ELECTRIC CHARGES AND FIELDS

1.1 Introduction :

Q. What is electrostatics ?

Solution : Electrostatics deals with the study of forces, fields and potentials arising from static charges (charges at rest).

1.2 Electric Charge :

Q. What is Electric Charge ?

Solution : Electric charge is possessed by material objects that makes it possible for them to exert electrical force and to respond to electrical force.

Q. What is the real cause of electrification when two bodies are rubbed with each other ?

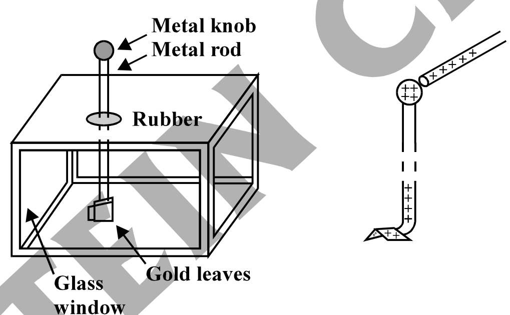
Solution : Due to rubbing of one body to another body there is a transfer of electrons from one body to another body. Thus a body will lose some of its electrons and acquired positive charge whereas another body will gain some electron and acquired negative charge. Hence by rubbing the two neutral bodies, both will get equal and opposite charge.

Q. When we rub a glass rod with silk, which one will get positive charge and which one negative charge ?

Solution : Glass rod gets positive charge and silk gets negative charge.

Q. Which appratus is used to detect the charge on a body ? Describe it.

Solution : Gold leaf electroscope is use to detect the charge on a body..



It consists of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergence is an indicator of the amount of charge.

Q. An Ebonite, Amber or rubber rod is rubbed with fur or wool. What type of charge do they acquire ?

Solution : Ebonite rod acquires negative charge and fur or wool an equal amount of positive charge.

Q. What does $q_1 + q_2 = 0$, signify in electrostatics ?

Solution : It signifies that both charges are equal in magnitude but opposite in sign.

Q. Due to electrification by rubbing, one body acquires charge q_1 and another body acquires charge q_2. What is the value of $q_1 + q_2$?

Solution : zero

1.3 Conductors and Insulators :

Q. What is difference between conductor and insulator ?

Solution : Some substances readily allow passage of electricity through them, others do not. Those which allow electricity to pass through them easily are called conductors. They have electric charges (electrons) that are comparatively free to move inside the material. Metals, human and animal bodies and earth are conductors. Most of the non-metals like glass, porcelain, plastic, nylon, wood offer high resistance to the passage of electicity through them. They are called insulators.

When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor. In contrast, if some chage is put on an insulator, it stays at the same place.

Q. What is earthing or grounding ? What is use of this ?

Solution : When we bring a charged body in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body). This process of sharing the charges with the earth is called ground or earthing. Earthing provides a safety measure for electrical circuits and appliances. A thick metal plate is buried deep into the earth and thick wires are drawn from this plate; these are used in buildings for the purpose of earthing near the mains supply. The electric wiring in our houses has three wires; live, neutral and earth. The first two carry electric currents from the power station and the third is earthed by connecting it to the buried metal plate. Metallic bodies of the electric appliances such as electric iron, refrigerator, TV are connected to the earth wire. When any fault occurs or live wire touches the metallic body, the charge flows to the earth without damaging the appliance and without causing any injury to the humans; this would have otherwise been unavoidable since the human body is a conductor of electricity.

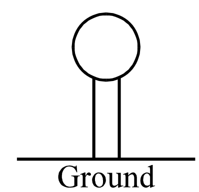
1.4 Charging by Induction :

Q. How the bodies can be charged by induction ? or How can you charge a metal sphere positively without touching it ? [NCERT solved example 1.1]

Solution : There are following steps to charge a metal sphere by induction.

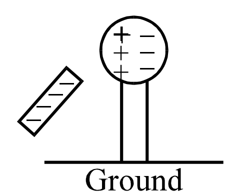
STEP I

A metal sphere supported on insulated stand.



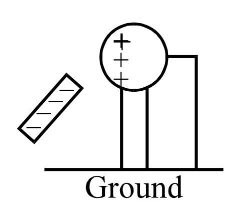
STEP II

A negatively charged rod is brought near it, without touching, the free electrons in the metal sphere are repelled by the excess of electrons of the rod and the left surface becomes +ve charge. But the sphere as whole is neutral.



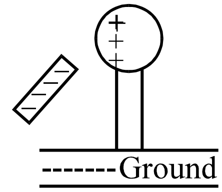
STEP III

Now the right surface of the sphere is earthed using a wire and the negative charges (electrons) flows to earth while the positive charges at the near end will remain held due to attractive force of negative charge on the rod.



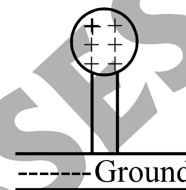
STEP IV

The wire is disconnected.



STEP V

Remove the rod and the sphere gets positive charged, spread uniformly over the sphere.



Note the following points for the charging by induction

(a) Rod (inducting body) neither gains nor loses charge.

(b) Rod is negatively charged whereas sphere gets positive charge i.e. nature of induced charge is always opposite to that of inducing charge.

(c) Remember that induced charge can be never greater than inducing charge.

1.5 Basic properties of Electric Charge :

Q. Write down the basic properties of electric charge ?

Solution : (i) Charge is a scalar quatity and it is additive i.e., total chage of a system is obtained simply by adding algebraically all the charges present anywhere on the system. Proper signs have to be used while adding the charge in a system.

(ii) Charge is conserved for an isolated system.

(iii) Charge is quantised i.e. $q = \pm ne$ where n = 1, 2, 3..... and $e = 1.6 \times 10^{-19}$ C.

Q. What is quantisation of charge and why is it so ?

Solution : Electric charge is always integral multiple of e is termed as quantisation of charge i.e. $q = \pm ne$ where n = 1, 2, 3..... and $e = 1.6 \times 10^{-19}$ C.

The cause of quantization is that only integral number of electrons can be transferred from one body to another.

Q. Calculate the charge carried by 12.5×10^{18} electrons.

Solution : 2 C

Q. If 10^9 electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body ? [NCERT solved example 1.2]

Solution : 198 years

Q. How much charge of (positive and negative) is there in a cup of water containing 250 mL ? [NCERT solved example 1.3]

Solution : 1.34×10^7 C

Q. In which situation a charged body is treated as the point charge or define point charge ?

Solution : If the size of charged bodies are very small as compared to the distances between them, we treat them as point charges.

Q. Is a charge of 2.8×10^{-18} C possible ?

Solution : No. From q = ne $\Rightarrow$ $\mathbf{n = \frac{2.8 \times 10^{-18}}{1.6 \times 10^{-19}} = 17.5}$, which is not integer.

Q. Write down the difference between the charge and mass.

Solution : (i) Charge on a body may be positive, negative or zero whereas mass of a body is always positive.

(ii) Charge is invariant i.e., it doesnot depends upon the velocity whereas mass depends upon the velocity of the moving body given by $\mathbf{m = \frac{m_0}{\sqrt{1 - v^2/c^2}}}$, where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.

(iii) Charge is quantized whereas quantization of mass is yet to be established.

(iv) Electric charge is always conserved whereas mass is not conserved as it can be changed into energy and vice-versa.

Q. Define one coulomb ?

Solution : One coulomb is the charge flowing through a wire in 1 s if the current is 1 A (ampere).

Q. Which is bigger, a coulomb or charge on an electron ? How many electronic charges form one coulomb of charge ?

Solution : Coulomb is bigger than the charge on the electron. One coulomb of charge is formed by 0.625×10^{19} of electrons.

Q. Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6.4 g (take the atomic weight of copper to be 64 g/mol).

Solution : 6.023×10^{22}

1.6 Coulomb's Law :

Q. Write down coulomb law and its limitation.

Solution : Coulomb measured the force between two point charges and found that it varied inversely as the square of the distance between the charges and was directly proportional to the product of the magnitude of the two charges and acted along the line joining the two charges. Thus, if two point charges q_1, q_2 are separated by a distance r in vacuum, the magnitude of the force **(F)** between them is given by $\mathbf{F = k\frac{|q_1 \times q_2|}{r^2}}$.

The constant $k = 1/4\pi\varepsilon_0$, ε_0 is called the permitivity of free space. The value of ε_0 in SI units is $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

Limitations : It is applicable only for point charges.

Q. Using which balance coulomb measured the force between two charged metallic sphere ?

Solution : Torsion balance, a sensitive device to measure force.

Q. Define one coulomb.

Solution : 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experiences an electrical force of repulsion of magnitude 9×10^9 N.

Q. Write down the dimensional formual of ε_0.

Solution : $\mathbf{\varepsilon_0 = \frac{1}{4\pi F}\frac{q_1 q_2}{r^2} = \frac{[AT][AT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]}$.

