity $v_0 = \sqrt{3gl}$ is imparted to it suddenly. Calculate the angle made by the string with the vertical when the acceleration of the particle is inclined to the string by 45^0.

20. Two bars of masses m_1 and m_2 connected by a non-deformed light spring rest on a horizontal plane. The coefficient of friction between the surface and the bars is equal to μ. What minimum constant force has to be applied in the horizontal direction to the bar of mass m_1 in order to shift the other bar ?

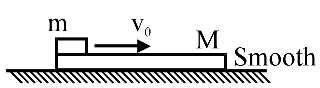
FINAL STEP EXERCISE (SUBJECTIVE)

1. A 0.5 kg block slides from the point A on a horizontal track with an initial speed 3 m/s toward a weightless horizontal spring of length 1 m and force constant 2 N/m. The part AB of the track is frictionless and the part BC has the coefficient of static and kinetic friction as 0.22 and 0.20 respectively. If the distances AB and BD are 2 m and 2.14 m respectively.



Find the total distance through which the block moves before it comes to rest completely. Take $g = 10\ m/s^2$.

2. A plank of mass M and Length L is placed at rest on a smooth horizontal surface. A small block of mass m is projected with a velocity v_0 from the left end of it as shown in fig. The coefficient of friction between the block and the plank is μ, and its value is such that the block becomes stationary with respect to the plank before it reaches the other end.

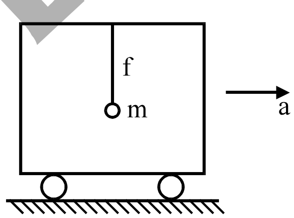


(i) Find the work done by the friction force on the block during the period it slides on the plank. Is the work positive or negative.

(ii) Calculate the work done on the plank during the same period. Is the work positive or negative ?

(iii) Also, determine the net work done by friction. Is the positive or negative ?

3. A pendulum of mass m and length *l* is suspended from the ceiling of a trolley which has a constant acceleration a in the horizontal direction as shown in the figure.



Find the maximum deflection θ of the pendulum from the vertical.

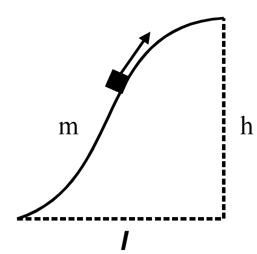
4. Two blocks of masses $m_1 = 10$ kg and $m_2 = 5$ kg, connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of the table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(a) Calculate the frictional force on m_1

(b) What should be the minimum angular speed of the turn table so that the masses will slip from this position ?

(c) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1 ?

5. A body of mass m was slowly hauled up the hill by a force F which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is h, the length of its base *l*, and the coefficient of friction μ.

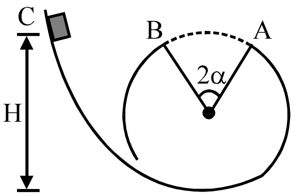


6. A balloon starts rising from the surface of the earth. The ascension rate is cosntant and equal to v_0. Due to the wind the balloon gathers a horizontal velocity component $v_x = ky$ where k is a constant and y is the height of ascent. Find how the following quantities depend on the height of ascent

(a) the horizontal drift of the balloon x(y),

(b) the total, tangential, and normal accelerations of the balloon.

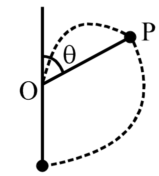
7. A small object slides without friction from the height H = 50 cm and then loops the vertical loop of radius r = 20 cm from which a symmetrical section of angle 2α has been removed.



Find angle α such that after loosing contact at A and flying through the air, the object will reach point B.

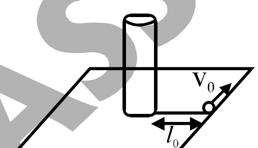
8. A chain of mass m = 0.8 kg and length L = 1.5 m rests on a rough table so that one of its ends hangs over the edge. The chain slides off the table all by itself when n = 1/8 of its length hangs. What will be the total work done by the frictional force acting on the chain by time slides completely off the table ?

9. A heavy particle is suspended by a string of length l. The particle is given a horizontal velocity v_0. The string becomes slack at some angle and the particle proceeds on a parabola.



Find the value of v_0 if the particle passes through the point of suspension.

10. A 'horizontal plane' supports a stationary vertical cylinder of radius R and a disc A attached to the cylinder by a horizontal thread AB of length l_0. An initial velocity v_0 is imparted to the disc as shown in figure. How long will it move along the plane until it strike against the cylinder ?



The friction is assumed to be absent.

ANSWERS SUBJECTIVE (INITIAL STEP EXERCISE)

1. $\frac{m}{l}R^2 g \sin\left(\frac{l}{R}\right)$ 2. 2.1 m 3. (a) 25 cm (b) 1.7 m/s

4. (a) $m\sqrt{kR}$ (b) $m\sqrt{kR + k^2t^4}$ (c) mkRt, $\frac{1}{2}$mkRt

5. $\frac{H}{2}$, H 6. 3 mg/k

7. 3.3 m/s 8. 0.67 m

10. (a) – mgr(1 – cosθ) (b) mgr(1 – cosθ) (c) 2g(1 – cosθ), g sin θ

(d) $\theta_0 = \cos^{-1}\left(\frac{2}{3}\right)$, greater than θ_0. 11. $\frac{mv^2}{2\pi R}$

12. $\sqrt{\frac{3mgR}{k}}$ 14. $\sqrt{\frac{k}{m}\left[\frac{L_0^2}{4} - (L_0 - x)^2\right]}$ 15. 60 m

16. $\frac{1}{2}a^2\sqrt{1+\frac{4s^2}{R^2}}$ 17. $\frac{R}{2}, \frac{1}{2}\sqrt{\mu_0 Rg}$ 18. $\omega_{max} = \sqrt{\frac{g\sin\theta + \mu\cos\theta}{r\cos\theta - \mu\sin\theta}}$, $\omega_{min} = \sqrt{\frac{g\sin\theta - \mu\cos\theta}{r\cos\theta + \mu\sin\theta}}$

19. π/2 20. $\mu\left(m_1 + \frac{m_2}{2}\right)g$

ANSWERS SUBJECTIVE (FINAL STEP EXERCISE)

1. 4.24 m

2. (i) $-\frac{1}{2}\frac{mM(M+2m)v_0^2}{(M+m)^2}$ (ii) $\frac{1}{2}\frac{m^2M}{(m+M)^2}v_0^2$

   (iii) $-\frac{1}{2}\frac{mM}{(m+M)^2}v_0^2$

3. $2\tan^{-1}\left(\frac{a}{g}\right)$

4. (a) 36 N (b) 11.67 rad/s (c) 0.2 m

5. mg(h + kl)

6. (a) $\frac{ky^2}{2v_0}$ (b) $\frac{kv_0^2}{\sqrt{v_0^2+k^2y^2}}$ 7. 60^0 8. – 1.3 J

9. $\sqrt{gl(2+\sqrt{3})}$ 10. $\frac{l_0^2}{2v_0R}$