Electrostatics

C1 Properties of charges :

(i) Two kinds of charges exist in nature, positive and negative with the property that unlike charges attract each other and like charges repel each other.

(ii) Excess of electrons means negative charge and deficiency of charge means positive charge.

(iii) Charge is conserved for an isolated system.

(iv) Charge is quantized i.e. $q = \pm ne$ where n = 1, 2, 3..... and $e = 1.6 \times 10^{-19}$ C

(v) Charge is invariant.

(vi) On charging a neutral body, the mass of the body will change.

C2 Electric field : The magnitude of electric field for a point charge q at the distance r is given by $\frac{1}{4\pi \in_0 \in_r}\frac{q}{r^2}$ and direction is towards the charge if it is positive and away from the charge if it is negative. $\in_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 is known as absolute permittivity constants and $\in_r$ is known as relative permittivity constant.

Principle of super position : The electric field due to a group of charges can be obtained using the superposition principle. That is, the total electric field equals the vector sum of the electric fields of all the charges at some point : $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3....$

Practice Problems :

1. A positive charge is placed at the origin. The minimum electric field produced at the point (3, 4, 0) is given by

(a) $2.6(-3\hat{i} + 4\hat{j}) \times 10^{-11}$ N / C **(b)** $2.6(-3\hat{i} - 4\hat{j}) \times 10^{-11}$ N / C

(c) $2.6(3\hat{i} - 4\hat{j}) \times 10^{-11}$ N / C **(d)** $2.6(3\hat{i} + 4\hat{j}) \times 10^{-11}$ N / C

2. Five point charges, each of value + q coul, are placed on five vertices of a regular hexagon of side L meters. The magnitude of the force on a point charge of value – q coul placed at the centre of the hexagon is

(a) 5kq^2/L^2 **(b)** 0 **(c)** kq^2/L^2 **(d)** 3kq^2/L^2

3. A cube of side b has a charge q at each of its vertices. The electric field due to this charge distribution at the centre of the cube is

(a) q/b^2 **(b)** q/2b^2 **(c)** 32q/b^2 **(d)** zero

[Answers : (1) d (2) c (3) d]

C3 Coulomb Force : $\vec{F} = q_0\vec{E}$, where q_0 is the test charge placed in the external electric field $\vec{E}$. Column Force is central, inverse square and conservative field. Coulamb's force is valid for distances from 10^{-13} cm to several kilometers. i.e. up to infinity.

Practice Problems :

1. A liquid drop of mass m and carrying n electrons can be balanced by applying an electric field. The direction and magnitude of the electric field is

(a) upward, $\frac{mg}{ne}$ **(b)** downward, $\frac{mg}{ne}$

(c) upward, $\frac{2mg}{ne}$ **(d)** downward, $\frac{2mg}{ne}$

2. **A charge Q is divided into two parts and the two parts are separated by a certain distance d. Then the maximum force between them will be**

(a) $\frac{kQ^2}{4d^2}$ **(b)** $\frac{kQ^2}{3d^2}$ **(c)** $\frac{kQ^2}{2d^2}$ **(d)** $\frac{kQ^2}{d^2}$

[Answers : (1) b (2) a]

C4 **Electric Field Lines :** A convenient specialized pictorial representation for visualizing electric field patterns is created by drawing lines showing the direction of the electric field vector at any point. These lines, called electric field lines, are related to the electric field in any region of space in the following manner :

- The electric field vector E is tangent to the electric field line at each point.
- The number of electric field lines per unit area through a surface that is perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, E is large where the field lines are close together and small where they are far apart.

For a positive point charge, the lines are directed radially outward and for a negative point charge, the lines are directed radially inward. The number of lines that originate from or terminate on a charge is proportional to the magnitude of the charge.

C5 **Motion of charged particles in electric field**

When a particle of charge q and mass m is placed in an electric field $\vec{E}$ then the force on the charge particles is $q\vec{E}$ and according to Newton's second law $\vec{F} = q\vec{E} = m\vec{a}$ $\Rightarrow$ $\vec{a} = \frac{q\vec{E}}{m}$ where $\vec{a}$ is the acceleration of the particle. If the particle is released from rest in uniform $\vec{E}$ or projected with certain speed along the direction of uniform $\vec{E}$ or opposite to the direction of uniform $\vec{E}$ then the path is straight line otherwise the path is parabolic in uniform $\vec{E}$.