Q. Write down the Coulomb's law in vector form.

Solution : Let the position vectors of charges q_1 and q_2 be $\vec{r}_1$ **and** $\vec{r}_2$ [see figure] respectively. We denote force on q_1 due to q_2 by $\vec{F}_{12}$ and force on q_2 due to q_1 by $\vec{F}_{21}$. The two point charges q_1 and q_2 have been numbered 1 and 2 for convenience and the vector leading from 1 to 2 is denoted by $\vec{r}_{21}$:

$$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$$

In the same way, the vector leading from 2 to 1 is denoted by $\vec{r}_{12}$:

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$$

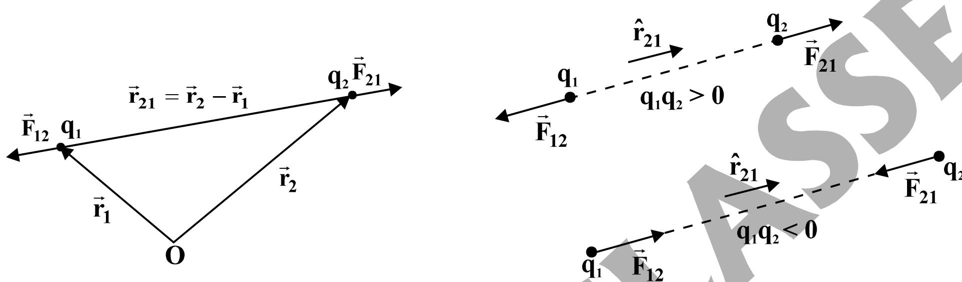
The magnitude of the vectors $\vec{\mathbf{r}}_{21}$ **and** $\vec{\mathbf{r}}_{12}$ is denoted by r_{21} and r_{12}, respectively ($r_{12} = r_{21}$). The direction of a vector is specified by a unit vector along the vector. To denote the direction from 1 to 2 (or from 2 to 1), we define the unit vectors :

$$\hat{\mathbf{r}}_{21} = \frac{\vec{\mathbf{r}}_{21}}{\mathbf{r}_{21}}, \hat{\mathbf{r}}_{12} = \frac{\vec{\mathbf{r}}_{12}}{\mathbf{r}_{12}}, \hat{\mathbf{r}}_{21} = \vec{\mathbf{r}}_{12}$$

Here, $\vec{\mathbf{r}}_{12} = |\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2|$ **and** $\vec{\mathbf{r}}_{21} = |\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|$

Coulomb's force law between two points charges q_1 and q_2 located at $\vec{\mathbf{r}}_1$ **and** $\vec{\mathbf{r}}_2$ is then expressed as

$$\vec{\mathbf{F}}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^2}\hat{\mathbf{r}}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{21}^3}|\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1|$$

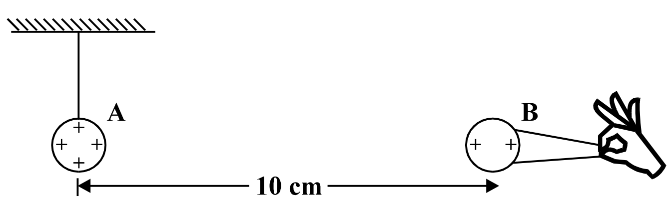


(a) (b)

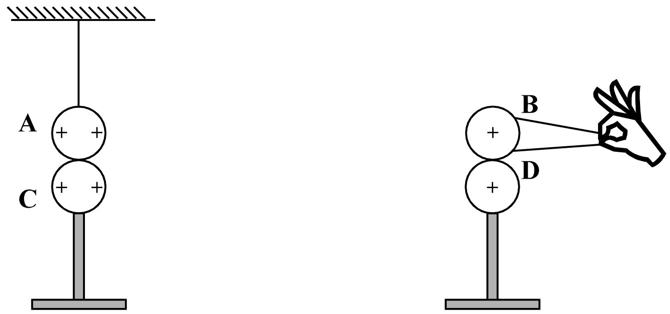
Q. Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse-square dependence on the distance between the charges/masses. (a) Compare the strength of these forces by determining the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons. (b) Estimate the accelerations of electron and proton due to the electrical force of their mutual attraction when they are 1 Å (= 10^{-10} m) apart ? (m_p =1.67 × 10^{-27} kg, m_e = 9.11 × 10^{-31} kg). [NCERT solved example 1.4]

Solution : (a) (i) 2.4 × 10^{39} (ii) 1.3 × 10^{36} (b) 2.5 × 10^{22} m/s^2, 1.4 × 10^{19} m/s^2

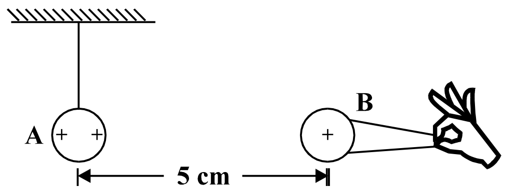
Q. A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating handle is brought close to A such that the distance between their centres is 10 cm, as shown. The resulting repulsion of A is noted (for example, by shining a beam of light and measuring the deflection of its shadow on a screen). Spheres A and B are touched by uncharged spheres C and D respectively, as shown. C and D are then removed and B is brought closer to A to a distance of 5.0 cm between their centres, as shown. What is the expected repulsion of A on the basis of Coulomb's law ? Spheres A and C and spheres B and D have identical sizes. Ignore the sizes of A and B in comparison to the separation between their centres. [NCERT solved example 1.5]



(a)



(b)



(c)

Solution : The electrostatic force on A, due to B, remains same in both situations

Q. Is Coulomb's law consistent with Newton's third law of motion ?

Solution : Yes, if we consider two point charges then the force exerted by the charges on each other are equal in magnitude but opposite in direction.

1.7 Force Between Multiple Charges :

Q. What is the principle of superposition of force between multiple charges ?

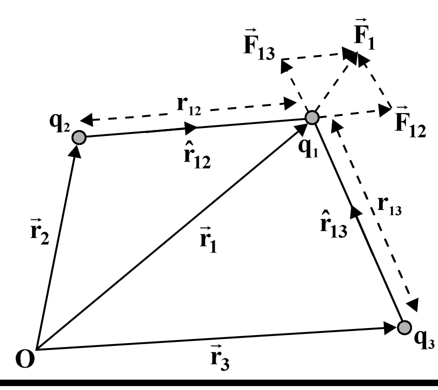
Solution : Force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to the other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges. This is termed as the principle of superposition.

Q. Consider the point charges q_1, q_2, q_3 with the position vector $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ respectively. Find the force on q_1 due to other two charges ? Express your answer in terms of unit vector.

OR

Find an expression for electrostatic force using Coulomb's law on a charge q_1 due to other charges q_2 and q_3 ?

Solution : Consider a system of three charges q_1, q_2 and q_3, as shown in figure. The force on one charge, say q_1, due to two other charges q_2, q_3 can therefore be obtained by performing a vector addition of the forces due to each one of these charges.



Thus, if the force on q_1 due to q_2 is denoted by $\vec{F}_{12}$, $\vec{F}_{12}$ is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|}$$

In the same way, the force on q_1 due to q_3, denoted by $\vec{F}_{13}$, is given by

$$\vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\hat{r}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|}$$

which again is the Coulomb force on q_1 due to q_3, even though other charge q_2 is present.

Thus the total force $\vec{F}_1$ on q_1 due to the two charges q_2 and q_3 is given as

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\hat{r}_{13}$$

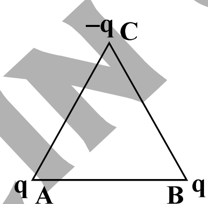
Q. Electrostatics is the consequence of which priciples ?

Solution : Electrostatics is a consequence of Coulomb's law and the superposition principle.

Q. Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side *l*. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, [NCERT solved example 1.6]

Solution : zero

Q. Consider the charges q, q and –q are placed at the vertices of an equilateral triangle of side length L. What is the force on each charge. [NCERT solved example 1.7]



Solution : $F_A = \frac{kq^2}{l^2}$, $F_B = \frac{kq^2}{l^2}$, $F_C = \frac{\sqrt{3}kq^2}{l^2}$, where $k = \frac{1}{4\pi\in_0}$

Q. Define the dielectric constant of the medium in terms of force acting between two charges.

Solution : The dielectric constant of the medium is the ratio of force between two point charge separated by a finite distance in free space or vacuum and force between the same charges separated by same distance in the given medium.

Q. Two identical point charges of charge 'Q' are kept at a distance 'r' from each other. A third point charge is placed on the line joining the above charges such that all the three charges are in equilibrium. Calculate the magnitude and locations of the third charge.

Solution : –Q/4, this charge is located at the mid point between the two above point charges.

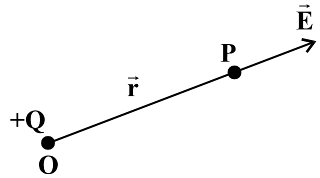
Q. Three point charges of + 2μC, –3 μC and –3 μC are kept at the vertices A, B and C, respectively of an equilateral triangle of side 20 cm. What should be the sign and magnitude of the charge to be placed at the mid-point (M) of side BC so that the charge at A remains in equilibrium ?

Solution : 3.93 μC

1.8 Electric Field :

Q. Define the term electric field intensity due to a point charge or intensity of electric field due to a point charge ? Also write down the unit ?

