Heat and Thermodynamics

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HEAT AND THERMODYNAMICS

C1 Zeroth law of thermodynamics

This law defines the concept of temperature and thermal equilibrium. When two bodies are in thermal equilibrium, their temperature are equal and vice versa. If bodies A and B are each in thermal equilibrium with a third body C, then A and B are in thermal equilibrium with each other.

C2 Measuring Temperature

Thermometer is used to measure the temperature using the above law. If the thermometric property at temperature at 0^0C, 100^0C and T_c ^{0}C is X_0, X_{100} and X respectively then $T_C = \frac{X - X_0}{X_{100} - X_0} \times 100^0 C$.

The relation between Celsius (C), Fahrenheit (F) and Kelvin (K) are given by $\frac{C}{5} = \frac{F-32}{9}$ and K = 273 + C.

Practice Problems :

1. A constant volume gas thermometer shows pressure readings of 50 cm and 90 cm of mercury at 0^0C and 100^0C, respectively. When the pressure reading is 60 cm of mercury, the temperature is

(a) 25^0C **(b)** 40^0C **(c)** 15^0C **(d)** 12.5^0C

[Answers : (1) (a)]

C3 Thermal Expansion

(i) Linear Expansion : $l = l_0(1 + \alpha\Delta T)$, where l_0 is the initial length and l is final length and α is the coefficient of the linear expansion and ΔT is the change in temperature.

(ii) Area expansion : $A = A_0(1 + \beta\Delta T)$, where A_0 is the initial area and A is the final area. Here $\beta = 2\alpha$ (for solid), is coefficient of area expansion.

(iii) Volume expansion : $V = V_0(1 + \gamma\Delta T)$, where V_0 is the initial volume, V is the final volume and $\gamma = 3\alpha$ (for liquid), is the coefficient of volume expansion.

Practice Problems :

1. Two rods of length l_1 and l_2 are made of materials whose coefficients of linear expansion are α_1 and α_2, respectively. The difference between their lengths will be independent of temperature if l_1/l_2 is equal to

(a) $\frac{\alpha_1}{\alpha_2}$ **(b)** $\frac{\alpha_2}{\alpha_1}$ **(c)** $\left(\frac{\alpha_1}{\alpha_2}\right)^{1/2}$ **(d)** $\left(\frac{\alpha_2}{\alpha_1}\right)^{1/2}$

(b)

2. A vessel of volume V and linear coefficient of expansion α contains a liquid. The level of liquid does not change on heating. The volume coefficient of real expansion of the liquid is

(a) $\frac{V+\alpha}{V}$ **(b)** $\frac{V-\alpha}{V}$ **(c)** $\frac{V}{V-\alpha}$ **(d)** 3α

(d)

[Answers : (1) b (2) d]

C4 Effect of temperature on density

$\rho = \frac{\rho_0}{1+\gamma\Delta T}$ where ρ_0 is the initial density and ρ is the final density

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C5 Thermal Stress

When a rod is heated or cooled, it expands or contracts. If it is prevented from the expansion or contraction, then stresses are produced in it corresponding to the thermal strain which are given by Thermal Strain = $\alpha\Delta T$, Thermal Stress = $Y\alpha\Delta T$, Force = $YA\alpha\Delta T$ where α is the coefficient of linear expansion, Y is the young's modulus of elasticity, A is the cross-sectional area of the rod and ΔT is the change in temperature.

Practice Problems :

1. A steel rod of length 25 cm has a cross-sectional area of 0.8 cm^2. The force required to stretch this rod by the same amount as the expansion produced by heating it through 10^0C is (coefficient of linear expansion of steel is $10^{-5}/^0C$ and Young's modulus of steel is 2×10^{10} N/m^2)

(a) 40 N (b) 80 N (c) 120 N (d) 160 N

[Answers : (1) d]

C6 Ideal Gas

An ideal gas is one for which the pressure P, volume V, and temperature T are related by

$$PV = nRT \text{ or } P = n_0kT \text{ or } \frac{P}{\rho} = \frac{RT}{M}.$$

Here n : no of moles, R : Gas constant, n_0 : No. of moles per unit volume

k : Boltzmann's constant, ρ : Density of Gas, M : Molar mass of Gas

Practice Problems :

1. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of

(a) P/8 (b) P (c) 2P (d) 8P

2. Two gases A and B having the same temperature T, same pressure P and same volume V are mixed. If the mixture is at the same temperature T and occupies a volume V, the pressure of the mixture is

(a) 2P (b) P (c) P/2 (d) 4P

[Answers : (1) c (2) a]

C7 Kinetic Interpretation of Pressure

The pressure exerted by an ideal gas, in terms of the speed of its molecules is

$$p = \frac{mv^2_{rms}}{3V} = \frac{1}{3}\rho v^2_{r.m.s} \qquad \left(\rho = \frac{m}{V}\right)$$

where v_{rms} is the root-means-square speed of the molecules of the gas. Here m is the mass of the gas.

Hence $v_{r.m.s.} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}}$. There is another two speeds of the gas molecules according to

Maxwell Speed Distribution :

Average speed, $v_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8kT}{\pi m}}$

Most probable Speed (the speed at which the number of molecules is maximum)

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

Here $v_{rms} > v_{avg} > v_{mp}$.

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Practice Problems :

1. **The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes**

   (a) 4v (b) 2v (c) v/2 (d) v/4

2. **At room temperature the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. The gas is**

   (a) H_2 (b) F_2 (c) O_2 (d) Cl_2

3. **If the rms velocity of oxygen molecule at certain temperature is 0.5 km/s, the rms velocity for hydrogen molecule at the same temperature will be**

   (a) 2 km/s (b) 4 km/s (c) 9 km/s (d) 16 km/s

4. **Find the percentage change in the speed of gas molecule if the percentage change in temperature is (a) 1% (b) 44%.**

5. **The rms speed of the gas molecule O_2 is v at the temperature T. If the temperature will become twice and molecule is dissociated into atoms then find new rms speed ?**

[Answers : (1) b (2) a (3) a (4) (a) ½% (b) 10% (5) 2v]

C8 Specific Heat Capacity and Calorimetry

Heat capacity per unit mass is known as specific heat capacity i.e., $\mathbf{s = \frac{dQ}{mdT}} \Rightarrow \int \mathbf{dQ = m\int sdT}$. If s is constant then $Q = ms(T_f - T_i)$.

Principle of Calorimetry : Heat lost = heat gained.

C9 Heat of Transformation

The amount of energy required per unit mass to change the phase (but not the temperature) of a particular material is its heat of transformation L. Thus. Q = Lm.

Practice Problems :

1. **One gm of ice at 0^0C is added to 5 gm of water at 10^0C. If the latent heat is 80 cal/gm, the final temperature of the mixture is**

   (a) 5^0C (b) 0^0C (c) -5^0C (d) None of the above

2. **200 gm of a solid ball at 20^0C is dropped in an equal amount of water at 80^0C. The resulting temperature is 60^0C. This means that specific heat of solid is**

   (a) One fourth of water (b) One half of water

   (c) Twice of water (d) Four times of water

3. **Calculate the final temperature of the mixture and mass of water in the following cases :**

   (a) 10 g of ice is mixed with 10 g of water at 10^0C

   (b) 10 g of ice is mixed with 10 g of water at 80^0C

   (c) 10 g of ice is mixed with 10 g of water at 90^0C

   (d) 10 g of ice is mixed with 100 g of water at 10^0C

[Answers : (1) b (2) b (3) (a) 0^0C, 11.25 g (b) 0^0C, 20 g (c) 5^0C, 20 g (d) 110g]

C10 Thermodynamic Work Done

The work performed by a system at pressure P expands from volume V_1 to V_2 is given by $W = \int_{V_1}^{V_2} PdV$.

Here P may be a constant or change during the volume change. The work done equals the area under the curve on a P-V diagram as shown in figure.

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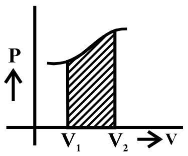
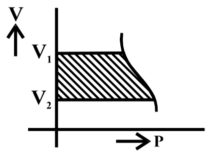




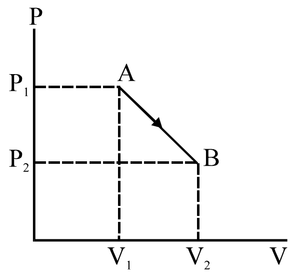
The work done in **cyclic process** (a process in which the thermodynamic variables periodically return to their original values) is equal to the area enclosed by the cycle.

