UNITS AND MEASUREMENT

2.1 Introduction :

Q. Define Unit ?

Solution : Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit.

Q. How do we make the choice of standard/unit of measurement or what are the essential requirement for the choice of a unit ?

Solution : The unit chosen for measuring any physical quantity should meet the following essential requirements : (i) it should be accurately define, easily accessible, easily reproducible and of suitable size. (ii) it must be invariable with time, temperature, pressure etc.

Q. What is fundamental units, derived units and system of units ?

Solution : The units for the fundamental or base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as combinations of the base units. Such units obtained for the derived quantities are called derived units. A complete set of these units, both the base units and derived units, is known as the system of units.

2.2 The International System of Units :

Q. What are the different system of units ?

Solution : In earlier time three such systems, the CGS, the FPS (or British) system and the MKS system were in use extensively till recently.

The base units of length, mass and time in these systems were as follows :

- In CGS system they were centimetre, gram and second respectively.
- In FPS system they were foot, pound and second respectively.
- In MKS system they were metre, kilogram and second respectively.

The system of units which is at present internationally accepted for measurement is the Systeme Internationale d' United (French for International System of Units), abbreviated as SI. The SI, with standard scheme of symbols, units and abbreviations, was developed and recommended by General Conference on Weights and Measures in 1971 for international usage in scientific, technical, industrial and commercial work.

Q. In SI, what are the seven fundamental (base) quantities ? Write down their units and define them.

Solution : In SI, there are seven base units which are :

(a) Length : The unit is metre (m). The metre is the length of the path travelled by light in vacuum during a time interval of 1/299, 792, 458 of a second. (1983)

(b) Mass : The unit is kilogram (kg). The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at international Bureau of Weight and Measures, at Serves, near Paris, France. (1889)

(c) Time : The unit is second (s). The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of cesium-133 atom. (1967)

(d) Electric current : The unit is ampere (A). The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (1948)

(e) Thermodynamic Temperature : The unit is kelvin (K). The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)

(f) Amount of substance : The unit is mole (mol). The mole is the amount of substance of a system, which contains as many elementary entities as there as atoms in 0.012 kilogram of carbon - 12. (1971)

(g) Luminous intensity : The unit is candela (cd). The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. (1979)

Q. What are the different supplementary physical quantities and their units on the SI system ?

Solution : There are two supplementary physical quantities :

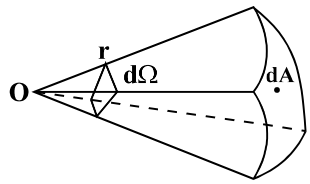
(i) Plane Angle : The unit of plane angle is radian and the symbol is rad.

(ii) Solid Angle : The unit of solid angle is steradian and the symbol is sr.

Q. Define solid angle ?

Solution : Solid angle $d\Omega$ as the ratio of the intercepted area dA of the spherical surface, described about the apex O as the centre, to the square of its radius r, as shown in figure.

$$d\Omega = \frac{dA}{r^2}$$



Q. What is the advantage of SI system over other system of units ?

Solution : (i) SI is a coherent system of units i.e., a system based on a certain set of fundamental units, from which all derived units are obtained. (ii) SI is a rational system of units, as it assigns only one unit to a particular physical quantity. For example, joule is the unit for all types of energy. (iii) SI units used decimal system, conversions within the system are quite simple and convenient.

Q. What is the value of (a) bar (b) curie (denoted by Ci) (c) roentgen (denoted by R) (d) barn (denoted by b) in SI unit ?

Solution : (a) 10^5 Pa (b) 3.7×10^{10} s^{-1} (c) 2.58×10^{-4} C/kg (d) 10^{-28} m^2

2.3 Measurement of Length :

Q. Which method is used to measure the large distances ?

Solution : Parallax method is used to measure the large distances.

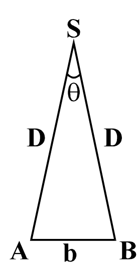
Q. What is the meaning of parallax and basis ? Give example.

Solution : Parallax is the name given to change in the position of an object with respect to the background, when the object is seen from two different positions. The distance between the two positions (i.e., points of observation) is called the basis.

When you hold a pencil in front of you against some specific point on the background (a wall) and look at the pencil first through your left eye A (closing the right eye) and then look at the pencil through your right eye B (closing the left eye), you would notice that the position of the pencil seems to change with respect to the point on the wall. This is called parallax. The distance between the two points of observation is called the basis. In this example, the basis is the distance between the eyes.

Q. Explain Parallax method to measure the large distance ? How the parallax method can employ to measure the size of a planet ?

Solution : To measure the distance D of a far away planet S by the parallax method, we observed it from two different positions (observations) A and B on the Earth, separated by distance AB = b at the same time as shown in the figure. We measure the angle between the two directions along which the planet is viewed at these two points. The $\angle ASB$ in figure represented by symbol θ is called the parallax angle or parallectic angle.



As the planet is very far away, $\frac{b}{D} << 1$, and therefore, θ is very small. Then we approximately take AB as an arc of length b of a circle with centre at S and the distance D as the radius AS = BS so that AB = b = D θ where θ is in radians, hence $D = \frac{b}{\theta}$.

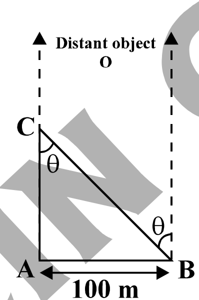
Having determined D, we can employ a similar method to determine the size or angular diameter of the planet. If d is a diameter of the planet and α the angular size of the planet (the angle subtended by d at the earth), we have α = d/D.