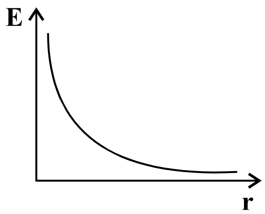
Solution : Let us consider a point charge Q placed in vacuum, at the origin O. This charge produces an electric field everywhere in the surrounding. The electric field or electric field intensity produced by the charge Q at a point P of position vector $\vec{r}$ is given by $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi \in_0}\frac{Q}{r^3}\vec{r} \quad \left(\hat{r} = \frac{\vec{r}}{r}\right)$



The SI unit of electric field is N/C and the alternate unit is V/m.

Q. Draw the graph for the variation of electric field due to the point charge with distance 'r'

Solution :



As the electric field is inversely proportional to r^2, hence the graph between E due to point charge with distance r is hyperbolic.

Q. If the force experienced by a point charge q in the presence of external enetric field $\vec{E}$ is $\vec{F}$ then what is the relation between $\vec{F}$ and $\vec{E}$?

Solution : $\vec{F} = q\vec{E}$

Q. Define electric field due to a charge.

Solution : The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point.

Q. Define source charge and test charge.

Solution : The charge Q, which is producing the electric field, is called a source charge and the charge q, which tests the effect of a source charge, is called a test charge.

Q. The electric field $\vec{E}$ due to a point charge at any point near it is defined as $\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q}$, where q is the test charge and $\vec{F}$ is the force acting on it. What is the physical significance of $\lim_{q \to 0}$ in this expression ?

OR

Why the value of test charge negligibly small while defining the electric field due to source charge ?

Solution : The source charge Q must remain at its original location to define the electric field due to this charge. However, if a charge q is brought at any point around Q, Q itself is bound to experience an electrical force due to q and will tend to move. A way out of this difficulty is to make q negligibly small. The force $\vec{F}$ is then negligibly small but the ratio $\vec{F}/q$ is finite and defines the electric field :

$$\vec{E} = \lim_{q\to 0}\left(\frac{\vec{F}}{q}\right)$$

Q. Does the electric field due to source charge depends on the value of test charge ?

Solution : No, this is because $\vec{F}$ is proportional to q,so the ratio $\vec{F}/q$ does not depend on q.

Q. What is the direction of electric field due to positive and negative charge ?

Solution : For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points radially inwards.

Q. What is the nature of symmetry of electric field due to a point charge ?

Solution : Spherical symmetry as $\mathbf{E} \propto \frac{1}{r^2}$.

Q. What is the meaning of spherical symmetry of electric field due to a point charge ?

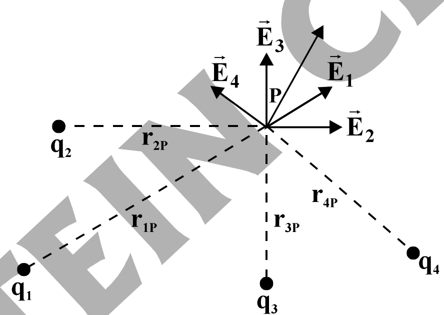
Solution : At equal distances from the charge, the magnitude of its electric field is same. The magnitude of electric field due to a point charge is thus same on a sphere with the point charge at its centre; in other words, it has a spherical symmetry.

Q. Derive the expression for the net electric field due to system of charges ?

OR

Consider the point charges q_1, q_2 ...... q_n with position vectors $\vec{r}_1, \vec{r}_2 .... \vec{r}_n$ w.r.t. origin O. Derive the expression of net electric field at the position $\vec{r}$.

Solution : Consider a system of charges q_1, q_2, ... q_n with position vectors $\vec{r}_1, \vec{r}_2 .... \vec{r}_n$ relative to some origin O. Like the electric field at a point in space due to a single charge, electric field at a point in space due to the system of charges is defined to be the force experienced by a unit test charge placed at that point, without disturbing the original positions of charges q_1, q_2,......q_n. We can use Coulomb's law and the superposition principle to determine this field at a point P denoted by position vector $\vec{r}$.



Electric field $\vec{E}_1$ at $\vec{r}$ due to q_1 at $\vec{r}_1$ is given by $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$, where $\hat{r}_{1P}$ is a unit vector in the direction from q_1 to P, and r_{1p} is the distance between q_1 and P.

In the same manner, electric field $\vec{E}_2$ at $\vec{r}$ due to q_2 at $\vec{r}_2$ is $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$ where $\vec{r}_{2P}$ is a unit vector in the direction from q_2 to P and r_{2P} is the distance between q_2 and P. Similarly expressions hold good for fields $\vec{E}_3, \vec{E}_4, ... \vec{E}_n$ due to charges q_3, q_4, ... q_n.

By the superposition principle, the electric field $\vec{E}$ at $\vec{r}$ due to the system of charges is (as shown in figure)

$$\vec{E}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + ... + \vec{E}_n(\vec{r})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P} + ... + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}^2}\hat{r}_{nP}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

Q. What is the physical significance of electric field ?

Solution : (1) Electric field characterises the electrical environment due to a system of charges

(2) The true physical significance of the concept of electric field, however, emerges only when we deal with time-dependent electromagnetic phenomena. Suppose we consider the force between two distant charges q_1, q_2 in accelerated motion. Now the greatest speed with which a signal or information can go from one point to another is c, the speed of light. Thus, the effect of any motion of q_1 and q_2 cannot arise instantaneously. There will be some time delay between the effect (force on q_2) and the cause (motion of q_1). It is precisely here that the notion of electric field (strictly, electromagnetic field) is natural and very useful. The field picture is this : the accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed c, reach q_2 and cause a force on q_2.

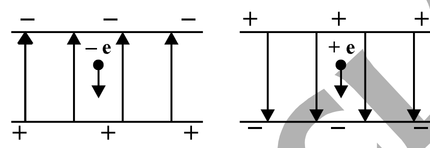
Q. What is the speed of the electrical signal ?

Solution : The speed of the electrical signal is 3×10^8 m/s

Q. Who was first to introduce the concept of field ?

Solution : Faraday

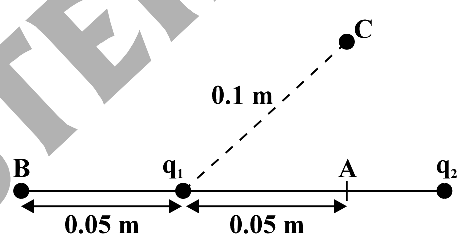
Q. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 N C^{-1}. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance. Compute the time of fall in each case. Neglect gravity. [NCERT solved example 1.8]



(a) (b)

Solution : (a) 2.9×10^{-9} s (b) 1.3×10^{-7} s

Q. Two point charges q_1 and q_2, of magnitude $+10^{-8}$C and -10^{-8} C, respectively, are placed 0.1 m apart. Calculate the electric fields at point A, B and C as shown in figure. [NCERT solved example 1.9]



Solution : $E_A = 7.2 \times 10^4$ N C^{-1} towards the right

$E_B = 3.2 \times 10^4$ N C^{-1} towards the left

$E_C = 9 \times 10^3$ N C^{-1} towards the right

Q. In the following cases, find the null point (the point at which net electric field is zero) on the line joining the two charges ?

(a) Two point charges +q and +q are separated by distance d

(b) Two point charges +q and +4q are separated by distance d

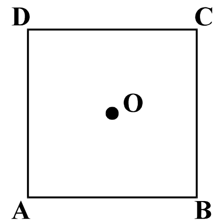
(c) Two point charges +q and –4q are separated by distance d

Solution : (a) at the mid-point (b) d/3 from +q between the two charges (c) d from +q, not between the two charges

Q. Two charges each +q are placed at (0, a) and (0, –a) respectively. Find the electric field at the point (x, 0) ? Also draw the variation of electric field with x. Find the value of x at which electric field is maximum.

Solution : $\dfrac{2kqx}{(x^2+a^2)^{3/2}}, x = \pm\dfrac{a}{\sqrt{2}}$

Q. Find the electric field at the point O (centre of the square ABCD of side length a) in the following cases :



(i) Each of the positive charge +q are placed at the corner.

(ii) Charges +q, –q, +q and –q are placed at A,B,C and D respectively.

(iii) Charges +q, –q, +2q and –2q are placed at A,B,C and D respectively.

Solution : (i) zero (ii) zero (iii) $\frac{2\sqrt{2}kq}{a^2}$

Q. Three equal charges are placed at the corner of an equilateral triangle. What is the electric field at the circumcentre of the triangle.

Solution : zero

Q. Two points electric charges of unknown magnitude and sign are placed a distance 'd' apart. The electric field intensity is zero at a point, not between the charges but on the line joining them. Write two essential conditions for this to happen.

Solution : (i) The two charges are of opposite signs i.e., one charge is positive and the other is negative.

(ii) The magnitude of charge nearer the null-point under consideration should be smaller than the magnitude of the other charge.

Q. n number of equal charges are placed at the corners of a regular polygon. What is the net electric field at the centre of the polygon ?

Solution : zero

Q. Five equal point charges are placed at the corner of a regular hexagon of side length a. If the magnitude of charge is q then what is the electric field at the centre of the hexagon ?

