Current Electricity

CURRENT ELECTRICITY

C1 Electric Current

Current is rate of flow of charge from one region to another. Mathematically, $I = \frac{dq}{dt}$ where dq is net charge transported at section during time dt. For steady state, $I = \frac{q}{t}$. The SI unit of current is ampere (A) . When 1A current flows through a conductor, then 6.25×10^{18} electrons flow per second.

<u>**Practice Problems :**</u>

1. If the current will constitute of α-particles then the number of α-particles flowing per second for 1μA current is

(a) 3.125×10^{12} **(b)** 6.25×10^{12} **(c)** 3.125×10^{18} **(d)** 6.25×10^{18}

2. If the current is given by at + b where a and b are constant, then the charge flown in time t is

(a) $\frac{at^2}{2} - bt$ **(b)** $\frac{at^2}{2} + bt$ **(c)** $at^2 + bt$ **(d)** $at^2 - bt$

3. If the charge flowing is given by exp (–at) then the current is

(a) ae^{-at} **(b)** $-ae^{-at}$ **(c)** e^{-at} **(d)** none

[Answers : (1) (a) (2) (b) (3) (b)]

C2A Current in Different Materials

Current in conductors is due to motion of electrons, in electrolytes due to motion of both positive and negative ions, in semi-conductors due to motion of electrons and holes.

C2B Electric Current in Conductors

There is a net current in the conductor is given by $I = nev_dA$ where n is number of electrons per unit volume, A is the cross-section area of conductor, e is the electric charge and v_d is its drift speed. The drift velocity is very small (~ 10^{-4} m/s) as compared to thermal speed of electrons at room temperature (~ 10^5 m/s). The current per unit cross section is called the current density $\vec{j}$, given by $\vec{j} = ne\vec{v}_d$ .Note that $\vec{j}$ is a vector whereas I is a scalar.

<u>**Practice Problems :**</u>

1. The average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A (Assume that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3, and its atomic mass is 63.5 u).

(a) 1.1 mm/s **(b)** 2.2 mm/s **(c)** 3.3 mm/s **(d)** 4.4 mm/s

[Answers : (1) (a)]

C3A Ohm's Law and Electrical Resistance

$I \propto V \Rightarrow I = \frac{V}{R}$ where R is the electrical resistance of the conductor. Resistance For a linear homogenous conductor of uniform cross section A and of the length l the resistance is given by $R = \rho \frac{l}{A}$ where ρ is specific resistance or resistivity of the material. Resistivity ρ depends on the material of the conductor and its temperature.

Practice Problems :

1. **If the percentage change in the length of wire is 2% on stretching then the percentage change in resistance is**

(a) 2 % **(b)** 3 % **(c)** 4 % **(d)** 5 %

2. **If the percentage change in the length of wire is 10% on stretching then the percentage change in resistance is**

(a) 20 % **(b)** 21 % **(c)** 22 % **(d)** 23 %

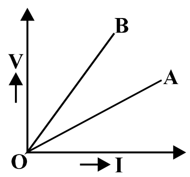
3. **If the percentage change in the cross-sectional radius of wire is 2% on stretching then the percentage change in resistance is**

(a) 4 % **(b)** 6 % **(c)** 8 % **(d)** –8 %

4. **A metallic wire of length 1 m is stretched to double its length. The ratio of its initial and final resistance assuming that there is no change in its density on stretching is**

(a) 1 : 2 **(b)** 1 : 4 **(c)** 2 : 1 **(d)** 4 : 1

5. **The voltage-current graphs for two resistors of the same material and same radii with lengths L_1 and L_2 are shown in figure. If $L_1 > L_2$, which of these graphs represent V – I for L_1 ?**



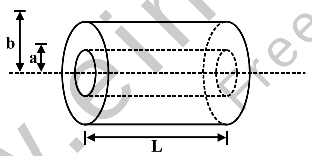
(a) graph A **(b)** graph B **(c)** both can be **(d)** none

[Answers : (1) c (2) b (3) c (4) b (5) b]

C3B Resistance of special configuration :

Configuration		**Resistance**	
1. Hollow cylinder		(a)	Across it ends

$$R = \frac{\rho L}{\pi(b^2 - a^2)}$$

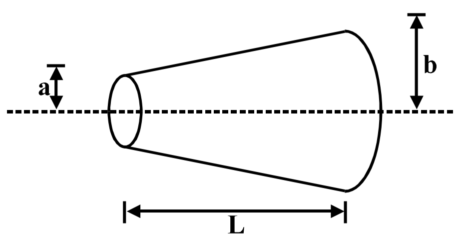


(b) Across its inner and outer surface

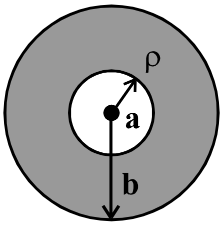
$$R = \frac{\rho}{2\pi L}\ln\left|\frac{b}{a}\right|$$

2. A Truncated Cone

$$R = \frac{\rho L}{\pi ab}$$



3. A sphere



$$R = \frac{\rho(b-a)}{4\pi ab}$$

Practice Problems :

1. The resistivity of a cylinder, of length l and radius r, changes with distance according to ax + b where a and b are constant quantity and x is the distance measured from left end. The resistance of cylinder if the potential drop across its ends

(a) $\left(\frac{al^2}{2}+bl\right)\frac{1}{\pi r^2}$ **(b)** $\left(al^2+bl\right)\frac{1}{\pi r^2}$ **(c)** $\left(\frac{al^2}{2}-bl\right)\frac{1}{\pi r^2}$ **(d)** **none**

2. Find the resistance for the following configurations :

(a) Cylinderical shell of inner radius a and outer radius b with length l consists of a material of resistivity ρ if the potential drop is between inner and outer shell.