The angle α can be measured from the same location on the earth. It is the angle between the two directions when two diametrically opposite points of the planet are viewed through the telescope. Since D is known, the diameter d of the planet can be determined by using α = d/D.

Q. Calculate the angle of (a) 1^0 (degree) (b) $1'$ (minute of arc or arcmin) and (c) $1''$ (second of arc or arc second) in radians. Use $360^0 = 2\pi$ rad, $1^0 = 60'$ and $1' = 60''$. [NCERT solved example]

Solution : (a) 1.745×10^{-2} rad (b) 2.91×10^{-4} rad (c) 4.85×10^{-6} rad

Q. A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distant object O in line with AC. He then walks perpendicular to AC up to B, a distance of 100 m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40^0 (θ is known as 'parallax') estimate the distance of the tower C from his original position A. [NCERT solved example]



Solution : 119 m

Q. The moon is observed from two diametrically opposite points A and B on Earth. The angle θ subtended at the moon by the two directions of observation is $1^0 54'$. Given the diameter of the Earth to be about 1.276×10^7 m, compute the distance of the moon from the Earth. [NCERT solved example]

Solution : 3.84×10^8 m

Q. The Sun's angular diameter is measured to be $1920''$. The distance D of the Sun from the Earth is 1.496×10^{11} m. What is the diameter of the Sun ? [NCERT solved example]

Solution : 1.39×10^9 m

Q. For a given base line, will a distant star show greater parallex or a near star ?

Solution : For a given base line, the nearer star will show greater parallax.

Q. Why parallex method cannot be used for measuring distances of stars more than 100 light years away or why parallex method for nearby stars only ?

Solution : As the distance of star increases, the parallex angle decreases, and a great degree of accuracy is required for its measurement. Keeping in view the practical limitation in measuring the parallex angle, the maximum distance of a star we can measure is limited to 100 light years.

Q. Write down the limitation of parallex method ?

Solution : This method cannot be used for measuring distances of stars more than 100 light years.

Q. How do you measure the size of a molecule of oleic acid ?

Solution : Oleic acid is a soapy liquid with large molecular size of the order of 10^{-9} m. The idea is to first form mono-molecular layer of oleic acid on water surface.

We dissolve 1 cm^3 of oleic acid in alcohol to make a solution of 20 cm^3. Then we take 1 cm^3 of this solution and dilute it to 20 cm^3, using alcohol. So, the concentration of the solution is equal to $\left(\frac{1}{20\times 20}\right)cm^3$ of oleic acid/cm^3 of solution. Next we lightly sprinkle some lycopodium powder on the surface of water in a large trough and we put one drop of this solution in the water. The oleic acid drop spreads into a thin, large and roughly circular film of molecular thickness on water surface. Then, we quickly measure the diameter of the thin film to get its area A. Suppose we have dropped n drops in the water. Initially, we determine the approximate volume of each drop (V cm^3).

Volume of n drops of solution = nV cm^3

Amount of oleic acid in this solution = $nV\left(\frac{1}{20\times 20}\right)cm^3$

This solution of oleic acid spreads very fast on the surface of water and forms a very thin layer of thickness t. If this spreads to form a film of area A cm^2, then the thickness of the film

$$t = \frac{\text{Volume of the film}}{\text{Area of the film}}$$

or, $$t = \frac{nV}{20\times 20A}\,cm$$

If we assume that the film has mono-molecular thickness, then this becomes the size of diameter of a molecule of oleic acid. The value of this thickness comes out to be of the order of 10^{-9} m.

Q. If the size of a nucleus (in the range of 10^{-15} to 10^{-14} m) is scaled up to the tip of a sharp pin, what roughly is the size of atom ? Assume tip of the pin to be in the range 10^{-5}m to 10^{-4}m.

Solution : The size of a nucleus is in the range of 10^{-15} m and 10^{-14} m. The tip of a sharp pin is taken to be in the range of 10^{-5} m and 10^{-4} m. Thus we are scaling up by a factor of 10^{10}. An atom roughly of size 10^{-10} m will be scaled up to a size of 1 m. Thus a nucleus in an atom is as small in size as the tip of a sharp pin placed at the centre of a sphere of radius about a metre long.

Q. Define the following terms : (a) 1 astronomical unit (b) 1 light year (c) 1 parsec.

Solution : (a) 1 astronomical unit (1 AU) is the average distance of the sun from the earth and it equals to 1.496×10^{11} m.

(b) 1 light year (1 ly) is the distance that light travels with a velocity of 3×10^8 m s^{-1} in 1 year and it equals to 9.46×10^{15} m.

(c) 1 parsec equals to the radius of a circle, at the centre of which an arc of length 1AU of the circle subtends an angle of 1 second ($1''$) and it equals to 3.08×10^{16} m.

Q. Write down some special units to measure the large distance.

Solution : 1 astronomocial unit, 1 light year and 1 parsec are some special units to measure the large distance.

Q. What is the value of 1 fermi (1 f) in m.

Solution : f = 10^{-15} m

Q. Do AU and Å represent the same unit of length ?

Solution : No, AU and Å represent two different units of length.

Q. What is the relation between AU and Å ?

Solution : 1 AU = 1.496×10^{11} m

1 Å = 10^{-10} m $\Rightarrow$ 1m = 10^{10} Å

Hence, 1 AU = 1.496×10^{21} Å

Q. What is the relation between (a) 1 AU and 1 ly (b) 1 AU and 1 parsec (c) 1 ly and 1 parsec (d) 1 fermi and 1 Å ?

Solution : 1 AU = 1.496×10^{11} m, 1 Å = 10^{-10} m, 1 ly = 9.46×10^{15} m, 1 parsec = 3.08×10^{16} m, 1 fermi = 10^{-15} m.