Solution : $\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}$

Q. Find the minimum value of electric field due to a charge at the distance 1m in vacuum ?

Solution : 14.4×10^{-10} N/C

1.9 Electric Field Lines :

Q. What is the concept of electric field lines ?

Solution : A convenient specialized pictorial representation for visualizing electric field patterns is created by drawing lines showing the direction of the electric field vector at any point. These lines, called electric field lines, are related to the electric field in any region of space in the following manner :

(i) The electric field vector $\vec{E}$ is tangent to the electric field line at each point.

(ii) The number of electric field lines per unit area through a surface that is perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, E is large where the field lines are close together and small where they are far apart.

Q. Write down the properties of field lines ?

Solution : (i) Field lines start from positive charge and end at negative charges. If there is a single charge, they may start or end at infinity.

(ii) In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.

(iii) Two field lines can never cross each other.

(iv) Electrostatic field lines do not form any closed loops.

Q. Why two field lines never crossed each other ?

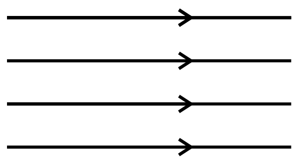
Solution : If they did, the field at the point of intersection will not have a unique direction, which is absurd.

Q. Why electrostatic field lines do not formed any closed loops ?

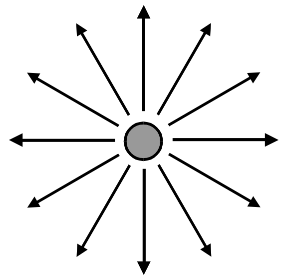
Solution : This follows from the conservative nature of electric field.

Q. Draw the lines of force to represent (i) uniform electric field (ii) positive charge (iii) negative charge (iv) two equal and opposite charges separated by some distance (v) two equal charges separated by some distance d.

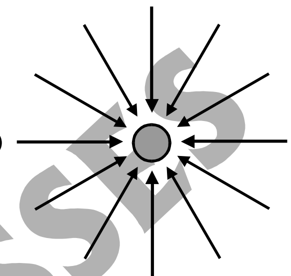
Solution : (i)



(ii)



(iii)



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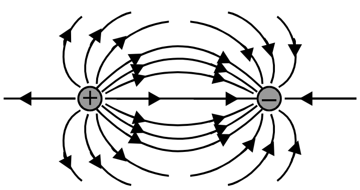

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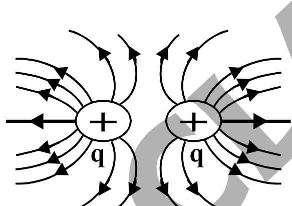
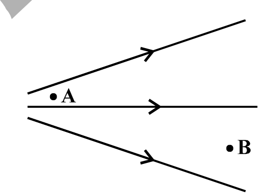


Q. Why the number of field lines crossing the enclosing area (electric flux) remains constant, whatever may be the distance of area from the charge ?

Solution : Since the electric field decreases as the square of the distance from a point charge and the area enclosing the charge increases as the square of the distance, the number of field lines crossing the enclosing area remains constant, whatever may be the distance of the area from the charge.

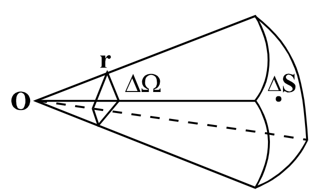
Q. At what point the electric field will be greater and why ?



Solution : The electric field at the point A is greater than the electric field at point B, since the field lines crowded where the field is strong and are spaced apart where it is weak.

Q. Define solid angle.

Solution : The solid angle is the angle subtended at O by a small perpendicular area ΔS, at a distance 'r' from O, can be written as $\Delta\Omega = \Delta S/r^2$.



Q. Is a field line a space curve or line curve ?

Solution : A field line is a space curve, i.e., a curve in three dimensions.

1.10 Electric Flux :

Q. Is area a vector or scalar quantity ?

Solution : Vector quantity, the direction of every area element of a closed surface is taken to be in the direction of outward normal.

Q. Define electric flux. Is it a vector or scalar quantity ?

Solution : Electric flux is proportional to the number of electric field lines that penetrate a surface. The electric flux through a surface is defined by the expression

$$\phi = \int \vec{E} . d\vec{A}$$

If $\vec{E}$ is uniform through out the surface then

$$\phi = \vec{E} . \vec{A} = EA \cos\theta$$

where θ is the angle that the electric field makes with the normal to the surface. It is a scalar quantity

Q. Write down the unit and dimension of electric flux.

Solution : The unit of electric flux is N C^{-1} m^2 and dimensional formula is $[ML^3T^{-3}A^{-1}]$.

Q. A square surface of sides √150 cm each is placed inside a uniform electric field of 25 Vm^{-1}, such that normal to the surface makes an angle of 60^0 with the direction of electric field. Find the flux of electric field through the square surface.

Solution : 0.18 Nm^2C^{-1}

Q. If the electric field is given by $(6\hat{i}+8\hat{j}+10\hat{k})$ unit, calculate the electric flux through a surface of area 40 units lying in Y – Z plane.

Solution : 240 units

Q. The electric field in a certain region of space is $(5\hat{i}+4\hat{j}-8\hat{k})\times 10^5$ N/C. Calculate electric flux due to this field over an area of $(2\hat{i}-4\hat{j})\times 10^{-2}m^2$.

Solution : 6×10^3 Nm^2C^{-1}

Q. What will be the angle between electric field and area vector of the surface such that the electric flux through the surface will be (a) maximum (b) minimum ?

Solution : (a) 0^0 i.e., the electric field vector is perependicular to the surface through which the electric flux will be determined (b) 90^0 i.e., electric field vector is in the plane of the surface through which the electric flux will be determined.

1.11 Electric Dipole :

Q. What is electric dipole ? What is the direction of electric dipole ?

Solution : An electric dipole is a pair of equal and opposite point charges q and –q, seperately by a distance 2a.

The line connecting the two charges defines a direction in space. By convention, the direction from –q to q is said to be the direction of the dipole.

Q. What is the centre of dipole ?

Solution : The mid-point of locations of –q and q is called the centre of the dipole.

Q. What is the total charge of electric dipole ?

Solution : zero

Q. Define electric deipole moment. Write down its unit and dimensional formula.

Solution : The dipole moment vector $\vec{p}$ of an electric dipole is defined by $\vec{p} = q \times 2a\,\hat{p}$ that is, it is a vector whose magnitude is charge q times the separation 2a (between the pair of charges q, –q) and the direction is along the line from –q to q. The unit of dipole moment is Cm and dimensional formula is [LTA].

Q. Derive an expression for the electric field intensity of an electric dipole at points (i) on the axis of the dipole (axial line of dipole) (ii) on the equatorial plane (on the perpendicular bisector of the line joining the two charges of dipole).

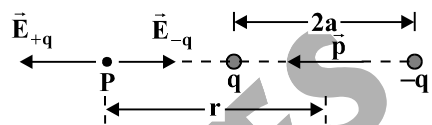
Solution : (i) For points on the axis.

Let the point P at distance r from the centre of the dipole on the side of the charge q.

$$\vec{E}_{-q} = -\left(\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{p}\right)$$

where $\hat{p}$ is the unit vector along the dipole axis (from – q to q). Also

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{p}$$



The total field at P is

$$\vec{E} = \vec{E}_{+q} + \vec{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

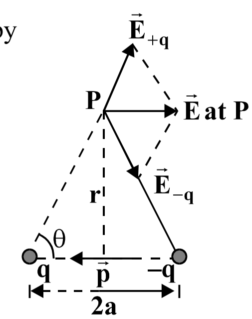
$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

For r > > a

$$\vec{E} = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3} (\because \vec{p} = q \times 2a\hat{p})$$

(ii) For points on the eqatorial plane

The magnitudes of the electric fields due to the two charges +q and –q are given by



$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2}$$

and are equal.

The directions of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are as shown in figure. Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to $\hat{p}$ .We have

$$\vec{E} = -(E_{+q} + E_{-q})\cos\theta\,\hat{p}$$

$$= -\frac{2qa}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}\hat{p}$$

At large distances (r >> a), this reduces to

$$\vec{E} = -\frac{2qa}{4\pi\varepsilon_0 r^3}\hat{p} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3} (\because \vec{p} = q \times 2a\hat{p})$$

Q. Write down the dependence of dipole field at very large distance from the dipole.

Solution : The electric field of a dipole at large distances is given by

(i) At a point on the dipole axis $\vec{E} = \dfrac{2\vec{p}}{4\pi\varepsilon_0 r^3}$ (r >> a)

(ii) At a point on the equatorial plane $\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3}$ (r >> a)

Hence, the dipole field at large distances falls off as $1/r^3$.

Q. On what factors the magnitude and direction of dipole field depends ?

Solution : The magnitude and the direction of the dipole field depends not only on the distance r but also on the angle between the position vector $\vec{r}$ and the dipole moment $\vec{p}$.

Q. What is point dipole ?

Solution : When the dipole size 2a approaches zero, the charge q approaches infinity in such a way that the product p = q × 2a is finite. Such a dipole is referred to as a point dipole.