(b) Cylinderical shell of inner radius a and outer radius b with length l consists of a material of resistivity ρ if the potential drop is across the ends.

(c) Spherical shell of inner radius a and outer radius b consists of a material of resistivity ρ if the potential drop is between inner and outer shell.

(d) A truncated cone.

[Answers : (1) a]

C3C Another form of ohm's Law

$\vec{J} = \sigma\vec{E} = \frac{1}{\rho}\vec{E}$ where σ is the conductivity, reciprocal of resistivity and $\vec{\mathbf{E}}$ is the electric field.

$\Rightarrow$ $\sigma = ne\,\frac{v_d}{E} = ne\mu$, μ is known as mobility of the charge carrier.

The conductivity is given by $\sigma = \frac{ne^2\tau}{m}$ where n is the number of electrons per unit volume, e is the electronic charge, τ is the relaxation time or collision time and m is the mass of electron.

C3D Dependence of resistivity and resistance on temperature

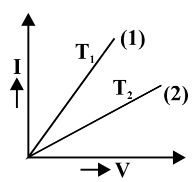
$$\rho_T = \rho_0\,[1 + \alpha(T - T_0)]$$

where ρ_0 is the resistivity at reference temperature T_0 and ρ_T is the resistivity at temperature T. The factor α is called the temperature coefficient of resistivity. In the same way

$$R_T = R_0[1 + \alpha(T - T_0)]$$

Practice Problems :

1. V – I graph for a given metallic wire at two temperatures are shown, which of these is for a higher temperature ?



(a)	**graph (1)**	**(b)**	**graph (2)**	**(c)**	**any graph**	**(d)**	**none**

2. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature (27.0 ^{0}C) is found to be 75.3 Ω. When the toaster is connected to a 230 V supply, the current settles, after a few seconds, to a steady value of 2.68 A. The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is $1.70 \times 10^{-4}\ {}^0C^{-1}$. The steady temperature of the nichrome element is

(a)	**547^{0}C**	**(b)**	**647^{0}C**	**(c)**	**747^{0}C**	**(d)**	**847^{0}C**

[Answers : (1) b (2) d]

C4 Resistivity of Various Materials

The conductors has the resistivity in the order of 10^{-8} Ωm, for carbon (graphite) has the resistivity 3.5×10^{-5}Ωm and has negative temperature coefficient for resistivity and for insulator the resistivity has the order greater than 10^5 Ωm. There are two types of resistors : wire bound resistors and carbon resistors. Wire bound resistors are made by winding the wires of an alloy, viz, manganin, constantan, nichrome or similar ones. The choice of these materials is dictated mostly by the fact that their resistivities are relatively insensitive to temperature. These resistances are typically in the range of a fraction of an ohm to a few hundren ohms.

Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits. Carbon resistors are small in size and their values are given using a colour code.

C5 Analysing the Electrical Circuit

Kirchoff's rules provides a general method for analyzing circuit networks.

Law I Kirchoff's junction rule of kirchoff's current law (KCL) : KCL is based on conservation of charge principle. This algebraic sum of the current at any junction is zero i.e. $\sum I = 0$.

Law II Kirchoff's loop rule of kirchoff's voltage Law (KVL) :

This law is based on conservation of energy principle. The algebraic sum of the potential difference in any closed loop across each elements of the circuit must equal zero. That is $\sum V = 0$.

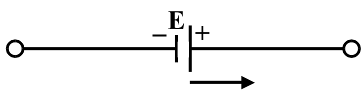
Problem solving strategy for electric curciut using Kirchoff's Laws :

1. Assume currents with their direction in each branch of the circuits keeping in mind the KCL. Often you will not known in advance the actual direction of an unknown current but this doesn't matter. Carry out your solution, using the assumed direction. If the actual direction of a particulay quantity is opposite to your assumption, the result will come out with a negative sign.

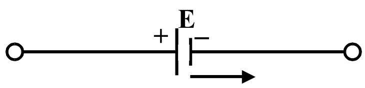
2. Choose any closed loop in the network, and designate a direction (clockwise or countercolckwise) to travel around the loop in applying the KVL. Travel, adding potential difference as you cross them and equate them to zero. Remember the following rules :

An emf is counted as positive when you transverse it from – to + and negative when you transverse it from + to – i.e.,

(a) If we transverse through a battery from the negative terminal to the positive terminal, the change in voltage is **+E**.

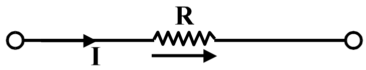


(b) If we transverse through a battery from the positive terminal to the negative terminal, the change in voltage is **–E**.

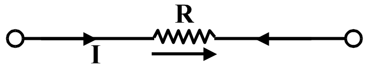


An IR term is negative if you travel through the resistor in the same direction as the assumed current and positive if you travel in the opposite direction, i.e.,

(a) If a resistor is transverse in the same direction as the assumed current flow, the change in potentials is **–IR**.



(b) If a resistor is transversed in the direction opposite from the assumed current flow, the change in potential is **+IR**.



Remember that the potential difference across the terminal of a battery of emf E and internal resistance r is given by (i) when the current i is drawn (discharging battery), V = E – ir (ii) when the current i is supplied (charging battery), V = E + ir.