Positive Work : If the cycle is clockwise on P – V and anticlockwise on V – P.

Negative Work : If the cycle is anticlockwise on P – V and clockwise on V – P.

Practice Problems :

1. **The work performed for the process shown in figure is**



(a) $\frac{1}{2}(P_1 + P_2)(V_2 - V_1)$ (b) $\frac{1}{2}(P_1 - P_2)(V_2 - V_1)$

(c) $\frac{1}{2}(P_1 - P_2)(V_2 + V_1)$ (d) $\frac{1}{2}(P_1 + P_2)(V_2 + V_1)$

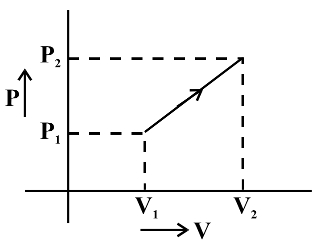
2. **If the volume will change in the following process from V_1 to V_2 then determine the work done in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) PV =K (iii) PT = K (iv) $PV^{\alpha} = K$ (v) VT = K (vi) V = KT where P, V and T are pressure volume and temperature of gas and K is a constant quantity.**

3. **If the pressure will change in the following process from P_1 to P_2 then determine the work done in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) PV =K (iii) PT = K (iv) $PV^{\alpha} = K$ (v) VT = K (vi) P = KT where P, V and T are pressure volume and temperature of gas and K is a constant quantity.**

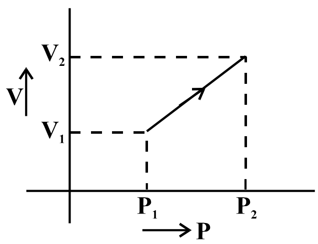
4. **If the temperature will change in the following process from T_1 to T_2 then determine the work done in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $P^2V = K$ (iii) PT = K (iv) $PV^{\alpha} = K$ (v) VT = K (vi) V = KT where P, V and T are pressure volume and temperature of gas and K is a constant quantity.**

5. **In the following process represented on PV diagram determine the work done**

(a)



(b)



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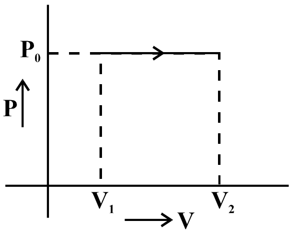
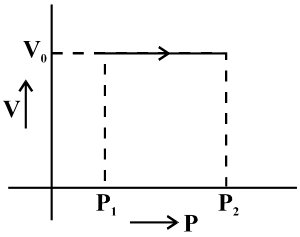
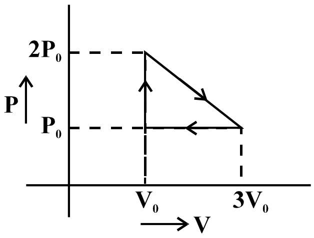
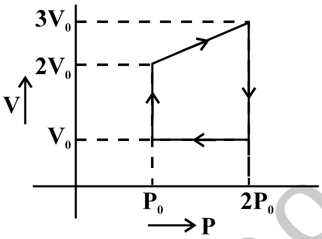
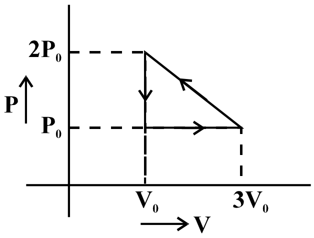
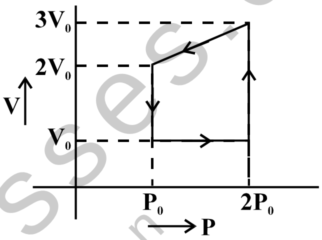
(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

[Answers : (1) a]

C11 Internal Energy

For an ideal gas, it is the sum of all types of kinetic energy associated with gas molecules. Remember the following points for internal energy of an ideal gas :

1. Internal energy at temperature T for an ideal gas of n moles is $U = n\frac{f}{2}RT$

   where f is the degrees of freedom of gas. The value of f for monoatomic gas is (He, Ne etc.) 3 whereas for diatomic gas (O_2, H_2, N_2 etc) is 5

2. Internal energy for mixture of gaseous is $U = n_1\frac{f_1}{2}RT + n_2\frac{f_2}{2}RT + ...$

3. The change in internal energy, $dU = nC_VdT$ for any thermodynamic process.

   Here C_V is the molar heat capacity at constant volume which equals to $\frac{f}{2}R$.