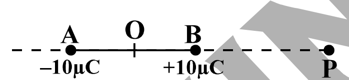
Q. Why the dipole moment of CO_2 and CH_4 equals to zero ?

Solution : In such molecules, the centres of positive charges and of negative charges lie at the same place. However, they develop a dipole moment when an electric field is applied. They are known as non-polar molecules.

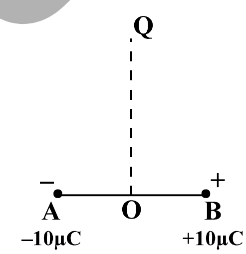
Q. What is polar molecules ? Give example.

Solution : Some molecules have the permanent electric dipole moment even in the absence of external electric field, known as polar molecules. Water is polar molecule.

Q. Two charges ± 10 μC are placed 5.0 mm apart. Determine the electric field at (a) a point P on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, as shown in figure (a) and (b) a point Q. 15 cm away from O on a line passing through O and normal to the axis of the dipole, as shown in figure (b). [NCERT solved example 1.10]



(a)



(b)

Solution : (a) 2.7×10^5 N C^{-1} along BP (b) 1.33×10^5 N C^{-1} along BA

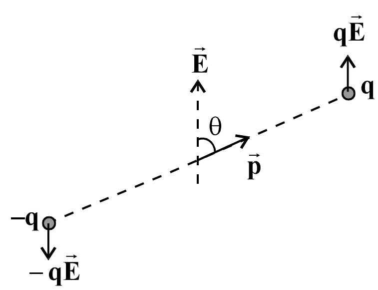
1.12 Dipole in a Uniform External Field :

Q. A permanent dipole of dipole moment $\vec{p}$ is placed in a uniform external field $\vec{E}$.

(a) What is force acting on the electric dipole and hence prove that no translatory motion of dipole is possible if it is released from rest ?

(b) Derive the expression for the torque acting on the dipole ?

Solution : (a) Consider a permanent dipole of dipole moment $\vec{p}$ in a uniform external field $\vec{E}$.



There is a force $q\vec{E}$ on q and a force $-q\vec{E}$ on –q. The net force on the dipole is zero, since $\vec{E}$ is uniform. Hence no translational motion is possible, if the dipole is released from rest.

(b) The charges are separated, so the forces act at different points, resulting in a torque on the dipole. When the net force is zero, the torque (couple) is independent of the origin. Its magnitude equals the magnitude of each force multiplied by the arm of the couple (perpendicular distance between the two antiparallel forces).

Magnitude of torque $= qE \times 2\,a \sin\theta$

$= 2q\,a\,E \sin\theta = pE \sin\theta$

Its direction is normal to the plane of the paper, coming out of it. The magnitude of $\vec{p}\times\vec{E}$ is also p E sin θ and its direction is normal to the paper, coming out of it. Thus, $\vec{\tau} = \vec{p}\times\vec{E}$.

Q. Which of the pair of vectors are always perpendicular in $\vec{\tau} = \vec{p}\times\vec{E}$? What is the value of $\vec{\tau}.\vec{p}$ and $\vec{\tau}.\vec{E}$?

Solution : $\vec{\tau}$ & $\vec{p}$ and $\vec{\tau}$ & $\vec{E}$ are always perpendicular.

$\vec{\tau}.\vec{p}$ **and** $\vec{\tau}.\vec{E}$ both equal to zero.

Q. How does a torque affect the dipole in an electric field ?

Solution : Torque tends to align the dipole along the field.

Q. What is the value of torque when dipole is parallel or antiparallel to $\vec{E}$?

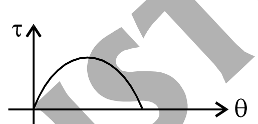
Solution : From $\tau = p\,E \sin\theta$, the torque on the dipole equals to zero when it is parallel or antiparallel to $\vec{E}$ because $\theta = 0$ when they are parallel and $\theta = \pi$ when they are anti-parallel.

Q. When is the torque on an electric dipole in a field is maximum and also give the maximum value ?

Solution : The torque is maximum when dipole is held at 90^0 to the field and its maximum value equals to pE.

Q. Draw the graph for the variation of torque acting on the dipole with angle θ between dipole and external field.

Solution :



Q. Which rule gives you the direction of torque acting on the dipole ?

Solution : Right handed screw rule.

Q. An electric dipole is held in a non uniform electric field. What is the force and torque acting on the dipole ?

Solution : The net force will be non-zero and the torque will be $\vec{p}\times\vec{E}$.

Q. If dipole is released from rest in uniform electric field then which of the motion is possible ?

Solution : Rotational motion is possible due to torque acting on the dipole.

Q. If a dipole is released from rest in uniform field such that dipole moment is parallel to field then which of the motion is possible : (i) translational motion (ii) rotational motion (iii) none. Explain it.

Solution : In uniform field, the force on the dipole equals to zero hence not translational motion is possible if it is released from rest.

If dipole moment and external field are parallel then torque acting on the dipole is also zero and hence no rotational motion is possible if it is released from rest.

Q. A dipole has given some angular velocity in external uniform field. Is the angular velocity of dipole variable and why ?

Solution : Yes, the angular velocity of dipole will change due to torque acting on the dipole.

Q. A comb run through dry hair attracts pieces of paper. The comb, as we know, acquires charge through friction. But the paper is not charged. What then explains the attractive force ?

Solution : The charge comb 'polarizes' the piece of paper, i.e., induced a net dipole moment in the direction of field. Further, the electric field due to the comb is not uniform.

1.13 Continuous Charge Distribution :

Q. Which law and the principle can be applied to find the electric field or force on a point charge due to discrete or continuous charge distribution or part discrete and part continuous charge distribution ?

Solution : Coulomb's law and the superposition principle.

Q. What is linear charge density and write down its unit ?

Solution : The linear charge density λ of a wire (linear charge distribution) is defined by $\lambda = \dfrac{\Delta Q}{\Delta l}$, where Δl is a small line element of wire on the macroscopic scale that includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that line element. The units for λ are C/m.

Q. What is surface charge density and write down its unit ?

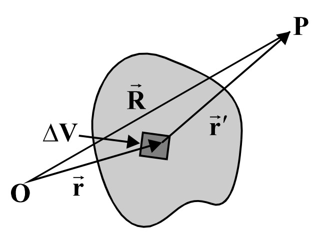
Solution : The surface charge density σ of a surface charge distribution is defined by $\sigma = \dfrac{\Delta Q}{\Delta S}$, where ΔS is area of small element of surface on the macroscopic scale that includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that area element. The units for σ are C/m^2.

Q. What is volume charge density and write down its unit ?

Solution : The volume charge density ρ of volume charge distribution is defined by $\rho = \dfrac{\Delta Q}{\Delta V}$, where ΔV is volume of small element on the macroscopic scale that includes a large number of microscopic charged constituents, and ΔQ is the charge contained in that volume element. The units for ρ are C/m^3.

Q. How do you apply superposition principle to obtain electric field at a point due to volume charge distribution at a time ?

Solution : Suppose a continuous charge distribution in space has a charge density ρ. Choose any convenient origin O and let the position vector of any point in the charge distribution be $\vec{r}$. The charge density ρ may vary from point to point i.e., it is function of $\vec{r}$. Divide the charge distribution into small volume elements of size ΔV. The charge in a volume element ΔV is ρΔV.



Now, consider any general point P (inside or outside the distribution) with position vector $\vec{R}$. Electric field due to the charge ρΔV is given by Coulomb's law : $\Delta\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\rho\Delta V}{r'^2}\hat{r}'$

where r' is the distance between the charge element and P, and $\hat{r}$ is a unit vector in the direction from the charge element to P. By the superposition principle, the total electric field due to the charge distribution is obtained by summing over electric fields due to the charge distribution is obtained by summing over electric fields due to different volume elements :

$$\vec{E} \cong \frac{1}{4\pi\varepsilon_0} \sum_{\text{all } \Delta V} \frac{\rho \Delta V}{r'^2} \hat{r}$$

Note that $\rho, r', \hat{r}'$ all can very from point to point. In a strict mathematical method, we should let $\Delta V \to 0$ and the sum then becomes an integral, given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\int \frac{\rho dV}{r'^2}\right)\hat{r}'$.

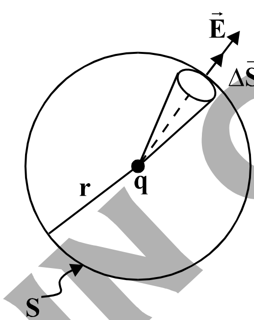
1.14 Gauss's Law :

Q. State Gauss's law in electrostatics.

Solution : Gauss's Law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge inside the surface divided by $\in_0$: $\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{in}}{\in_0}$.

Q. Using the Coulomb's law derive the Gauss law.

Solution : Let us consider the total flux through a sphere of radius r, which encloses a point charge q at its centre. Divide the sphere into small area elements, as shown in figure.