<u>Practice Problems :</u>

1. The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4 Ω, the maximum current that can be drawn from the battery is

(a) 25 A (b) 28 A (c) 30 A (d) 32 A

2. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A. The terminal voltage of the battery when the circuit is closed is

(a) 5.5 V (b) 6.5 V (c) 7.5 V (d) 8.5 V

[Answers : (1) c (2) d]

C6 Combination of Resistances and Equivalent Resistance

(a) Series combination (Potential divider) : In general for any number of resistors in series, $R_{eq} = \sum_{I=1}^{n} R_i$

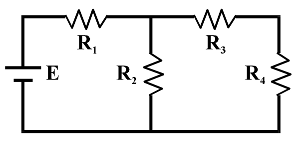
(b) Parallel Combination (Current divider) : For any combination of resistors in parallel, $\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$

<u>Practice Problems :</u>

1. A wire of resistance 4 R is cut into 4 identical parts. Each part is stretched to twice the original length. If all the four wires after stretching are connected in parallel. The new resistance of the combination is

(a) R (b) 2R (c) 3R (d) 4R

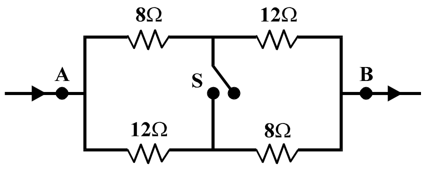
2. In the circuit given below with E = 65 V, R_1 = 50 Ω, R_2 = 100 Ω, R_3 = 100 Ω and R_4 = 300 Ω.



The voltage drop across the resistor R_1 is

(a) 10 V (b) 15 V (c) 20 V (d) 25 V

3. Consider the following circuit when the switch S is closed.



The current passing through the connecting wire if the battery of 10 V is connected across A and B

(a) 1 A (b) 2 A (c) 3 A (d) none

4. A set of n identical resistors, each of resistance R Ω, when connected in series have an effective resistance X Ω and when the resistors are connected in parallel, their effective resistance is Y Ω. The relation between R, X and Y is

(a) $R = \sqrt{XY}$ (b) $2R = \sqrt{XY}$ (c) $3R = \sqrt{XY}$ (d) $4R = \sqrt{XY}$

[Answers : (1) a (2) d (3) d (4) a]

C7 Combination of Cells

(a) Series combination

Here $E = E_1 + E_2 + E_3 + ...$

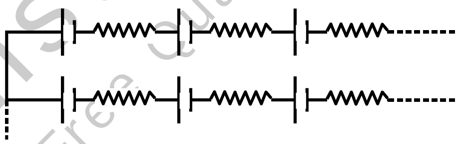
and $r = r_1 + r_2 + r_3 + ...$

(b) Parallel combination

Here $E = \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}}$ and $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$

(c) Mixed Combination

Let mn identical cells are connected as shown in figure. Here



E = emf of each cell

r = internal resistance of each cell

The combination of cells is equivalent to a single cell of (a) emf = mE and (b) internal resistance $\dfrac{mr}{n}$

For the maximum current the external resistance should be $R = \dfrac{mr}{n}$ is connected

<u>Practice Problems :</u>

1. Two identical cells of emf 1.5 V each joined in parallel provide supply to an external circuit consisting of two resistors of 17Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1.4 V. The internal resistance of each cell is

(a) 1.21 Ω (b) 1.41 Ω (c) 1.51 Ω (d) 1.61 Ω

2. **Three cells of emf 2.0 V, 1.8 V and 1.5 V are connected in series. Their internal resistances are 0.05 Ω, 0.7 Ω and 1.0 Ω, respectively. If this battery is connected to an external resistor of 4 Ω. The potential difference across the terminals of the cell of emf 1.5 V while in use is**

   (a) **0.28 V** **(b)** **0.38 V** **(c)** **0.48 V** **(d)** **0.58 V**

3. **Two cells of emf 1.5 V and 2 V and internal resistance 1 Ω and 2 Ω respectively are connected in parallel to pass current in the same direction through an external resistance of 5Ω. The potential difference across the 5 Ω resistor.**

   (a) $\frac{25}{17}$ **V** **(b)** $\frac{26}{17}$ **V** **(c)** $\frac{27}{17}$ **V** **(d)** $\frac{28}{17}$ **V**

   [Answers : (1) a (2) d (3) a]

C8 Electrical Instruments

1. Galvanometer

Galvanometer detects the presence of current in the branch where it is connected. It does not measure current.

2. Ammeter

- It is always connected in series with the branch in which it has to measure the current.
- Its resistance should be small enough so that its presence does not significantly alter the current in the branch.
- The resistance of an ideal ammeter is zero.
- A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt) in parallel to the galvanometer S.

The magnitude of shunt S resistance is given by $S = G\frac{I_g}{I - I_g}$

where G = resistance of the galvanometer

I_g = full scale deflection current of the galvanometer

I = maximum current to the measured by the ammeter

3. Voltmeter

- It is always connected in parallel with the branch across which it has to measure the voltage.
- Its resistance should be large enough so that it may draw very small current from the circuit; and the disturbance produces is significant.
- An ideal voltmeter has infinite resistance.
- A galvanometer may be converted into a voltmeter by connecting a large resistance in series with the galvanometer. The magnitude of the high resistance is given by $R = \frac{V}{I_g} - G$, where V is the maximum voltage to be measure by the voltmeter.