(a) $\dfrac{1\text{ly}}{1\text{AU}} = \dfrac{9.46\times10^{15}}{1.496\times10^{11}} \Rightarrow$ 1 ly = (63.2) AU

(b) $\dfrac{1\,\text{parsec}}{1\text{AU}} = \dfrac{3.08\times10^{16}}{1.496\times10^{11}} \Rightarrow$ 1 parsec = (2.06×10^5) AU

(c) $\dfrac{1\,\text{ly}}{1\,\text{parsec}} = \dfrac{9.46\times10^{15}}{3.08\times10^{16}} \Rightarrow$ 1 ly = 0.307 parsec

(d) $\dfrac{1\,\text{fermi}}{1\,\text{Å}} = \dfrac{10^{-15}}{10^{-10}} \Rightarrow$ 1 fermi = (10^{-5}) Å

Q. Which one is largest unit to measure the large distance among 1 AU, 1 parsec and 1 ly ?

Solution : 1 parsec.

2.4 Measurement of Mass :

Q. Using which method, large masses in the universe like planet, stars is measured ?

Solution : Large masses in the universe like planets, stars etc., based on Newton's law of gravitation can be measured by using gravitational method.

Q. Using which method, mass of atom/sub-atomic particles is measured ?

Solution : For measurement of small masses of atomic/sub-atomic particles etc., we make use of mass spectrograph in which radius of the trajectory is proportional to the mass of a charged particle moving in uniform electric and magnetic field.

Q. Are inertial and gravitational mass of a body different from one another ?

Solution : No, they are equivalent.

Q. Define unified atomic mass unit.

Solution : unifined atomic mass unit (u), which has be established for expressing the mass of atoms as

1 unified atomic mass unit = 1u

= (1/12) of the mass of an atom of carbon-12 isotope $\left({}^{12}_{6}\text{C}\right)$ including the mass of electrons

= 1.66×10^{-27} kg.

2.5 Measurement of Time :

Q. Fill in the blank : Atomic standard of time is based on the periodic vibrations produced in a ____________ atom

Solution : cesium

2.6 Accuracy, Precision of Instruments and Errors in Measurement :

Q. Define Error and discuss types of errors.

Solution : The measurement of any physical quantity by any measuring instrument contains some uncertainity, this uncertainity is called error. The errors in measurement is broady classified as (a) systematic errors and (b) random errors.

Q. What is systematic error and discuss some sources of systematic errors ?

Solution : The systematic errors are those errors that tend to be in one direction, either positive or negative i.e., the causes of systematic errors are known and hence such errors can be minimised.

Some sources of systematic errors are :

(a) Instrumental errors : that arise from the errors due to imperfect design or calibration of the measuring instrument, zero error in the instrument.

(b) Imperfection in experimental technique or procedure

(c) Personal errors : that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions.

Q. How the systematic errors can be minimised ?

Solution : Systematic errors can be minimised by improving experimental techniques, selecting better instrument and removing personal errors as far as possible.

Q. What is random error and how it can be minimised ?

Solution : These can arise due to random and unpredictable fluctuations in experimental conditions, personal errors by the observer taking readings.

The random errors can be minimised by repeating the observation a large number of times and taking the arithmetic mean of all the observations.

Q. What is the difference between accuracy and precision ? Give example.

Solution : The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured.

The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument. For example, suppose the true value of a certain length is near 3.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.38 cm. The first measurement has more accuracy (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

Q. What is least count and least count error ?

Solution : The smallest value that can be measured by the measured instrument is called its least count. All the readings or measured values are good only up to this value. For e.g., the least count of vernier callipers is 0.01 cm i.e., using this instrument we can measure the minimum value of length upto 0.01 cm.

The least count error is the error associated with the resolution of the instrument.

Least count error belongs to the category of random errors but within a limited size, it occurs with both systematic and random errors.

Q. How the least count error can be reduced ?

Solution : Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.

Q. Explain Absolute error, Relative error and Percentage error ?

Solution : Suppose the values obtained in several measurements are a_1, a_2, a_3.... then

(i) |Absolute error| = True value – individual measurement value

$$\text{True value} = a_{mean} = \frac{\sum_{i=1}^{n} a_i}{n}$$

Error in the measurement of a_1, a_2 .... are given by

$|\Delta a_1| = a_{mean} - a_1$. $|\Delta a_2| = a_{mean} - a_2$ ....

(ii) $$\text{Mean absolute error} = \frac{|\Delta a_1| + |\Delta a_2| + ...}{n}$$

(iii) $$\text{Relative error} = \frac{\Delta a_{mean}}{a_{mean}}.$$

(iv) $$\text{Percentage error} = \delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

Q. Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are :

	Clock 1	**Clock 2**
Monday	**12:00:05**	**10:15:06**
Tuesday	**12:01:15**	**10:14:59**
Wednesday	**11:59:08**	**10:15:18**
Thrusday	**12:01:50**	**10:15:07**
Friday	**11:59:15**	**10:14:53**
Saturday	**12:01:30**	**10:15:24**
Sunday	**12:01:19**	**10:15:11**

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer ? [NCERT solved example]

Solution : Clock 2 is to be preffered to Clock 1

Q. We measure the period of oscillation of a simple pendulum. In successive measurements, the readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71 s and 2.80 s. Calculate the absolute errors, relative error or percentage error. [NCERT solved example]

Solution : The absolute error is 0.11 s and the percentage error is 4%

Q. Discuss the different combination of error ? or Discuss how errors propagate in sum, difference, product and division of quantities.