4. Internal energy is state dependent function and hence for a cyclic process, dU = 0.

<u>**Practice Problems :**</u>

1. Find the internal energy of ideal monoatomic gas of 5 mol. at 27^{0}C.

2. Find the internal energy of the gaseous mixture consists of 1 mol. of H_2 gas and 3 moles of He gas at the temperature T.

3. Find the internal energy of the gaseous mixture consists of N_1 molecules of H_2 gas and N_2 molecules of He gas at the temperature T.

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4. **If the volume will change in the following process from V_1 to V_2 then determine the change in internal energy in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $PV = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $V = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

5. **If the pressure will change in the following process from P_1 to P_2 then determine the change in internal energy in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $PV = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $P = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

6. **If the temperature will change in the following process from T_1 to T_2 then determine the change in internal energy in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $P^2V = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $V = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

C12 First Law of Thermodynamics: This law is based on conservation of energy and in mathematical form the law is given by $dQ = dU + dW$

Heat and work are path dependent whereas internal energy is a state dependent.

Signs for heat and work in thermodynamics

(a) When heat is added to a system or absorbed by system, Q is positive

(b) When heat is transferred out of the system or rejected by system Q is negative

(c) When work is done by the system W is positive

(d) When work is done on the system W is negative.

Practice Problems :

1. **If the volume will change in the following process from V_1 to V_2 then determine the amount of heat in each case assume that the gas is ideal on which the process is performed :(i) $P = KV^2$ (ii) $PV = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $V = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

2. **If the pressure will change in the following process from P_1 to P_2 then determine the amount of heat in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $PV = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $P = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

3. **If the temperature will change in the following process from T_1 to T_2 then determine the amount of heat in each case assume that the gas is ideal on which the process is performed : (i) $P = KV^2$ (ii) $P^2V = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $V = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity. The degrees of freedom of the gas is f.**

4. **Whether the following statements are true or false. Give reasons**
   - **(i) The temperature must rise if the heat is supplied to the gas**
   - **(ii) The temperature must rise if the heat is supplied to solid or liquid**
   - **(iii) The gas must expand if the heat is supplied and temperature will decrease**
   - **(iv) The gas must expand if the heat is supplied and temperature will increase**
   - **(v) Without involving the amount of heat, the temperature of the gas may change.**

C13 Molar Heat Capacity

Now we define the molar heat capacity of two special thermodynamic process :

(i) Molar heat capacity at constant volume (C_V) : It is defined as the heat required to raise the temperature by 1K of 1 mole of gas at constant volume. Mathematically

$$C_V = \frac{(dQ)_V}{ndT} \Rightarrow (dQ)_V = nC_V dT$$

Here $(dQ)_V$ is the required heat for this process i.e. for isochoric process.

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(ii) Molar heat capacity at constant pressure (C_P) : It is defined as the heat required to raise the temperature by 1 K of 1 mole of gas at constant pressure. Mathematically

$$C_P = \frac{(dQ)_P}{ndT} \Rightarrow (dQ)_P = nC_P dT$$

Here $(dQ)_p$ is the required heat for this process i.e. for isobaric process to raise the temperature by dT on n moles of gas.