4. Potentiometer

Uses of Potentiometer

Comparison of emf's

The emf of an unknown battery E_2 may be obtained by comparing its length l_2 for zero deflection to that of the unknown battery E_1 with balance point located at l_1. Thus, $\frac{\mathbf{E_2}}{\mathbf{E_1}} = \frac{l_2}{l_1}$.

Measurement of Internal Resistance

The balance point l_0 is obtained with only emf E connected. A second balance point l is obtained by connection of external resistance R. Then internal resistance is given by $\mathbf{r = \left(\frac{l_0}{l} - 1\right)R}$

<u>**Practice Problems :**</u>

1. **In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, the emf of the second cell is**

(a) **1.25 V** **(b)** **1.50 V** **(c)** **2.25 V** **(d)** **2.5 V**

2. **A 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. The internal resistance of the cell is**

(a) **1.5 Ω** **(b)** **1.7 Ω** **(c)** **1.9 Ω** **(d)** **2.1 Ω**

3. **A 10 m long wire of uniform cross-sectional and 20 Ω resistance is used in a potentiometer. The wire is connected in series with a battery of 5 V along with an external resistance of 480 Ω. If an unknown emf E is balanced at 6.0 m length of the wire. The potential gradient of the potentiometer wire is**

(a) **2×10^{-2}V/m** **(b)** **3×10^{-2}V/m** **(c)** **4×10^{-2}V/m** **(d)** **5×10^{-2}V/m**

4. **In the above problem, the value of unknown emf is**

(a) **0.10 V** **(b)** **0.12 V** **(c)** **0.14 V** **(d)** **0.16 V**

5. **The length of the potentiometer wire is 600 cm and it carries a current of 40 mA. For a cell of emf 2 V and internal resistance 10 Ω, the null point is found to be at 500 cm. If a voltmeter is connected across the cell, the balancing length is decreased by 10 cm. The resistance of the voltmeter is**

(a) **450 Ω** **(b)** **490 Ω** **(c)** **530 Ω** **(d)** **550 Ω**

6. **Four identical cells, each of emf 2V, are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the cell as read by an ideal voltmeter is 1.6 V. The internal resistance of each cell is**

(a) **1 Ω** **(b)** **2 Ω** **(c)** **3 Ω** **(d)** **none**

7. **A voltmeter V of resistance 400 Ω is used to measure the potential difference across a 100 Ω resistor in the circuit in which an emf of 84 V is connected with 200 Ω resistor in series. The reading on the voltmeter is**

(a) **20 V** **(b)** **22 V** **(c)** **24 V** **(d)** **26 V**

[Answers : (1) c (2) b (3) a (4) b (5) b (6) d (7) c]

C9 Wheatstone Bridge

Let us assume that R_2 is unknown and R_3 can be varies and it is adjusted until no current flows in the galvanometer. In this condition, wheatstone bridge is said to be balanced, and the resistances satisfy the ratio $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

C10 Meter Bridge

It is used to measure the unknown value of a resistor which is based on Wheatstone bridge.

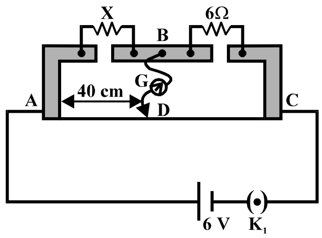
The four arms AB, BC, DA and CD [with resistances R, S, λ l and λ(100 – l)] form a Wheatstone bridge where λ is known as resistance per unit length of the wire. If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current. Let the distance of the jockey from the end A at the balance point be $l = l_1$. The balance condition gives $\mathbf{R = S\frac{l_1}{100 - l_1}}$.

<u>Practice Problems :</u>

1. **In a meter bridge, the null point is found at a distance of 33.7 cm. If now a resistance of 12Ω is connected in parallel with S, the null point occurs at 51.9 cm. The value of R is**

   (a) 6.86 Ω (b) 13.5 Ω (c) 7.94 Ω (d) none

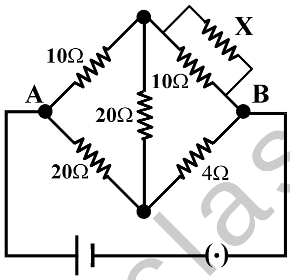
2. **In the circuit, a metre bridge is shown in its balanced state.**



   The metre bridge wire has a resistance of 1 ohm/cm. The value of the unknown resistance is

   (a) 2Ω (b) 4Ω (c) 6Ω (d) 8Ω

3.



   For the balance wheat stone bridge, the value of X is

   (a) 1.5 Ω (b) 2.5 Ω (c) 3.5 Ω (d) 4.5 Ω

   (b)

   [Answers : (1) a (2) b (3) b]

C11 Energy and Power In Electrical Circuits

Rate of delivering electrical energy or power input to a circuit element, $P = V_{ab} I$

Power delivered to a resistor = $VI = I^2R = \dfrac{V^2}{R}$

Power output of a source = $V_{ab}I = E I - I^2r$

Power input to a source : $P = V_{ab} I = (EI + I^2r)$

Maximum Power Transfer Theorem

The maximum power is transferred to an external resistor if its resistance is equal to the internal resistance of the cell of battery.