Solution : If we do an experiment involving several measurements, we must know how the errors in all the measurements combine.To make such estimates, we should learn how errors combine in various mathematical operations. For this, we use the following procedure :

(a) Error in sum of two or more quantities : Let two quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum

$$Z = A + B$$

We have by addition, $Z \pm \Delta Z$

$$= (A \pm \Delta A) + (B \pm \Delta B) = (A + B) \pm \Delta A \pm \Delta B$$

The maximum possible error in Z

$$\Delta Z = \Delta A + \Delta B$$

Hence the rule : When two or more quantities are in addition, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

(b) Error in a difference : Let two quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the difference

$$Z = A - B$$

We have by addition, $Z \pm \Delta Z$

$$= (A \pm \Delta A) - (B \pm \Delta B) = (A - B) \pm \Delta A \pm \Delta B$$

The maximum possible error in Z

$$\Delta Z = \Delta A + \Delta B$$

Hence the rule : When two quantities are in difference, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

(c) Errors in a product : Let two quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the product $Z = AB$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B) = AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B.$$

Dividing LHS by Z and RHS by AB we have,

$1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B).$

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/Z = (\Delta A/A) + (\Delta B/B)$

Hence the rule : When two quantities are in product, the maximum relative error is the sum of the relative (fractional) errors in the individual quantities.

(d) Error in a quotient : Let two quantities A and B have measured values $A \pm \Delta A$, $B \pm \Delta B$ respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the division $Z = A/B$. Then

$$Z \pm \Delta Z = (A \pm \Delta A)/(B \pm \Delta B)$$

$$Z\left[1 \pm \frac{\Delta Z}{Z}\right] = \frac{A\left[1 \pm \frac{\Delta A}{A}\right]}{B\left[1 \pm \frac{\Delta B}{B}\right]} \qquad \Rightarrow \qquad 1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$$

$\because \quad \frac{\Delta B}{B} << 1$, from binomial approximate $(1 + x)^n \simeq 1 + nx$ for $x << 1$

$$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \mp \frac{\Delta B}{B}\right) = 1 \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B} \pm \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right)$$

since ΔA and ΔB are small, we shall ignore their product

Hence $\pm \frac{\Delta Z}{Z} = \pm \frac{\Delta A}{A} \pm \frac{\Delta B}{B}$

Hence the rule : When two quantities are in division, the maximum relative error is the sum of the relative (fractional) errors in the individual quantities.

(e) Error in case of a measured quantity raised to a power :

$$Z = \frac{A^p B^q}{C^r}$$

Taking log of both side

$$\log Z = \log \frac{A^p B^q}{C^r} \Rightarrow \log Z = p \log A + q \log B - r \log C$$

Differentiating both side, we get

$$\frac{dZ}{Z} = p\frac{dA}{A} + q\frac{dB}{B} - r\frac{dC}{C}$$

In terms of relative error we may write this equation as $\pm \frac{\Delta Z}{Z} = \pm p\frac{\Delta A}{A} \pm q\frac{\Delta B}{B} \mp r\frac{\Delta C}{C}$.

Hence the rule is : if $Z = \frac{A^p B^q}{C^r}$, then maximum possible relative error is $\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$

Q. The temperature of two bodies measured by a thermometer are $t_1 = 20^0C \pm 0.5^0C$ and $t_2 = 50^0C \pm 0.5^0C$. Calculate the temperature difference and the error theirin. [NCERT solved example]

Solution : $30^0C \pm 1^0C$

Q. The resistance R = V/I where V = (100 ± 5)V and I = (10 ± 0.2)A. Find the percentage error in R. [NCERT solved example]

Solution : 7%

Q. Two resistors of the resistances $R_1 = 100 \pm 3$ ohm and $R_2 = 200 \pm 4$ ohm are connected (a) in series, (b) in parallel. Find the equivalent resistance of the (a) series combination, (b) parallel combination. Use for (a) the relation $R = R_1 + R_2$ and for (b) $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2}$ and $\frac{\Delta R'}{R'^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$.

[NCERT solved example]

Solution : (a) (300 ± 7) ohm (b) (66.7 ± 1.8) ohm

Q. Find the relative error in Z, if $Z = A^4B^{1/3}/CD^{3/2}$. [NCERT solved example]

Solution : $4(\Delta A/A) + (1/3)(\Delta B/B) + (\Delta C/C) + (3/2)(\Delta D/D)$

Q. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g ? [NCERT solved example]

Solution : 3%

2.7 Significant Figures :

Q. What is significant figure ?

Solution : Every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as significant digits or significant figures. Take the example that the period of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain while the digit 2 is uncertain.

Q. Write down the rules for significan figure.

Solution : Rules for determining the number of siginificant figures :

Rule 1 : All the non-zero digits are significant.

Rule 2 : All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

Rule 3 : If the number is less than 1, the zeros on the right of decimal point but to the left of the first non-zero digit are not significant. For example $\underline{0}.\underline{00}2308$, the underlined zeros are not significant.

Rule 4 : The terminal or trailing zeros in a number without a decimal point are not significant. e.g. 123 m = 12300 cm = 123000 mm has three significant figures.

Rule 5 : The trailing zeros in a number with a decimal point are significant. e.g. 3.500 has four significant figure.

Rule 6 : A choice of change of different units does not change the number of significant figures in a measurement. For example A length is reported to be 4.700 m. Now suppose we change units, then 4.700 m = 470.0 cm = 4700 mm = 0.004700 km numbers of significant figures in this remains 4. To avoid above confusion, use scientific notation. 4.700m = 4.700×10^2 cm = 4.700×10^3 mm. The power of 10 is irrelevent to the determination of significant figures.