Remember the following points :

(i) C_P is always greater than C_V

(ii) $C_P – C_V = R$ for ideal gas

(iii) The ratio $\frac{C_P}{C_V}$ is known as adiabatic exponent (γ)

(iv) $C_V = \frac{R}{\gamma - 1}$ and $C_P = \frac{R\gamma}{\gamma - 1}$

(v) In terms of degree of freedom (f) $C_V = \frac{f}{2}R$, $C_P = \left(1 + \frac{f}{2}\right)R$ and $\gamma = \left(1 + \frac{2}{f}\right)$

(vi) C_V for mixture of gases $C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2} + n_3 C_{V_3} + ...}{n_1 + n_2 + n_3}$

(vii) γ for mixture of gases $\frac{n}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} + ...$ Here $n = n_1 + n_2 + n_3 + ...$

(viii) Molecular weight of mixture of gases $\frac{m_1 + m_2 + ...}{M} = \frac{m_1}{M_1} + \frac{m_2}{M_2} + ...$

Practice Problems :

1. Find the molar heat capacity for the following process performed on ideal monoatomic gas : (i) $P = KV^2$ (ii) $P^2V = K$ (iii) $PT = K$ (iv) $PV^\alpha = K$ (v) $VT = K$ (vi) $V = KT$ where P, V and T are pressure volume and temperature of gas and K is a constant quantity.

2. Find the adiabatic exponent and molar mass of the following gaseous mixture :

(i) 1 mol. of O_2 mixed with 2 moles of He

(ii) 1 mol. of He mixed with 2 moles of H_2

C14 Application of the First Law of Thermodynamics

(i) Isochoric Process (Constant Volume Process) : $W = 0$, $Q = \Delta U$

(ii) Isobaric Process (Constant Pressure Process) : $W = P(V_2 – V_1) = nR(T_2 – T_1)$

$Q = nC_P(T_2 – T_1)$

(iii) Isothermal Process (Constant Temperature Process)

$$\Delta U = 0,\ Q = W = nRT \ln \frac{V_2}{V_1} = nRT \ln \frac{\rho_1}{\rho_2} = nRT \ln \frac{P_1}{P_2}.$$

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(iv) Adiabatic Process

PV^{γ} = constant, $TV^{\gamma-1}$ = constant, $P^{1-\gamma}T^{\gamma}$ = constant.

For this process Q = 0, $\Delta U = \frac{nR}{\gamma - 1}(T_2 - T_1) = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$ and W = –ΔU

(v) Cyclic Process

There is process in which, after certain interchanges of heat and work, the system is restored to its initial state, named as cyclic process. In this case ΔU = 0 and Q = W

(vi) Free expansions

These are adiabatic process in which no transfer of heat occurs between the system and its environment and no work is done on or by the system. Thus, Q = W = 0 and hence from the first law thermodynamics ΔU = 0.

<u>Practice Problems :</u>

1. A monatomic gas (γ = 5/3) is suddenly compressed to (1/8) of its initial volume adiabatically, then the pressure of the gas will change to :

(a) 24/5 (b) 8 (c) 40/3 (d) 32

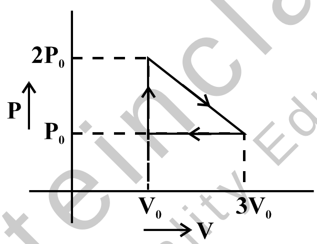
2. In an adiabatic change, the pressure P and temperature T of a diatomic gas are related by the relation $P \propto T^C$ where c equals

(a) 5/3 (b) 2/5 (c) 3/5 (d) 7/2

3. One mole of an ideal gas requires 207 J heat to raise the temperature by 10 K when heated at constant pressure. If the same gas is heated at constant volume to raise the temperature by the same 10 K, the heat required is : [R = 8.3 J/mol K]

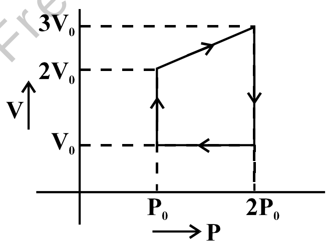
(a) 198.7 J (b) 29 J (c) 215.3 J (d) 124 J

4. The following cyclic process is performed on one mole of monoatomic gas.



Find W, ΔU and Q in the individual process and in the complete process ?

5. The following cyclic process is performed on one mole of monoatomic gas.



Find W, ΔU and Q in the individual process and in the complete process ?