<u>Practice Problems :</u>

1. **Which one has the greatest resistance ?**

   (a) 220 V, 60 W bulb (b) 220 V, 100 W bulb

   (c) 220 V, 1 kW heater (d) all have same resistances

2. **Two bulbs whose resistances are in the ratio of 1 : 2 are connected in parallel to a source of constant voltage. The ratio of power dissipation in these bulbs is**

   (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

3. **Two heater wires of the same dimensions are first connected in series and then in parallel to a source of supply. The ratio of heat produced in the two cases is**

   (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

4. **An electric motor operating on a 50 V d.c. supply draws a current of 12 A. If the efficiency of the motor is 30%, the resistance of the windings of the motor is**

   (a) 2.9 Ω (b) 3.9 Ω (c) 4.9 Ω (d) none

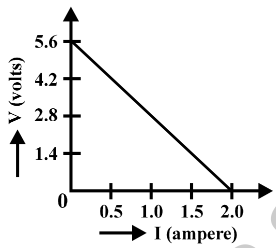
5. **Three identical resistors, each of resistance R, when connected in series a d.c. source, dissipate power X. If the resistors are connected in parallel to the same d.c. source, then the power will be dissipated is**

   (a) 9X (b) 6X (c) 3X (d) X

6. **A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel to the same source of 200 V. The energy liberated per second in the new combination is**

   (a) 100 W (b) 200 W (c) 300 W (d) 400 W

7. **Four cells of identical emf E, internal resistance r, are connected in series to a variable resistor. The graph shows the variation of terminal voltage of the combination with the current output :**



   The current from the cells for which maximum power dissipation occur in the circuit is

   (a) 0.5 A (b) 0.75 A (c) 1 A (d) 1.25 A

   [Answers : (1) a (2) b (3) c (4) a (5) a (6) d (7) c]

C12 Capacitance and type of Capacitors :

The capacitance C of a capacitor is defined from q = CV, where V is the potential difference the conductors. The SI unit of capacitance is the farad, (1 farad = 1 F = 1CV^{-1}).

(i) **Parallel Plate Capacitors without dielectric :** $C = \dfrac{\varepsilon_0 A}{d}$

(ii) **Parallel Plate Capacitors with dielectric :** $C = \dfrac{K\varepsilon_0 A}{d}$

where K is dielectric constant of dielectric within the capacitor, A is the area of the plate and d is the distance between the plates.

(iii) **Spherical Capacitor :** $C = \dfrac{4\pi\varepsilon_0 ab}{b - a}$

If the radius of the outer sphere tends to infinity $b \to \infty$, the capacitance reduces to $C = 4\pi\varepsilon_0 a$ which is called the capacitance of an isolated sphere. Here a and b are the radius of inner and outer sphere.

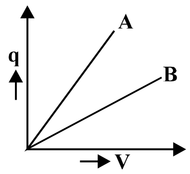
(iv) **Cylindrical Capacitor :** $C = \dfrac{2\pi\varepsilon_0 l}{\ln\left|\frac{b}{a}\right|}$ **where a and b are inner and outer radius of the cylinder and *l* is length of the cylinder.**

Practice Problems :

1. **The ratio of electric field at any two points in the parallel plate capacitor is**

(a) **1 : 1** **(b)** **1 : 2** **(c)** **2 : 1** **(d)** **2 : 3**

2. **The given graph shows the variation of charge q versus potential difference V for two capacitors C_1 and C_2. The two capacitors have same plate sepration but the plate area of C_2 is double than that of C_1.**



Which of the lines in the graph correspond to C_1 ?

(a) **A** **(b)** **B** **(c)** **both may be** **(d)** **none**

[Answers : (1) a (2) b]

C13 Batteries and Capacitors

A battery maintains a certain potential difference its terminals. Thus, when capacitors is connected to a battery, charge flows between the capacitor and the battery until the capacitor has the same potential across it as the battery.

When a charged capacitor disconnected from the battery and then connected to a uncharged capacitor, the potential difference across the charged capacitor is changed. But the total charge of the capacitors system is constant.

Practice Problems :

1. **You are given an isolated parallel plate capacitor of capacitance C charged to a potential difference V. Which of the following quantity remains same when the separation between the plates is doubled with the help of insulating handles attahced to the plates ?**

(a) **potential difference across the plates** **(b)** **field between the plates**

(c) **energy stored in the capacitor** **(d)** **none**

[Answers : (1) d]

C14 Combination of Capacitors

Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, the potential difference V is applied across each capacitor. The total charge q stored on the capacitor is the sum of the charges stored on all the capacitors.

Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same total charge q and the same potential difference V as the actual capacitors. C_{eq} for n capacitor in parallel is given by

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Capacitors in Series

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of potential differences across all the capacitors is equal to the applied potential difference V.

Capacitor that connected in series can be replaced with an equivalent capacitor that has the same charge q and the same potential difference V as the actual series capacitors. C_{eq} for n capacitors in series is given by

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Practice Problems :

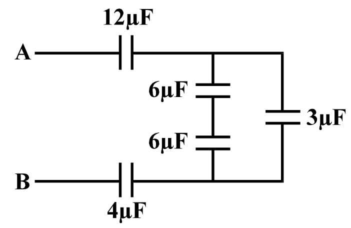
1. **Three capacitors of equal capacitance, when connected in series, have a net capacitance of C_1 and when connected in parallel, have a capacitance of C_2. The value of C_1/C_2 is**

(a) 1 : 9 **(b)** 9 : 1 **(c)** 1 : 3 **(d)** 3 : 1

2. **When two capacitors of capacitance C_1 and C_2 are connected in series the net capacitance is 3 μF; when connected in parallel its value is 16 μF. The value of C_1 is**