Rule 7 : The digit 0 conventionally put on the left of a decimal for a number less than 1 (like 0.1250) is never significant. However, the zeroes at the end of such number are significant in a measurement.

Rule 8 : The multiplying or dividing factors which are neither rounded numbers nor numbers representing measured values, are exact and have infinite number of significant digits. For example in $r = \frac{d}{2}$ or $s = 2\pi r$, the factor 2 is an exact number and it can be written as 2.0, 2.00 or 2.0000 as required.

Q. Write down the number of significant figures in the following : (i) 5729 N (ii) 5.729 N (iii) 5729.00 N (iv) 5729×10^5 N (v) 5700 N (vi) 57.000 N (vii) 0.02370 N (viii) 0.02307 N (ix) 5.700×10^3 N

Solution : (i) 4 (ii) 4 (iii) 6 (iv) 4 (v) 2 (vi) 5 (vii) 4 (viii) 4 (ix) 4

Q. Write down the different rules for Arithmetic Operations with significant figues.

Solution : (1) In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures. For example, if mass of an object is measured to be, say, 4.237 g (four significant figures) and its volume is measured to be 2.51 cm^3 (3 significant figures) then its density will contain 3 significant figure i.e, $\mathbf{Density = \frac{4.237\,g}{2.51\,cm^3} = 1.69\,g\,cm^{-3}}$.

(2) In addition or subtraction, the final result should retain as many decimal place as are there in the number with the least decimal places. For e.g., the sum of the numbers 436.32 g, 227.2 g and 0.301 g by mere arithmetic addition, is 663.821 g. The final result should, therefore, be rounded off to 663.8 g.

Q. Write down the rules for rounding off the uncertain digits.

Solution : Rule 1 : The preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in the case) is more than 5, and is left unchanged if the latter is less than 5. For e.g., A number 2.746 rounded off to three significant figures is 2.75, while the number 2.743 would be 2.74.

Rule 2 : If the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1. Then, the number 2.745 rounded off to three significant figures becomes 2.74. On the other hand, the number 2.735 rounded off to three significant figures becomes 2.74 since the preceding digit is odd.

Q. Each side of a cube is 7.203 m. Find total surface area and total volume. [NCERT solved example]

Solution : 311.3 m^2, 373.7 m^3

Q. 5.74 g of the substance occupies 1.2 cm^3. Find the density. [NCERT solved example]

Solution : 4.8 g cm^{-3}

Q. Write down the rules for determining the uncertainity in the result of Arithmatic Calculations.

Solution : The rules for determining the uncertainty or error in the number/measured quantity in arithmetic operations can be understood from the following examples.

(1) If the length and breadth of a thin rectangular sheet are measured, using a metre scale as 16.2 cm and, 10.1 cm respectively, there are three significant figures in each measurement. It means that the length l may be written as

l = 16.2 ± 0.1 cm

= 16.2 cm ± 0.6 %

Similarly, the breadth b may be written as

b = 10.1 ± 0.1 cm

= 10.1 cm ± 1 %

Then, the error of the product of two (or more) experimental values, using the combinations of errors rule, will be

l b = 163.62 cm^2 ± 1.6 %

= 163.62 ± 2.6 cm^2

This leads us to quote the final result as

l b = 164 ± 3 cm^2

Here 3 cm^2 is the uncertainty or error in the estimation of area of rectangular sheet.

(2) If a set of experimental data is specified to n significant figures, a result obtained by combining the data will also be valid to n significant figures.

However, if data are subtracted, the number of significant figures can be reduced.

For example, 12.9 g – 7.06 g, both specified to three significant figures, cannot properly be evaluated as 5.84 g but only as 5.8 g, as uncertainties in substraction or addition combine in a different fashion (smallest number of decimal places rather than the number of significant figures in any of the number added or subtracted).

(3) The relative error of a value of number specified to significant figures depends not only on n but also on the number itself.

For example, the accuracy in measurement of mass 1.02 g is ± 0.01 g whereas another measurement 9.89 g is also accurate to ± 0.01 g. The relative error in 1.02 g is

$= (\pm 0.01/1.02) \times 100\ \%$

$= \pm 1\ \%$

Similarly, the relative error in 9.89 g is

$= (\pm 0.01/9.89) \times 100\ \%$

$= \pm 0.1\ \%$

Finally, remember that intermediate results in a multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.

These should be justified by the data and then the arithmetic operations may be carried out; otherwise rounding errors can build up. For example, the reciprocal of 9.58, calculated (after rounding off) to the same number of significant figures (three) is 0.104, but the reciprocal of 0.104 calculated to three significant figures, we would have retrieved the original value of 9.58.

This example justifies the idea to retain one more extra digit (than the number of digits in the least precise measurement) in intermediate steps of the complex multi-step calculations in order to avoid additional errors in the process of rounding off the numbers.

2.8 Dimensions of Physical Quantity :

Q. What is dimensions of a physical quantity ?

Solution : The dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity. Note that using the square brackets [ ] round a quantity means that we are dealing with **'the dimensions of'** the quantity.

Q. Write down the dimensions of the following fundamental physical quantities : (i) length (ii) mass (iii) time (iv) electric current (v) thermodynamic temperature (vi) luminous intensity (vii) amount of substance.

Solution : (i) [L] (ii) [M] (iii) [T] (iv) [A] (v) [K] (vi) [cd] (vii) [mol].

2.9 Dimensional Formulae and Dimensional Equation :

Q. What is dimensional formulae and dimensional equation.

Solution : The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity. For example, the dimensional formula of the volume is $[M^0L^3T^0]$.

An equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the physical quantity. For e.g., the dimensional equations of volume [V] may be expressed as $[V] = [M^0L^3T^0]$.

Q. Derive the dimensional formulae for the following physical quantities : (i) volume (ii) speed and velocity (iii) acceleration or acceleration due to gravity (g) (iv) force (v) density (vi) area (vii) linear momentum (viii) Impulse (ix) pressure (x) Universal constant of gravitation (G).

Solution : (i) volume = (length) (breadth) (height) = $[L]\,[L]\,[L] = [L^3] = [M^0L^3T^0]$

(ii) speed = $\dfrac{\textbf{length}}{\textbf{time}} = \dfrac{[L]}{[T]} = [LT^{-1}] = [M^0L^1T^{-1}]$

velocity = $\dfrac{\textbf{displacement}}{\textbf{time}} = \dfrac{[L]}{[T]} = [LT^{-1}] = [M^0L^1T^{-1}]$

(iii) Acceleration = $\dfrac{\textbf{change in velocity}}{\textbf{time}} = \dfrac{[LT^{-1}]}{[T]} = [LT^{-2}] = [M^0L^1T^{-2}]$

(iv) Force = mass acceleration = $[M]\,[LT^{-2}] = [MLT^{-2}]$

(v) Density = $\dfrac{\textbf{mass}}{\textbf{volume}} = \dfrac{[M]}{[L^3]} = [ML^{-3}]$

(vi) Area = $[L^2]$

(vii) Linear momentum = (mass) (velocity) = $[MLT^{-1}]$

(viii) Impulse = (force) (time) = $[MLT^{-2}]$ $[T]$ = $[MLT^{-1}]$

(ix) pressure = $\dfrac{\text{force}}{\text{area}} = \dfrac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(x) Universal constant of gravitation (G)

From Newton's law of gravitation $F = \dfrac{Gm_1m_2}{r^2}$ or $G = \dfrac{Fr^2}{m_1m_2}$, where F is force between masses m_1, m_2 at a distance r

$$G = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

2.10 Dimensional Analysis and Its Applications :

Q. Write down the various applications of dimensional analysis ? or write down the uses of dimensional equations.

Solution : Following are the three uses of dimensional equations :

(1) Conversion of one system of units into another

(2) Checking the correctness of various formulae or checking the dimensional consistency of equations

(3) Derivation of formulae or deducing relation among the physical quantities

Q. Write down the principle of homogeneity of dimensions and write down its use.

Solution : We can divide or multiply quantities having different dimensions but we can not add or subtract the quantities having different dimensions. Thus velocity cannot be added to force but they can be multiplied or divided. This principle called the principle of homogeneity of dimensions in an equation is extremely useful in checking the correctness of an equation.

Q. Check the dimensional consistency of the following equation : $x = x_0 + v_0t + (1/2)\ at^2$, where x is position at time t, x_0 is the initial position, v_0 is the initial velocity and a is the acceleration.

Solution : The dimensions of each term may be written as

$[x] = [L]$

$[x_0] = [L]$

$[v_0t] = [L\ T^{-1}]\ [T] = [L]$

$[(1/2)\ a\ t^2] = [L\ T^{-2}]\ [T^2] = [L]$

As each term on the right hand side of this equation has the same dimension, hence this equation is a dimensionally correct equation.

Q. What is the limitation of the principle of homogeneity of dimensions ?

Solution : A dimensionally correct equation need not be actually an exact (correct) equation, but a dimensionally wrong (incorrect) or inconsistent equation must be wrong.

Q. Let us consider an equation $\dfrac{1}{2}mv^2 = mgh$ where m is the mass of the body, v its velocity, g is the acceleration due to gravity and h is the height. Check whether this equation is dimensionally correct. [NCERT solved example]

Solution : Dimensionally correct

Q. The SI unit of energy is $J = kg\ m^2s^{-2}$; that of speed v is $m\ s^{-1}$ and of acceleration a is $m\ s^{-2}$. Which of the formulae for kinetic energy (K) given below can you rule out on the basis of dimensional arguments (m stands for the mass of the body) :

(a) $K = m^2v^3$ (b) $K = (1/2)\ mv^2$ (c) $K = ma$ (d) $K = (3/16)mv^2$ (e) $K = (1/2)mv^2 + ma$ [NCERT solved example]

Solution : On the basis of dimensional arguments formula (a), (c) and (e) are ruled out

Q. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on its length (*l*), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions. [NCERT solved example]

Solution : $T = k\sqrt{\frac{l}{g}}$

Q. Write down the various limitations of dimensional analysis.

Solution : (1) This method gives us no inforation about the dimensionless constants in the formula.

(2) If a quantity depends on more than three factors, having dimensions, the formula cannot be derived.

(3) We cannot derive the formulae containing trigonometrical functions, exponential functions, log functions etc., which have no dimensions.

(4) The method of dimensions cannot be used to derive an exact form of relation, when it consists of more that one part any side. For example, the exact form of the formula $s = ut + \frac{1}{2}at^2$ cannot be obtained.

(5) It gives no information whether a physical quantity is a scalar or a vector.

Q. Convert (i) 1 newton into dyne (ii) 1 joule into ergs

Solution : (i) M_1 = 1 kg; L_1 = 1 m; T_1 = 1 s and M_2 = 1 g; L_2 = 1 cm; T_2 = 1 s

n_1 = 1 newton; n_2 (number of dynes) = ?

$$n_2 = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)\left(\frac{T_1}{T_2}\right)^{-2} = n_2 = 1\left(\frac{1kg}{1g}\right)^1\left(\frac{1m}{1cm}\right)^1\left(\frac{1s}{1s}\right)^{-2} = 1\left(\frac{10^3 g}{1g}\right)^1\left(\frac{10^2 cm}{1cm}\right)^1 \times 1$$

$n^2 = 10^3 \times 10^2 = 10^5$

Hence, 1 newton = 10^5 dyne.