The flux through an area element $\Delta\vec{S}$ is $\Delta\phi = \vec{E}.\Delta\vec{S} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}.\Delta\vec{S}$, where we have used Coulomb's law for the electric field due to a single charge q. The unit vector $\hat{r}$ is along the radius vector from the centre to the area element. Now, since the normal to a sphere at every point is along the radius vector at that point, the area element $\Delta\vec{S}$ and $\hat{r}$ have the same direction. Therefore, $\Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$ since the magnitude of a unit vector is 1.

The total flux through the sphere is obtained by adding up flux through all the different area elements :

$$\phi = \sum_{\text{all } \Delta S} \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$$

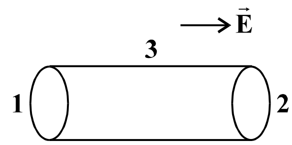
Since each area element of the sphere is at the same distance r from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2}\sum_{\text{all } \Delta S}\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}S$$

Now S, the total area of the sphere, equal $4\pi r^2$. Thus,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

Q. A cylinder is placed in a uniform electric field as shown in figure. Find the flux through the circular face and curved surface of the cylinder and also calculate the total flux through the cylinder.



Solution : The total flux ϕ through the surface is $\phi = \phi_1 + \phi_2 + \phi_3$, where ϕ_1 and ϕ_2 represents the flux through the surfaces 1 and 2 of the cylinder and ϕ_3 is the flux through the curved cylindrical part of the closed surface. Now the normal to the surface 3 at every point is perpendicular to $\vec{E}$, so by definition of flux, $\phi_3 = 0$. Further, the outward normal to 2 is along $\vec{E}$ while the outward normal to 1 is opposite to $\vec{E}$. Therefore,

$$\phi_1 = -E\,S_1, \qquad \phi_2 = +E\,S_2$$

$$S_1 = S_2 = S$$

where S is the area of circular cross-section. Thus, the total flux is zero as expected by Gauss's law.

Q. If the net electric flux through a closed surface is zero then can we conclude that the total charge contained in the close surface is also zero ?

Solution : Yes, from Gauss's Law.

Q. Is the Gauss's Law true for any type of closed surface, no matter what its shape or size ?

Solution : Yes

Q. What is Gaussian surface ?

Solution : The surface that we choose for the application of Gauss's law is called the Gaussian surface.

Q. Although we can take any Gaussian surface to apply Gauss Law. However, we have to take care not to let the Gaussian surface pass through any discrete charge. Why ?

Solution : This is because electric field due to a system of discrete charges is not well defined at the location of any charge. However, the Gaussian surface can pass through a continuous charge distribution.

Q. What is the use of Gauss's Law ?

Solution : (i) This law is fundamentally used to calculate the electrostatic flux through the closed surface.

(ii) This law is often useful to calculate the electric field when the system of charges has some symmetry which can be faciliated by the choice of suitable Gaussian surface.

Q. What is the significance of Gauss's Law ?

Solution : (i) Gauss's Law is true for any type of closed surface, no matter what its shape or size and independent of the location of the charge enclosed by the surface.

(ii) In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field is due to all the charges, both inside and outside S but the flux due to all the charges depends only on the total charge inside S.

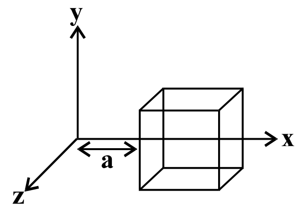
(iii) This law is often useful to calculate the electric field when the system of charges has some symmetry which can be faciliated by the choice of suitable Gaussian surface.

(iv) Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

Q. If Coulomb's law had $\frac{1}{r^n}$ dependence where $n \neq 2$, would Gauss's theorem had been still valid ?

Solution : No, Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

Q. The electric field components are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which α = 800 N/C $m^{1/2}$.



Calculate (a) the flux through the cube, and (b) the charge within the cube. Assume that a = 0.1 m. The length of the side of the cube is a. [NCERT solved example 1.11]

Solution : (a) 1.05 Nm^2/C (b) 9.27×10^{-12}C

Q. An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x. It is given that $\vec{E} = 200\hat{i}$ N/C for x > 0 and $\vec{E} = -200\hat{i}$ N/C for x < 0. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at x = +10 cm and other is at x = –10 cm. (a) What is the net outward flux through each flat face ? (b) What is the flux through the side of the cylinder ? (c) What is the net outward flux through the cylinder ? (d) What is the net charge inside the cylinder ? [NCERT solved example 1.12]

Solution : (a) 1.57 Nm^2/C (b) 0 (c) 3.14 Nm^2/C (d) 2.78×10^{-11}C

1.15 Applications of Gauss's Law :

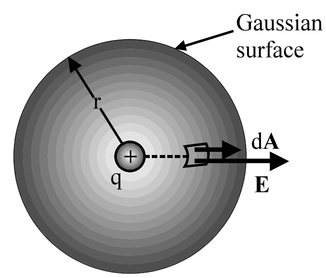
Q. For which type of charge configuration, Gauss's law is used to determine the electric field ?

Solution : For symmetric charge configuration, Gauss's law is used to determine the electric field.

Q. Derive Coulomb's law using the Gauss's law or find the electric field due to a point charge q at the distance 'r' from the charge ?

Solution : We choose a spherical gaussian surface of radius r centered on the point charge, as in figure. The electric field of a positive point charge is radial outward by symmetry and is therefore normal to the surface at every point. **E** is therefore parallel to d**A** at each point on the surface, as so

E . d**A** = E dA and Gauss's law gives $\Phi_E = \oint \mathbf{E}.d\mathbf{A} = \oint E dA = \frac{q}{\epsilon_0}$



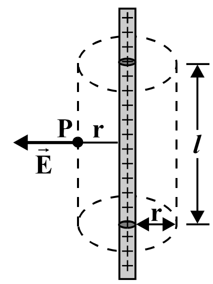
The electric field at each point of the surface remains constant and hence

$$E\oint dA = \frac{q}{\epsilon_0} \Rightarrow E(4\pi r^2) = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{4\pi \epsilon_0 r^2}$$

which is the familiar electric field of a point charge that we developed from Coulomb's law.

Q. Apply Gauss's law to calculate the field intensity at a point from a infinite line charge of charge per unit length λ. Also draw the variation of electric field with distance.

Solution : Consider an infinitely long thin straight wire with uniform linear charge density λ.



To calculate the field, imagine a cylindrical Gaussian surface, as shown in figure. Since the field is everywhere radial, flux through the two ends of the cylindrical Gaussian surface is zero. At the cylindrical part of the surface, $\vec{\mathbf{E}}$ is normal to the surface at every point, and its magnitude is constant, since it depends only on r. The surface area of the curved part is $2\pi r l$, where l is the length of the cylinder.

Flux through the Gaussian surface

= flux through the curved cylindrical part of the surface

= $E \times 2\pi r l$

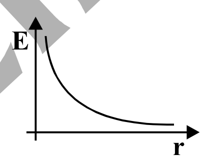
The surface includes charge equal to $\lambda\, l$. Gauss's law then gives $E \times 2\pi r l = \lambda l/\varepsilon_0$

i.e., $$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Vectorially, $\vec{\mathbf{E}}$ at any point is given by $\vec{\mathbf{E}} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{n}}$

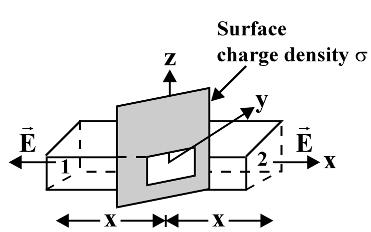
where $\hat{\mathbf{n}}$ is the radial unit vector in the plane normal to the wire passing through the point. $\vec{\mathbf{E}}$ is directed outwards if λ is positive and inward if λ is negative.

As E is inversely proportional to r hence the graph of E v/s r is hyperbolic which is given by :



Q. Apply Gauss's law to calculate the field intensity at a point near an infinite plane sheet of charge of uniform charge density σ or prove that the field at a point near an infinite plane sheet of charge does not depend upon the distance or find the field due to uniformly charged infinite plane sheet placed in the yz plane at a point on the x-axis or find the field due to a finite larger planar sheet in the middle region of the planar sheet, away from the ends . Also draw the variation of electric field with distance.

Solution : Let σ be the uniform surface charge density of an infinite plane sheet. We take the x-axis normal to the given plane. By symmetry, the electric field will not depend on y and z coordinates and its direction at every point must be parallel to the x-direction.



We can take the Gaussian surface to be a rectangular parallelopiped of cross sectional area A, as shown. (A cylindrical surface will also do.) As seen from the figure, only the two faces 1 and 2 will contribute to the flux; electric field lines are parallel to the other faces and they, therefore, do not contribute to the total flux.

The unit vector normal to surface 1 is in –x direction while the unit vector normal to surface 2 is in the +x direction. Therefore, flux $\vec{E}.\Delta\vec{S}$ through both the surfaces are equal and add up. Therefore the net flux through the Gaussian surface is 2 EA. The charge enclosed by the closed surface is σA. Therefore by Gauss's law,

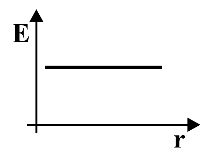
$$2\,EA = \sigma A/\varepsilon_0$$

$$\text{or, } E = \sigma/2\varepsilon_0$$

$$\text{Vectorically, } \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

where $\hat{n}$ is a unit vector normal to the plane and going away from it.