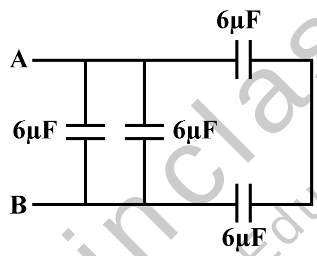
(a) 1 μF **(b)** 2 μF **(c)** 3 μF **(d)** none

3. **The equivalent capacitance of the following network between A and B is**



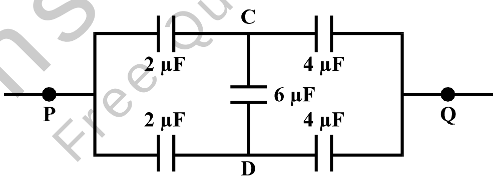
(a) 0.5 μF **(b)** 15 μF **(c)** 2.66 μF **(d)** none

4. **The equivalent capacitance of the following network between A and B is**



(a) 0.5 μF **(b)** 15 μF **(c)** 2.66 μF **(d)** none

5. **The equivalent capacitance of the following network between A and B is**

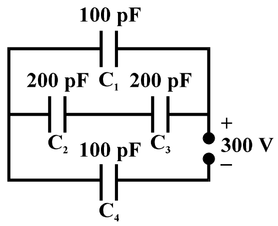


(a) 0.5 μF **(b)** 15 μF **(c)** 2.66 μF **(d)** none

6. **An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors. The minimum number of capacitors is**

(a) 12 **(b)** 18 **(c)** 24 **(d)** 30

7.



The potential difference across C_4 is

(a) 100 V **(b)** 200 V **(c)** 250 V **(d)** none

8. **A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32. The capacitance of the capacitor is**

(a) 2.5 nF **(b)** 3.5 nF **(c)** 4.5 nF **(d)** 5.5 nF

9. **In the above problem the potential of the inner sphere is**

(a) 25 mV **(b)** 35 mV **(c)** 45 mV **(d)** 55 mV

10. **The ratio of the capacitance of this capacitor with that of an isolated sphere of radius 12 cm is**

(a) 123 **(b)** 223 **(c)** 323 **(d)** 423

11. **A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. The potential of the inner cylinder is**

(a) 2.9×10^4V **(b)** 3.9×10^4V **(c)** 4.9×10^4V **(d)** 5.9×10^4V

12. **A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 V m^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown i.e., without starting to conduct electricity through partial ionisation). For safety, we should like the field never to exceed, say 10% of the dielectric strength. The minimum area of the plates is required to have a capacitance of 50 pF is**

(a) 17 cm^2 **(b)** 18 cm^2 **(c)** 19 cm^2 **(d)** 20 cm^2

[Answers : (1) a (2) d (3) a (4) b (5) c (6) b (7) b (8) d (9) c (10) d (11) a (12) c]

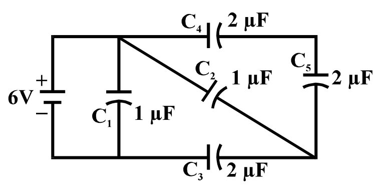
C15 Energy Stored in Capacitor

The energy stored in a capacitor is given by

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

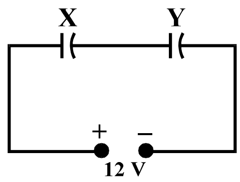
Practice Problems :

1. **The total energy stored in the capacitors in the given network is**



(a) 1.6×10^{-5} J **(b)** 2.6×10^{-5} J **(c)** 3.6×10^{-5} J **(d)** 4.6×10^{-5} J

2. X and Y are two parallel plate capacitors having the same area of plates and same separation between the plates. X has air between the plates and Y contains a dielectric medium of $\in_r = 5$.



The ratio of electrostatic energy stored in X and Y is

(a) 2 : 1 (b) 3 : 1 (c) 4 : 1 (d) 5 : 1

[Answers : (1) c (2) d]

C16 Capacitor with a Dielectric

When the space between the conductors is filled with a dielectric material, the capacitance increased by a factor K, called the dielectric constant of the material i.e. $C = K C_0$

(a) Capacitor with dielectric insertion when battery is disconnected

(i) Charge remains constant i.e. $q = q_0$

(ii) Potential between the plates decreases i.e. $V = \frac{V_0}{K}$

(iii) Field between the plates decreases i.e. $E = \frac{E_0}{K}$

(iv) Energy stored in the capacitor decreases i.e. $U = \frac{U_0}{K}$

(b) Capacitor with dielectric insertion when battery is connected.

(i) Potential difference remains constant.

(ii) Charge on plates increases i.e. $q = Kq_0$

(iii) Electric field remains constant

(iv) Energy stored in the capacitor increases i.e. $U = KU_0$

Practice Problems :

1. In a parallel plate capacitor, the capacitance increase from 4 microfarad to 80 microfarad, on introducing a dielectric medium between the plates. The dielectric constant of the medium is

(a) 10 (b) 20 (c) 30 (d) 40

[Answers : (1) b]

C17 Dielectrics and polarization

If the medium between the plates of a capacitor is filled with an dielectric (insulating substance), the electric field due to charged plates induces a net dipole moment in the dielectric. This effect is called polarisation. Although the net charge in the dielectric is zero but an electric field rises in the dielectric due to polarisation which is opposite to the external field E. The electric field in the dielectric is E/K where K is the dielectric constant of the dielectric. Hence the potential difference between the plates is reduced and that's why the capacitance will increase due to introduction of the dielectric by the factor K. The magnitude of induced charge on dielectric is given by $q_P = q\left(1-\frac{1}{K}\right)$, where q_p is the induced charge on dielectric and q is the charge on the plate of the capacitor.