6. The following cyclic process is performed on one mole of diatomic gas.

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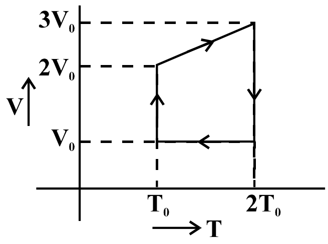
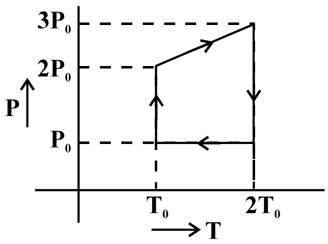


Find W, ΔU and Q in the individual process and in the complete process ?

7. **The following cyclic process is performed on one mole of diaatomic gas.**



Find W, ΔU and Q in the individual process and in the complete process ?

[Answers : (1) d (2) d (3) d]

C15 Efficiency of a Themodynamic Cycle

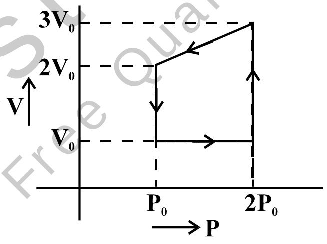
The efficiency of a thermodynamic cycle is defined as

$$\eta = \frac{W_{net}}{Q_{in}}$$

where W_{net} is the net work done by the cycle, and Q_{in} is total heat input of the cycle.

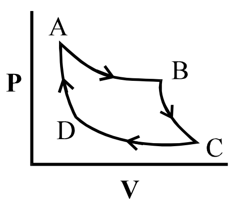
Practice Problems :

1. **Following cyclic process is performed on one mole of monoatomic gas.**



Is the efficiency of the cycle is defined ? If yes, find

C16 Carnot Cycle : This cycle consists of four processes given in the figure : (i) AB is isothermal expansion at temperature T_1 (ii) BC is adiabatic expansion (iii) CD is isothermal compression at temperature T_2 (iv) DA is adiabatic compression.



The efficiency of the cycle is $1 - T_2/T_1$, where $T_2 < T_1$.

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<u>Practice Problems :</u>

1. A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

(a) 800 J (b) 1600 J (c) 3500 J (d) 6400 J

[Answers : (1) b]

C17 Second Law of Thermodynamics

(i) Kelvin Planck Statement : No process is possible whose sole result is the absorption of heat from a reservoir and the conversion of the heat into work.

(ii) Clausius' Statement : No process is possible whose sole result is the transfer of heat from a colder object to a hotter object. According to this law no engine has the efficiency equals to 1, it is always less than 1

C18 Entropy : Entropy is a measure of disorder of the molecular motion of a system. The greater the disorder, the greater is the entropy. The change in entropy is given by $dS = \frac{dQ}{T}$.

C19 Heat Transfer

There are three mechanisms for heat transfer : Conduction, Convection and Radiation.

Conduction : Conduction occurs in solids. If the ends of a rod of thermal conductivity k is kept at the temperature T_1 and T_2 then heat flowing per unit time through the rod is given by $(T_1 - T_2)/R$, where R is the thermal resistance of the rod. If the length of the rod is l and cross-sectional area A then $R = l/kA$.

<u>Practice Problems :</u>

1. Two ends of rods of length L and radius r of the same material are kept at the same temperature. Which of the following rods conducts most heat

(a) L = 50 cm, r = 1 cm (b) L = 100 cm, r = 2 cm

(c) L = 25 cm, r = 0.5 cm (d) L = 75 cm, r = 1.5 cm

2. Heat is flowing through two cylindrical rods of the same material. The diameter of the rods are in the ratio 1 : 2 and their lengths are in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of the amounts of heat conducted through them per unit time will be

(a) 1 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 8

[Answers : (1) b (2) d]

Convection : It occurs in fluids.

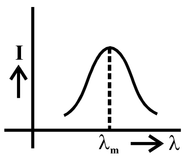
Heat Radiation : Heat radiation is electro-magnetic energy transfer in the form of electromagnetic waves (infrared waves) through any medium. Heat radiation has the same character as the electromagnetic wave. This transfer does not require any material medium. The surface of any material medium emits heat radiations if its temperature is above 0 K.

Black Body

A perfect black body is one which absorbs all the radiations (from $\lambda = 0$ to $\lambda = \infty$) incident on it.