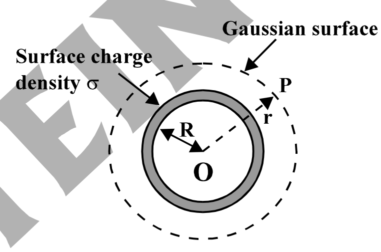
In this case, the electric field is independent of distance and the graph of field with distance r is given by :



Q. Apply Gauss's law to calculate the field intensity at any point outside due to uniformally charged thin spherical shell or Prove that for points outside the shell, field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre or Prove that for points outside the shell, field due to a uniformly charged shell is same as the field produced by point charge.

Solution : Let σ be the uniform surface charge density of a thin spherical shell of radius R.

Consider a point P outside the shell will radius vector $\vec{r}$. To calculate $\vec{E}$ at P, we take the Gaussian surface to be a sphere of radius r and with centre O, passing through P. All points on this sphere are equivalent relative to the given charged configuration. (That is what we mean by spherical symmetry.)



The electric field at each point of the Gaussian surface, therefore, has the same magnitude E and is along the radius vector at each point. Thus $\vec{E}$ and $\Delta\vec{S}$ at every point are parallel and the flux through each element is EΔS. Summing over all ΔS, the flux through the Gaussian surface is $E \times 4\pi r^2$. The charge enclosed is $\sigma \times 4\pi R^2$. By Gauss's law

$$E \times 4\pi r^2 = \frac{\sigma}{\varepsilon_0}4\pi R^2$$

Or, $$E = \frac{\sigma R^2}{\varepsilon_0 r^2} = \frac{q}{4\pi\varepsilon_0 r^2}$$

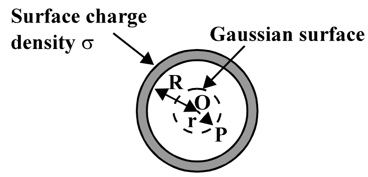
where $q = 4\pi R^2\sigma$ is the total charge on the spherical shell. Vectorially,

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

Thus for points outside the shell, the field due to a uniformly charged shell is as if the entire charge of the shell is concentrated at its centre. This, however, is exactly the field produced by a charge q placed at the centre O.

Q. Apply Gauss's law to calculate the field intensity at any point inside due to uniformally charged thin spherical shell or prove that the field due to a uniformly charged thin spherical shell is zero at all points inside the shell.

Solution : In figure, the point P is inside the shell. The Gaussian surface is again a sphere through P centred at O.



The flux through the Gaussian surface, calculated as before, is $E \times 4\pi r^2$. However, in this case, the Gaussian surface encloses no charge. Gauss's law then gives

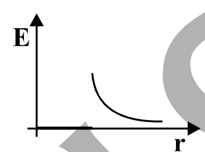
$E \times 4\pi r^2 = 0$

i.e., $E = 0$ $(r < R)$

that is, the field due to a uniformly charged thin shell is zero at all points inside the shell.

Q. Draw the graph for variation of field intensity at any point inside and outside due to uniformally charged thin spherical shell.

Solution : The graph for the variation of electric field with distance 'r' from the center of the spherical shell is given by :



Q. An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus (inside and outside the nucleus). [NCERT solved example 1.13]

Solution : $E = \dfrac{Ze}{4\pi\epsilon_0}\left(\dfrac{1}{r^2} - \dfrac{r}{R^3}\right)$; $r < R$, $E = 0$; $r > R$

NCERT EXERCISE

1.1 **What is the force between two small charged spheres having charges of 2×10^{-7}C and 3×10^{-7}C placed 30 cm apart in air ?**

1.2 **The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge –0.8 μC in air is 0.2 N.**

(a) **What is the distance between the two spheres ?**

(b) **What is the force on the second sphere due to the first ?**

1.3 **Check that the ratio $\dfrac{ke^2}{Gm_e m_p}$ is dimensionless. What does the ratio signify ?**

1.4 **(a) Explain the meaning of the statement 'electric charge of a body is quantised'.**

(b) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges ?

1.5 **When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.**

1.6 Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\ \mu C$ and $q_D = -5\ \mu C$ are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square ?

1.7 (a) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not ?

(b) Explain why two field lines never cross each other at any point ?

1.8 Two point charges $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum.

(a) What is the electric field at the midpoint O of the line AB joining the two charges ?

(b) If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge ?

1.9 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, –15 cm) and B : (0, 0, +15 cm), respectively. What are the total charge and electric dipole moment of the system ?

1.10 An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30^0 with the direction of a uniform electric field of magnitude $5 \times 10^4\ NC^{-1}$. Calculate the magnitude of the torque acting on the dipole.

1.11 A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which ?)

(b) Is there a transfer of mass from wool to polythene ?

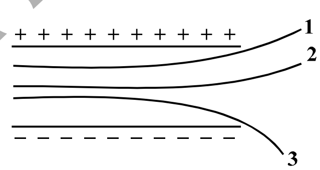
1.12 Two insulated small charged sphere A and B have their centres separated by distance of 50 cm. Each sphere has the charge 0.65 μC.

(a) What is the ratio of final force to initial force if the distance between them will become halved and charge will become doubled ?

(b) Suppose the spheres A and B have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B ?

1.13 Suppose the spheres A and B in exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B ?

1.14 Figure shows tracks of four particles in a uniform electrostatic field. Give the signs of each charges. Which particle has the highest charge to mass ratio ?

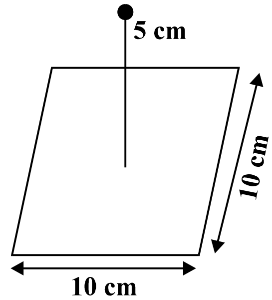


1.15 Consider a uniform electric field $\vec{E} = 3 \times 10^3\ \hat{i}$ N/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz plane ? (b) What is the flux through the same square if the normal to its plane makes a 60^0 angle with the x-axis ? (c) What is the net flux of the uniform electric field through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes ?

1.16 What is the net flux of the uniform electric field of exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes ?

1.17 Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3\ Nm^2/C$. (a) What is the net charge inside the box ? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box ? Why or Why not ?

1.18 A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm. What is the magnitude of the electric flux through the square ?



1.19 A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface ?

1.20 A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centred on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface ? (b) What is the value of the point charge ?

1.21 A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere ?

1.22 A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 μC/m^2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere ?

1.23 An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density.

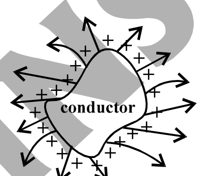
1.24 Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-12} C/m^2. What is $\vec{E}$: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates ?

<u>ADDITIONAL EXERCISES</u>

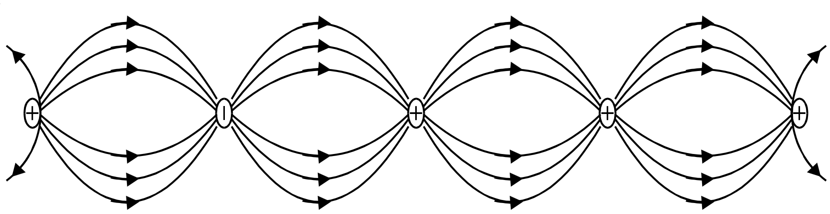
1.25 An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 NC^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81$ m s^{-2}; $e = 1.60 \times 10^{-19}$ C).

1.26 Why the following pattern does not represent the electric field ?

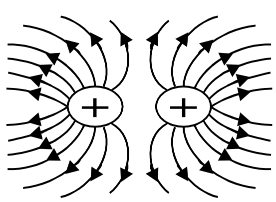
(a)



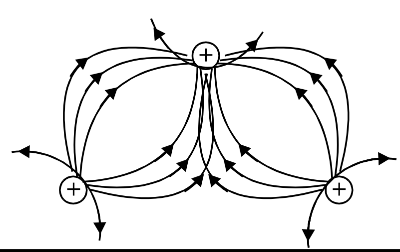
(b)

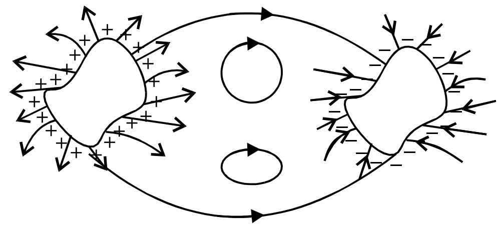


(c)



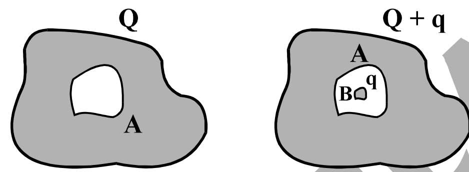
(d)



(e) 

1.27 In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive z-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative z-direction ?

1.28. (a) A conductor A with a cavity as shown in figure (a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is Q + q [fig. b]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.



(a) (b)

1.29 A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $(\sigma/2\varepsilon_0)\ \hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

1.30 Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Hint : Use Coulomb's law directly and evaluate the necessary integral]

1.31 It is now believed that protons and neutrons (which constitute nuclei or ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge + (2/3) e, and the 'down' quark (denoted by d) of charge (–1/3) e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter). Suggest a possible quark composition of a proton and neutron.