C18 Force Between the Plates of Capacitor

The plates of parallel plate capacitor attracts each other with a force given $F = \dfrac{q^2}{2 \in_0 A}$.

Electrostatic stress of force per unit area acting on either capacitor plate is given by $\dfrac{1}{2} \in_0 E^2$.

Practice Problems :

1. A conducting plate A of surface area 0.01 m^2 is suspended from one arm of a sensitive balance such that it is parallel to another similar horizontal plate B and is at a height of 2 mm above B. The plate A in this situation is balanced by some weight in the other arm of the balance. A potential difference of 100 V is established between plates A and B. The additional mass should be added to the other arm to keep the system balanced is

(a) 0.0113 gm (b) 0.0213 gm (c) 0.0313 gm (d) 0.0414 gm

2. A parallel plate capacitor of plate area A and charge q is connected to a spring of spring constant k. The other end of the spring is connected to the wall. The extension in the spring at equilibrium is

(a) $\dfrac{q^2}{2 \in_0 AK}$ **(b)** $\dfrac{q^2}{3 \in_0 AK}$ **(c)** $\dfrac{q^2}{4 \in_0 AK}$ **(d)** $\dfrac{q^2}{5 \in_0 AK}$

[Answers : (1) a (2) a]

C19 Force on Dielectrics

A charged parallel plate capacitor, with battery disconnected or battery connected, pulled the dielectric inwards.

(i) The force acting on a dielectric slab while battery connected is given by

$$F = \frac{1}{2} V^2 \frac{dC}{dx}$$

(ii) The force acting on the dielectric slab while battery disconnected is given by

$$F = \frac{q^2}{2C^2} \frac{dC}{dx}$$

Practice Problems :

1. A parallel plate capacitor with plates width b and length L, the separation between the plates d is held constant and the plates are connected to a battery of potential difference V. A dielectric slab of relative permittivity K and thickness equal to the separation between the plates is inserted. Calculate

(a) the electrostatic energy of the system and

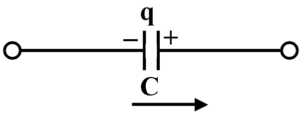
(b) force on the slab when x length of the slab is already inserted into the space between the plates.

[Answers : (1) (a) $\dfrac{\in_0 bV^2}{2d}[L + x(K-1)]$ **(b)** $\dfrac{\in_0 bV^2}{2d}(K-1)$ **]**

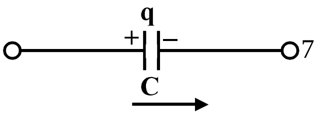
C20 Capacitors with Electric Circuit

Applying Kirchoff's rule for circuits with capacitors :

1. If we traverse through a capacitor from the negative terminal to the positive terminal, the change in potential is $\frac{+q}{C}$.

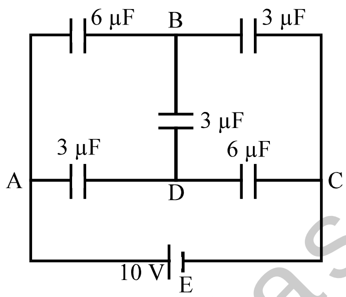


2. If we traverse through a capacitor from the positive terminal to the negative terminal, the change in potential is $\frac{-q}{C}$.



<u>Practice Problems :</u>

1. In the circuit shown find the charge on each capacitor.

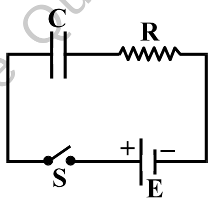


[Answers : (1) 24μC, 18μC, 6μC]

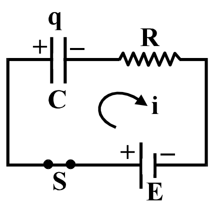
C21 Circuits with Resistors and Capacitors :

Charging of a Capacitor

The figure shows a circuit arrangement in which a capacitor in series with a resistor is connected across a battery of emf E through a switch S.



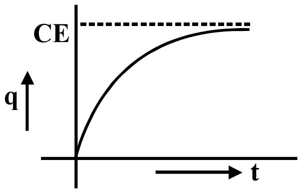
When the switch S is closed the current starts flowing and the capacitor starts charging. If i be the instantaneous current in the circuit and q be the instantaneous charge on capacitor, then applying Kirchoff's Voltage Law, we get $-\frac{q}{C} - iR + E = 0$



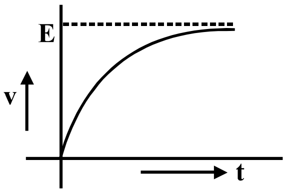
After solving the equation we get the following results.

