Differential Equation

C1 Order and Degree of a Differential Equation :

Order : Order is the highest differential appearing in a differential equation.

Degree : It is determined by the degree of the highest order derivative present in it after the differential equation is cleared of radicals and fractions so far as the derivatives are concerned.

$$f_1(x,y)\left[\frac{d^m y}{dx^m}\right]^{n_1} + f_2(x,y)\left[\frac{d^{m-1} y}{dx^{m-1}}\right]^{n_2} + ...f_k(x,y)\left[\frac{dy}{dx}\right]^{n_k} = 0$$

The above differential equation has the order m and degree n_1.

Practice Problems :

1. The order of the differential equation whose general solution is given by $y = (c_1 + c_2)e^x + c_3 e^{x+c_4}$ is

(a) 4 (b) 3 (c) 2 (d) 1

2. The degree of the differential equation, of which $y^2 = 4a(x + a)$ is a solution, is

(a) 1 (b) 2 (c) 3 (d) none of these

3. The order of the differential equation whose general solution is given by :

$y = c_1\cos(2x + c_2) - (c_3 + c_4)\, a^{x+c_5} + c_6 \sin(x - c_7)$ is

(a) 3 (b) 4 (c) 5 (d) 2

[Answers : (1) d (2) b (3) c]

C2 Formation of Differential Equation :

The differential equation corresponding to a family of curve can be obtained by using the following steps :

(a) Identify the number of essential arbitrary constant in equation of curve.

If arbitrary constants appear in addition, subtraction, multiplication or division, then we can club them to reduce into one new arbitrary constant.

(b) Differentiate the equation of curve till the required order.

(c) Eliminate the arbitrary constant from the equation of curve and additional equation obtained in step (b) above.

1. The differential equation of the family of curves $y = e^x$ (A cos x + B sin x), where A and B are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ **(b)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 2y = 0$

(c) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ **(d) none of these**

[Answers : (1) c]

C3 Solution of a Differentible Equation :

The solution or the integral of a differential equation is, therefore, a relation between dependent and independent variables (free from derivatives) such that it satisfies the given differential equation.

Practice Problems :

1. **The solution of the differential equation, y dx – x dy + xy^{2}dx = 0 is**

(a) $\frac{x}{y} + \frac{x^2}{2}$ = constant (b) $\frac{x}{y} - \frac{x^2}{2}$ = constant

(c) $\frac{x}{y^2} + \frac{x^2}{2}$ = constant (d) none

2. **The solution of the differential equation; x dy (y^2 e^{xy} + $e^{x/y}$) = y dx ($e^{x/y}$ – y^2 e^{xy}) is**

(a) $x/y = \ln(e^{x/y} + \lambda)$ (b) $xy = \ln(e^{x/y} + \lambda)$

(c) $x + y = \ln(e^{x/y} + \lambda)$ (d) $x - y = \ln(e^{x/y} + \lambda)$

[Answers : (1) a (2) b]

C4 Solution of elementary types of first order and first degree differential equations :

Equations Reducible to the Variables Separable form : Its general form is $\frac{dy}{dx}$ = f(ax + by + c) a, b ≠ 0. To solve this, put ax + by + c = t.

Homogeneous Differential Equation :

A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f and g are homogeneous function of x and y, and of the same degree, is called homogeneous differential equation and can be solved easily by putting y = vx.

C5 Equations Reduciable to the Homogeneous form

Equations of the form $\frac{dy}{dx} = \frac{ax+by+c}{Ax+By+C}$

can be made homogeneous (in new variables X and Y) be substituting x = X + h and y = Y + k, where h and k are constants.

Now, h and k are chosen such that ah + bk + c = 0, and Ah + Bk + C = 0; the differential equation can now be solved by putting Y = vX.

C6 Linear differential equations of first order :

The differential equation $\frac{dy}{dx} + Py = Q$, is linear in y, where P and Q are functions of x.

Integrating Factor (I.F.) : It is an expression which when multiplied to a differential equation converts it into an exact form.

I.F. for linear differential equation = $e^{\int P dx}$ (constant of integration will not be considered)

After multiplying above equation by I.F. both side, it becomes $\frac{d}{dx}(y.e^{\int Pdx}) = Qe^{\int Pdx}$ ⇒

$y.e^{\int Pdx} = \int Q.e^{\int Pdx}$ dx + c.

Some times differential equation becomes linear if x is taken as the dependent variable and y as independent variable. The differential equation has then the following form :

$\frac{dx}{dy} + P_1 x = Q_1$. where P_1 and Q_1 are functions of y. pThe I.F. now is $e^{\int P_1 dy}$

Practice Problems :

1. **Solution of the differential equation $(x+2y^3)\frac{dy}{dx} = y$ is**

   (a) $x = y^2(c + y^2)$ **(b)** $x = y(c - y^2)$ **(c)** $x = 2y(c - y^2)$ **(d)** $x = y(c + y^2)$

   (d)

2. **The equation of the curve whose tangent at any point (x, y) makes an angle $\tan^{-1}(2x + 3y)$ with x-axis and which passes through (1, 2) is**

   (a) $6x + 9y + 2 = 26e^{3(x-1)}$ **(b)** $6x - 9y + 2 = 26e^{3(x-1)}$

   (c) $6x + 9y - 2 = 26e^{3(x-1)}$ **(d)** **None of these**

   (a)

   [Answers : (1) d (2) a]

C7 Time saving tips :

1. The equation of the form $\frac{dy}{dx} + Py = Qy^n$ where P and Q are functions of x only and n is constant {n $\neq 0, \neq 1$} can be reduced to linear form by $y^{-n}.\frac{dy}{dx} + P.y^{1-n} = Q$ and put $y^{1-n} = v$.

2. Some important integers factors (I.F.) for the quick solutions

   (i) $d\left(\tan^{-1}\left(\frac{x}{y}\right)\right) = \frac{ydx - xdy}{x^2 + y^2}$ (ii) $d\left(\log\left(\frac{x}{y}\right)\right) = \frac{ydx - xdy}{xy}$

   (iii) $d(x^m y^n) = x^{m-1}.y^{n-1}.(mydx + nxdy)$ (iv) $d\left(\frac{1}{2}\log\left(\frac{x+y}{x-y}\right)\right) = \frac{xdy - ydx}{x^2 - y^2}$