Black Body Radiation

The graph is plotted between intensity of heat radiation I and wave length λ of heat radiation emitted by the black body as shown in figure



There is particular wavelength λ_m at which the intensity of emitted heat radiation is maximum, this wave length is relates with the temperature of the black body using the following law $\lambda_m T = b$ = constant. This law is known as **Wien's Displacement law.** Here b is known as Wien's constant having value 0.29 cm-K.

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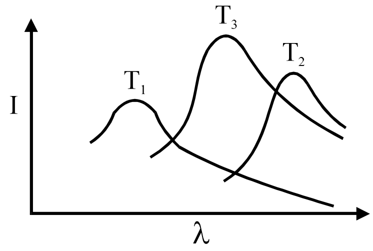
Practice Problems :

1. The intensity of radiation emitted by the Sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperature of the Sun and the North Star is

(a) 1.46 (b) 0.69 (c) 1.21 (d) 0.83

2. The plots of intensity versus wavelength for three black bodies at temperatures T_1, T_2 and T_3 respectively are as shown in figure. Their temperature are such that



(a) $T_1 > T_2 > T_3$ (b) $T_1 > T_3 > T_2$ (c) $T_2 > T_3 > T_1$ (d) $T_3 > T_2 > T_1$

[Answers : (1) b (2) b]

Stefan's Boltzmann Law

The energy of heat radiation emitted per unit time E is directly proportional to the fourth power of absolute temperature of the body i.e., $E = e\sigma(T^4 - T_0^4)$

where e is the emissivity of the surface defined as the ratio of emissive power of the surface to the emissive power of black body surface at the same temperature. Its value lies between 0 and 1. For black body e = 1. σ is known as Stefan's constants, its numerical value is 5.68×10^{-8} Wm^{-2} K^{-4}. T_0 is the surrounding temperature in which body is placed. If A is the surface area of the body, then the rate of heat emitted by the body is

$$\frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$$

As dQ = msdT then the rate of cooling if $T > T_0$

$$\mathbf{\frac{dT}{dt} = -\frac{e\sigma A}{ms}(T^4 - T_0^4)}$$

If $T_0 = 0$ or $T >> T_0$ then $\frac{dQ}{dt} = e\sigma T^4$ and $\frac{dT}{dt} = -\frac{e\sigma A}{ms}T^4$

Newton's Law of Cooling

When the temperature difference between the body and is surrounding is not very large, i.e. $T - T_0 = \Delta T$ is small then the rate of cooling is given by $\frac{dT}{dt} = -k(T - T_0)$. This law is known as **Newton's Law of Cooling** which is derived from **Stefan's Law.** There is another way to express the **Newton's Law Cooling**

$$\left[\frac{T_1 - T_2}{t}\right] = K\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right]$$

Here T_1 : Initial temperature of the body, T_2 : Temperature of the body after time t

T_0 : Surrounding temperature, K : A constant

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Practice Problems :

1. **A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiates in watt would be**

   (a) 225 (b) 450 (c) 900 (d) 1800

2. **A sphere, a cube and a thin circular plate all made of the same mass and finish are heated to a temperature of 200^0C; which of these objects will cool slowest when left in air at room temperature**

   (a) The sphere (b) The cube

   (c) The circular plate (d) All will cool at the same rate

3. **A ball A has twice the diameter as another ball B of the same material and with same surface finish. A and B are both heated to the same temperature and allowed to cool radiatively; then**

   (a) Rate of cooling of A is same as that of B

   (b) Rate of cooling of A is twice that of B

   (c) Rate of cooling of A is half that of B

   (d) Rate of cooling of A is four times that of B

4. **The temperature of a body is increased from 27^0C to 127^0C. The radiation emitted by it increases by a factor of**

   (a) (256/81) (b) (15/9) (c) (4/3) (d) (12/27)

5. **A liquid cools in 6 minutes from 80^0C to 60^0C. Take the temperature of surrounding to be 30^0C and assume that Newton's law of cooling is applicable throughout the process. Its temperature after 10 minutes is**

   (a) 48.2^0C (b) 42.8^0C (c) 37.5^0C (d) 32.5^0C

[Answers : (1) d (2) a (3) c (4) a (5) b]

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