$q = Q_{max}(1 - e^{-t/\tau})$, $v = V_{max}(1 - e^{-t/\tau})$, $i = I_{max}\, e^{-t/\tau}$

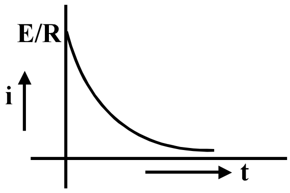
where Q_{max} = CE; V_{max} = E; I_{max} = E/R and τ = RC (time constant)



(a)



(b)



(c)

(a) Charge on a capacitor is increasing with time $q = CE(1 - e^{-t/RC})$.
At t = 0, q is zero and as t = ∞, q is Q_{max} = CE

(b) Potential difference across the capacitor increases with time $v = E(1 - e^{-t/RC})$.
At t = 0, v is zero and at t = ∞, v is V_{max} = F

(c) Current through the capacitor decreases with time

$$\mathbf{i = \frac{E}{R}}\, e^{-t/RC}. \text{ At } t = 0,\ i = I_{max} = E/R \text{ and } t = \infty,\ i = 0$$

The **time constant** τ is the time during which charge on the capacitor rises to 0.63 times the maximum value Mathematically,

when t = τ, $q = q_{max}(1 - e^{-1})$ or $q = 0.63\, q_{max}$ $\left(\because \mathbf{\frac{1}{e} = 0.37}\right)$

<u>**Practice Problems :**</u>

1. For the charging RC-circuit draw the graph between $l$ni vs. time t and what is the physical meaning of slope of this graph.

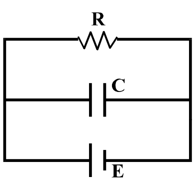
2. A 15.0 kΩ resistor and a capacitor are connected in series and then a 12.0 V potential difference is suddenly applied across them. The potential difference across the capacitor rises to 5.00 V in 1.30 μs. (a) Calculate the time constant of the circuit. (b) Find the capacitance of the capacitor.

3. A 3.00 MΩ resistor and a 1.00 μF capacitor are connected in series with an ideal battery of emf ε = 4.00 V. At 1.00 s after the connection is made, what are the rates at which (a) the charge of the capacitor is increasing, (b) energy is being stored in the capacitor, (c) thermal energy is appearing in the resistor, and (d) energy is being delivered by the battery ?

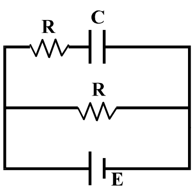
4. An initially uncharged capacitor C is fully charged by a device of constant emf ε connected in series with a resistor R. (a) Show that the final energy stored in the capacitor is half the energy supplied by the emf device. (b) By direct integration of i^2R over the charging time, show that the thermal energy dissipated by the resistor is also half the energy supplied by the emf device.

5. In the following circuit initially the capacitor is uncharged. Find the current passing through the battery at any time t. Also find the current through the battery at t = 0 and t = ∞.

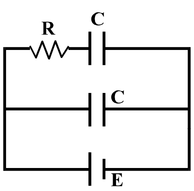
(a)



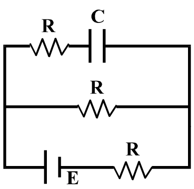
(b)



(c)



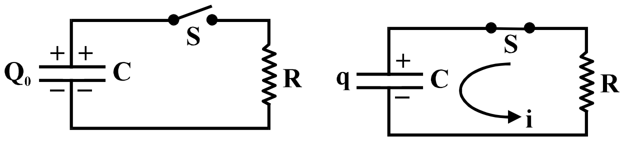
(d)



[Answers : (2) (a) 2.41 μs (b) 161 pF (3) (a) 0.955 μC/s (b) 1.08 μW (c) 2.74 μW (d) 3.82 μW]

Discharging of a Capacitor :

The figure shows a capacitor C with an initial charge Q_0 connected in series with a resistor. When the switch is closed the capacitor starts discharging. If i be the instantaneous current and q be the instantaneous charge on the capacitor then, using Kirchoff's voltage law

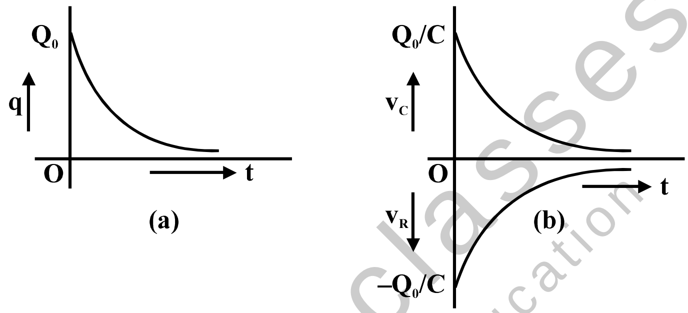


$$\frac{q}{C} + R\frac{dq}{dt} = 0 \text{ or } \frac{dq}{q} = -\frac{dt}{RC}$$

After solving the above equation, we get

$$q = Q_0e^{-t/\tau},\ i = -I_0e^{-t/\tau},\ V = V_0e^{-t/\tau}$$

where τ = RC is the time constant, Q_0, I_0 and V_0 are the initial charge, current and potential difference across the capacitor.



Practice Problems :

1. **A 1.0 μF capacitor with an initial stored energy of 0.50 J is discharged through a 1.0 MΩ resistor. (a) What is the initial charge on the capacitor ? (b) What is the current through the resistor when the discharge starts ? (c) Determine V_C, the potential difference across the capacitors, and V_R, the potential difference across the resistor, as functions of time. (d) Express the production rate of thermal energy in the resistor as a function of time.**
2. **Consider a discharging RC circuit. Draw the lnq vs. time graph and what is the physical meaning of the slope.**
3. **How many time constants will elapse before the charge on a capacitor falls to 0.1% of its maximum value in a discharging RC circuit ?**

[Answers : (1) (a) 1.0×10^{-3} C (b) 10^{-3} A (c) $V_C = 10^3e^{-t}$V, $V_R = 10^3e^{-t}$V; (d) $P = e^{-2t}$W]