1.32 (a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where $\vec{E}$ = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

1.33 A particle of mass m and charge (–q) enters the region between the two charged plates initially moving along x-axis with speed v_x . The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at far edge of the plate is $qEL^2/(2m\ v_x^2)$.

1.34 Suppose that the particle is an electron projected with velocity $v_x = 2.0 \times 10^6$ m s^{-1}. If E between the plates separated by 0.5 cm is 9.1×10^2 N/C, where will the electron strike the upper plate ?

($|e| = 1.6 \times 10^{-19}$C, $m_e = 9.1 \times 10^{-31}$ kg).

ANSWERS

1.1 6×10^{-3} N (repulsive)

1.2 (a) 12 cm (b) 0.2 N (attractive)

1.3 2.4×10^{39}. This is the ratio of electric force of the gravitational force (at the same distance) between an electron and a proton.

1.5 Charge is not created or destroyed. It is merely transferred from one body to another.

1.6 Zero N

1.8 (a) 5.4×10^{6} N C^{-1} along OB (b) 8.1×10^{-3} N along OA

1.9 Total charge is zero. Dipole moment = 7.5×10^{-8} C m along z-axis.

1.10 10^{-4} N m

1.11 (a) 2×10^{12}, from wool to polythene. (b) Yes, but of a negligible amount (= 2×10^{-18} kg in the example)

1.12 (a) 1.5×10^{-2} N (b) 0.24 N

1.13 5.7×10^{-3} N

1.14 Charges 1 and 2 are negligible, charge 3 is positive. Particle 3 has the highest charge to mass ratio.

1.15 (a) 30 Nm^2/C, (b) 15 Nm^2/C

1.16 Zero. The number of lines entering the cube is the same as the number of lines leaving the cube.

1.17 (a) 0.07 μC (b) No, only that the net charge inside is zero.

1.18 2.2×10^{5} N m^2/C

1.19 1.9×10^{5} N m^2/C

1.20 (a) -10^{3} N m^2/C; because the charge enclosed is the same in the two cases.

(b) –8.8 nC

1.21 –6.67 nC

1.22 (a) 1.45×10^{-3} C (b) 1.6×10^{8} Nm^2/C

1.23 10 μC/m

1.24 (a) Zero, (b) Zero, (c) 1.9 N/C

1.25 9.81×10^{-4} mm

1.26 Only (c) is right; the rest cannot represent electrostatic field lines, (a) is wrong because field lines must be normal to a conductor, (b) is wrong because field lines cannot start from a negative charge, (d) is wrong because field lines cannot intersect each other, (e) is wrong because electrostatic field lines cannot form closed loops.

1.27 The force is 10^{-2} N in the negative z-direction, that is, in the direction of decreasing electric field. You can check that this is also the direction of decreasing potential energy of the dipole; torque is zero.

1.28 (a) Hint : Choose a Gaussian surface lying wholly within the conductor and enclosing the cavity.

(b) Gauss's law on the same surface as in (a) shows that q must induce –q on the inner surface of the conductor.

(c) Enclose the instrument fully by a metallic surface.

1.29 Hint : Consider the conductor with the hole filled up. Then the field just outside is $(\sigma/\varepsilon_0)\,\hat{n}$ and is zero inside. View this field as a superposition of the field due to the filled up hole plus the field due to the rest of the charged conductor. Inside the conductor, these fields are equal and opposite. Outside they are equal both in magnitude and direction. Hence, the field due to the rest of the conductor is

$$\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{n}.$$

1.31 p;uud; n;udd

1.32 **(a) Hint : Prove it by contradiction. Suppose the equilibrium is stable; then the test charge displaced slightly in any direction will experienced a restoring force towards the null-point. That is, all field lines near the null point should be directed inwards towards the null-point. That is, there is a net inward flux of electric field through a closed surface around the null-point. But by Gauss's law, the flux of electric field through a surface, not enclosing any charge, must be zero. Hence, the equilibrium cannot be stable.**

(b) The mid-point of the line joining the two charges is a null-point. Displace a test charge from the null-point slightly along the line. There is a restoring force. But displace it, say, normal to the line. You will see that the net force takes it away from the null-point. Remember, stability of equilibrium needs restoring force in all directions.

1.34 **1.6 cm**

ADDITIONAL QUESTIONS AND PROBLEMS

Q. **A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.**

(a) **Estimate the number of electrons transferred (from which to which)**

(b) **Is there a transfer of mass from wool to polythene ?**

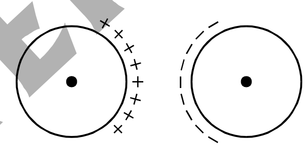
A. (a) 2×10^{12} (b) yes

Q. **Find the electric field due to a uniformly charged ring of total charge q at the distance 'a' on the axis of the ring (radius 'r') from the centre ?**

A. $\dfrac{qr}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}$

Q. **Two charged spherical conductors, each of radius R, are distant d (d > 2R). They carry charges +q and –q. Will the force of attraction between them be exactly $\dfrac{q^2}{4\pi \in_0 d^2}$.**

A. No, the force of attraction between charged spherical conductors will be more than $\dfrac{q^2}{4\pi\varepsilon_0 d^2}$.



On account of mutual attractive force there will be a redistribution of charges on the spheres as shown.As a result, effective distance between them is reduced and the force increases.

Q. **A charge Q is broken into two parts and placed at certain distance d such that the force between them is maximum, find the maximum force ?**

A. $\dfrac{kQ^2}{4d^2}, k = \dfrac{1}{4\pi\varepsilon_0}$

Q. **Two identical pith balls, each of mass m and charge +q, are suspended from a point with threads of length *l* each. If θ be the angle which each thread makes with vertical in equilibrium, find value of charge on each ball.**

A. $2l \sin\theta \sqrt{4\pi \in_0 mg \tan\theta}$

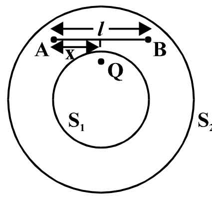
Q. **In a uniform horizontal electric field E in the vertical plane, a positive point charge q and mass m is suspended by a thread. The other end of the thread is fixed. If acceleration due to gravity is g then find the tension in the string at equilibrium and angle made by the string with vertical ? If electric field will be vertically downward then find the tension in the string.**

A. $T = \sqrt{(qE)^2 + (mg)^2}$, $\theta = \tan^{-1}\dfrac{qE}{mg}$, $T = qE + mg$

Q. **In a uniform electric field E, a particle of charge q and mass m is released. Find the distance travelled by the particle in time t. Also calculate the kinetic energy of the particle at this moment ?**

A. $\frac{q^2E^2t^2}{2m}$

Q. **In figure, calclate the total flux of the electrostatic field through the spheres S_1 and S_2. The wire AB, shown here, has a linear charge density λ given by $\lambda = kx$, where x is the distance measured along the wire from the end A.**

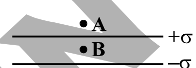


A. $\frac{Q}{\epsilon_0}, \frac{Q + \frac{kl^2}{2}}{\epsilon_0}$

Q. **Define the term electric flux. State its unit. Is electric flux a scalar quantity ?**
A sphere S_1 of radius r_1 enclose a charge Q. If there is another concentric sphere S_2 of radius r_2 ($r_2 > r_1$) and there be no additional charges between S_1 and S_2, find the ratio of electric flux through S_1 and S_2. If there is an additional charge –2Q between S_1 and S_2 then find the ratio of electric flux through them. If there is another charge +3Q outside S_2 then find the ratio of electric flux through them.

A. 1 : 1, 1 : –1, 1 : –1

Q. **Two plane sheets of charge densities $+\sigma$ and $-\sigma$ are kept in air as shown in the figure. What are the electric field intensities at point A and B ?**



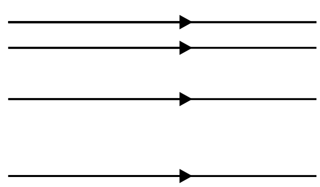
A. **0, $\frac{\sigma}{\epsilon_0}$**

5. **There is a point charge q each at the four corners of a square. What will be the ratio of force between the charges at adjacent corners and the charges at opposite corners of the square ?**

A. 2 : 1

6. **Can any set of parallel field represent a electrostatic field ? Justify your answer.**

A. No, for e.g., such type of parallel field lines which are not equi spaced represents electrostatic field.



7. **Two like point charges q each at a separation R in air exert a force of magnitude F. They are now immersed in a non-conducting oil of dielectric constant 16. What should the new separation between the charges be so that the force between them remains unchanged ?**

A. R/4

Q. **Apply Gauss's law to calculate the field intensity at any point inside and outside of the solid non-conducting charged sphere. Also draw the variation of electric field with distance.**

A. $E = \frac{\rho r}{3\varepsilon_0} r < R, E = \frac{q}{4\pi\varepsilon_0 r^2} r \geq R \left(q = \rho \frac{4}{3}\pi R^3 \right)$, **$\rho$ is volume charge density**