(ii) The dimensional formula of energy is $[M^1L^2T^{-2}]$.

M_1 = 1 kg,	M_2 = 1 g
L_1 = 1 m,	L_2 = 1 cm
T_1 = 1 s,	T_2 = 1 s
n_1 = 1 (joule)	n_2 (no. of ergs) = ?

As, $$n_2 = n_1\left(\frac{M_1}{M_2}\right)^1\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2}$$

$$\therefore \quad n_2 = 1\left(\frac{1kg}{1g}\right)^1\left(\frac{1m}{1cm}\right)^2\left(\frac{1s}{1s}\right)^{-2} = 1\left(\frac{10^3 g}{1g}\right)^1\left(\frac{10^2 cm}{1cm}\right)^2 \times 1 = 10^3 \times 10^4 = 10^7$$

Hence, 1 joule = 10^7 ergs

NCERT EXERCISE

2.1 Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to .....m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to .....$(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers .....m in 1 s

(d) The relative density of lead is 11.3. Its density is .....g cm^{-3} or .....kg, m^{-3}.

2.2 Fill in the blanks by suitable conversion of units

(a) 1 kg m^2 s^{-2} = ...g cm^2s^{-2}

(b) 1 m = ... 1y

(c) 3.0 m s^{-2} = ... km h^{-2}

(d) G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$ = ... $(cm)^3$ s^{-2} g^{-1}.

2.3 A calorie is a unit of heat of energy and it equals about 4.2 J where 1J = 1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of tims is γ s. Show that a calorie has a magnitude 4.2 α^{-1} β^{-2} γ^2 in terms of the new units.

2.4 Explain this statement clearly :

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary :

(a) atoms are very small objects

(b) a jet plane moves with great speed

(c) the mass of Jupiter is very large

(d) the air inside this room contains a large number of molecules

(e) a proton is much more massive than an electron

(f) the speed of sound is much smaller than the speed of light

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance ?

2.6 Which of the following is the most precise device for measuring length :

(a) a vernier callipers with 20 divisions on the sliding scale

(b) a screw gauge of pitch 1 mm and 100 divisions on the sliding scale

(c) an optical instrument that can measure length to within a wavelength of light ?

2.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair ?

2.8 Answer the following :

(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread ?

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale ?

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only ?

2.9 The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement.

2.10 State the number of significant figures in the following :

(a) 0.007 m^2 (b) 2.64 × 10^{24} kg (c) 0.2370 g cm^{-3} (d) 6.320 J (e) 6.032 N m^{-2} (f) 0.0006032 m^2

2.11 **The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.**

2.12 **The mass of box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures ?**

2.13 **A physical quantity P is related to four observables a, b, c and d as follows :**

$P = a^3b^2 / (\sqrt{c}\, d)$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P ? If the value of P calculated using the above relation turns out to be 3.763 to what value should you round off the result ?

2.14 **A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion :**

(a) $y = a \sin 2\pi t/T$

(b) $y = a \sin vt$

(c) $y = (a/T) \sin t/a$

(d) $y = (a/\sqrt{2}) (\sin 2\pi t/T + \cos 2\pi t/T)$

(a = maximum displacement of the particle, v = speed of the particle. T = time-period of motion). Rule out the wrong formulas on dimensional grounds.

2.15 **A famous relation in physics relates 'moving mass' m to the 'rest mass' m_0 of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :** $m = \dfrac{m_0}{(1 - v^2)^{1/2}}$.

Guess where to put the missing c.

2.16 **The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å : 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms ?**

2.17 **One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen ? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large ?**

2.18 **Explain this common observation clearly : If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).**

2.19 **The principle of 'parallax' in section 2.3.1 is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit = 3×10^{11} m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres ?**

2.20 **The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs ? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun ?**

2.21 **Precise measurements of physical quantities are a need to science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.**

2.22 **Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity) :**

(a) the total mass of rain-bearing clouds over India during the Monsoon

(b) the mass of an elephant

(c) the wind speed during a storm

(d) the number of strands or hair on your head

(e) the number of air molecules in your classroom.

2.23 **The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases ? Check if your guess is correct from the following data : mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.**

2.24 **When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72" of arc. Calculate the diameter of Jupiter.**

ADDITIONAL EXERCISES

2.25 **A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v : tan θ = v and checks that the relation has a correct limit : as v → 0, θ → 0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct ? If not, guess the correct relation.**

2.26 **It is claimed that two cesium clocks, if allowed to run for 100 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s ?**

2.27 **Estimate the average mass density of a sodium atom assuming its size to be about 2.8 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase : 970 kg m^{-3}. Are the two densities of the same order of magnitude ? If so, why ?**

2.28 **The unit of length convenient on the nuclear scale is a fermi : 1 f = 10^{-15} m. Nuclear sizes obey roughly the following empirical relation : $r = r_0 A^{1/3}$, where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in exercise. 2.27.**

2.29 **A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth ?**

2.30 **A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine ? (Speed of sound in water = 1450 m s^{-1}).**

2.31 **The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us ?**

2.32 **It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From the fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.**

2.33 **A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G,**

he could arrive at a number with the dimension of time. Further, it was very large number, its magnitude being close to the present estimate on the age of universe (~ 15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants ?

ANSWERS

2.1 (a) 10^{-6} ; (b) 1.5×10^4 ; (c) 5 ; (d) 11.3, 1.13×10^4

2.2 (a) 10^7; (b) 10^{-16} ; (c) 3.9×10^4 ; (d) 6.67×10^{-8}

2.5 500

2.6 (c)

2.7 0.035 mm

2.9 94.1

2.10 (a) 1 ; (b) 3 ; (c) 4 ; (d) 4 ; (e) 4 ; (f) 4

2.11 8.72 m^2; 0.0855 m^3

2.12 (a) 2.3 kg ; (b) 0.02 g

2.13 13%; 3.8

2.14 (b) and (c) are wrong on dimensional grounds. Hint : The argument of a trigonometric function must always be dimensionless.

2.15 The correct formula is $m = m_0 (1 - v^2/c^2)^{-1/2}$

2.16 $\cong 3 \times 10^{-7}$ m^3

2.17 $\cong 10^4$; intermolecular separation in a gas is much larger than the size of a molecule.

2.18 Near objects make greater angle than distant (far off) objects at the eye of the observer. When you are moving, the angular change is less for distant objects than nearer objects. So, these distant objects seem to move along with you, but the nearer objects in opposite direction.

2.19 $\cong 3 \times 10^{16}$ m; as a unit of lenth 1 parsec is defined to be equal to 3.084×10^{16} m.

2.20 1.32 parsec; 2.64" (second of arc)

2.23 1.4×10^3 kg m^{-3}; the mass density of the Sun is in the range of densities of liquids / solids and not gases. This high density arises due to inward gravitational attraction on outer layers due to inner layers of the Sun.

2.24 1.429×10^5 km

2.25 Hint : tan θ must be dimensionless. The correct formula is $\tan\theta = v/v'$ where v' is the speed of rainfall.

2.26 Accuracy of 1 part in 10^{11} to 10^{12}

2.27 $\cong 0.7 \times 10^3$ kg m^{-3}. In the solid phase atoms are tightly packed, so the atomic mass density is close to the mass density of the solid.

2.28 $\cong 0.3 \times 10^{18}$ kg m^{-3} – Nuclear density is typically 10^{15} times atomic density of matter.

2.29 3.84×10^8 m

2.30 55.8 km

2.31 2.8×10^{22} km

2.32 3.581 km

2.33 Hint : the quantity $e^4/(16\pi^2\varepsilon_0^2 m_p m_e^2 c^3 G)$ has the dimension of time

ADDITIONAL QUESTIONS AND PROBLEMS

Q. **Find the dimensions of the following quantities :**
(1) angle (2) work (3) energy (4) moment of force or torque (5) power (6) surface tension (7) surface energy (8) force constant (9) thrust (10) tension (11) stress (12) strain (13) modulus of elasticity (14) energy per unit volume (15) radius of gyration (16) moment of inertia (17) angular displacement (18) angular velocity (19) angular acceleration (20) angular momentum (21) wavelength (22) frequency (23) velocity gradient (24) plank's constant (25) Reynold number (26) Rydbrg constant (27) coefficient of viscosity (28) gas constant (29) Boltzmann constant (30) coefficient of thermal conductivity (31) Stefan's constant (32) Weins constant

Q. **The value of a force on a body is 20 N in SI units. What is the value of this force in cgs units, that is, dynes ?**

A. 20×10^5

Q. **Check by the method of dimensions whether the following relations are true.**

(i) $t = 2\pi\sqrt{\frac{l}{g}}$ **(ii)** $v = \sqrt{\frac{P}{D}}$

where v = velocity of sound and P = pressure, D = density of medium.

(iii) $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$ **where n = frequency of vibration, *l* = length of the string,**

F = stretching force, m = mass per unit length of the string.

Q. **Assuming that the critical velocity of flow of a liquid through a narrow tube depends on the radius of the tube, density of the liquid and viscosity of the liquid, find an expression for critical velocity.**

Q. **What is the difference between the inertial and gravitational mass ? How are they measured.**

Q. **If density (D), acceleration due to gravity (g) and frequency (n) are taken as base quantities, find the dimensions of force in terms of above quantities ?**

A. Dg^4n^{-6}

Q. **Find the dimensions of a and b in the formula** $\left[P + \frac{a}{V^2}\right](V - b) = RT$.

A. ML^5T^{-2}, L^3

Q. **A gas bubble, form an explosion under water, oscillates with a period proportional to $P^a d^b E^c$, where P is the static pressure, d is the density and E is the total energy of the explosion. Find the values of a, b and c.**

A. $a = -\frac{5}{6}, b = \frac{1}{2}$ **and** $c = \frac{1}{3}$

Q. **A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis find the expression for period of oscillation in terms of radius of star R, mean density of fluid ρ and universal gravitational constant G.**

A. $T = K\frac{1}{\sqrt{G\rho}}$

Q. **The energy E of an oscillating body in simple harmonic motion depends on its mass m, frequency n and amplitude a. Using the method of dimensional analysis find the relation between E, m, n and a.**

A. kmn^2a^2 (k = constant)

Q. **The speed V of a moving particles varies with time t as $V = At^2 + Bt$, where V is in ms^{-1} and t is in second. Find the units and dimensions of A and B.**

A. m/s^3, m/s^2, LT^{-3}, LT^{-2}

Q. **If the velocity of light c, the gravitational constant G, and Plank's constant h are chosen as fundamental units, find the dimensions of time, mass and length in the new system.**

A. $T = [c^{-5/2}h^{1/2}G^{1/2}]$, $M = [h^{1/2}c^{1/2}G^{-1/2}]$, $L = [G^{1/2}c^{-3/2}h^{1/2}]$