<u>Practice Problems :</u>

1. **A charged particle of mass m and charge q is released from rest in a uniform electric field E. The kinetic energy of the particle after time t is**

(a) $\frac{2E^2t^2}{mq}$ **(b)** $\frac{Eq^2m}{2t^2}$ **(c)** $\frac{E^2q^2t^2}{2m}$ **(d)** $\frac{Eqm}{2t}$

[Answers : (1) c]

C6 **Electric flux :** Electric flux is proportional to the number of electric field lines that penetrate a surface. The electric flux through a surface is defined by the expression $\phi = \int \vec{E}.d\vec{A}$ If $\vec{E}$ is surface uniform then $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ is the angle that the electric field makes with the normal to the surface.

C7 **Gauss's Law :** Gauss's Law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge inside the surface divided by $\in_0$: $\Phi_E = \oint E.dA = \frac{q_{in}}{\in_0}$. Using Gauss's law, one can calculate the electric field due to various symmetric charge distributions.

Practice Problems :

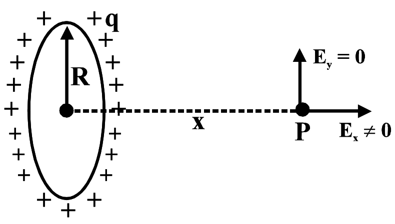
1. **The electric charge is placed at the centre of a cube of side a. The electric flux through one of its faces will be**

(a) $\frac{q}{6\in_0}$ **(b)** $\frac{q}{\in_0 a^2}$ **(c)** $\frac{q}{4\pi\in_0 a^2}$ **(d)** $\frac{q}{\in_0}$

[Answers : (1) a]

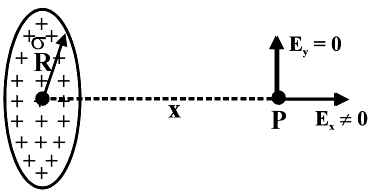
C8 Electric Field due to special type of configuration :

1. A Ring of Charge



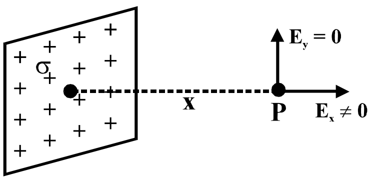
$$E_y = 0,\ E_x = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

2. A Disc of Charge



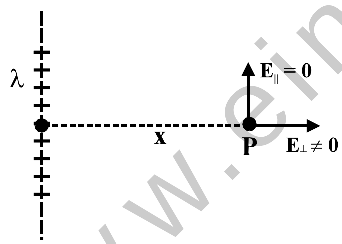
$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2+R^2}}\right]$$

3. Infinite Sheet of Charge



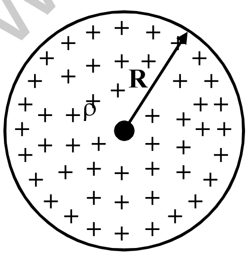
$E_x = \frac{\sigma}{2\varepsilon_0}$, **where σ is the surface charge density**

4. Infinitely Long Line of Charge



$E_{\parallel} = 0, E_{\perp} = \frac{\lambda}{2\pi\varepsilon_0 x}$, **where λ is the linear charge density**

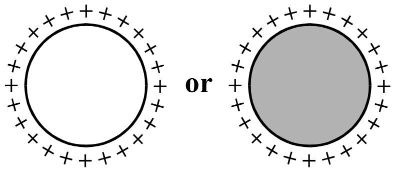
5. A Spherical Volume Charge Distribution (non-conducting solid sphere)



$$E = \frac{\rho r}{3\varepsilon_0}\, r < R,\ E = \frac{q}{4\pi\varepsilon_0 r^2}\, r \ge R\left(q = \rho\frac{4}{3}\pi R^3\right)$$

ρ is volume charge density

6. Spherical Conductor (Hollow or Solid)



E = 0 r < R, $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ r ≥ R

Practice Problems :

1. A sphere of radius R has a uniform volume charge distribution. At a distance x from its centre, for x < R, the electric field is directly proportional to

(a) $1/x^2$ **(b)** $1/x$ **(c)** x **(d)** x^2

(c)

2. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of 4.0×10^{-6}C. In equilibrium, the thread makes and angle of 60^0 with the vertical. The surface charge density on the plate in μC/m² is

(a) 0.75 **(b)** 7.5 **(c)** 75 **(d)** 750

[Answers : (1) c (2) d]

C9 Electric Potential Energy : The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by a potential-energy function U : $W_{a \to b} = U_a - U_b$.

C10 Electric Potential Energy of two point charges : The potential energy for two point charges q and q_0 separated by a distance r is $U = \dfrac{1}{4\pi \in_0}\dfrac{qq_0}{r}$

C11 Electric potential energy of a point charge in the electric field of several charges : The potential energy for a charge q_0 in the electric field of a collection of charges q_i is given by

$$U = \frac{q_0}{4\pi \in_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ...\right) = \frac{q_0}{4\pi \in_0}\sum_i \frac{q_i}{r_i}$$ where r_i is the distance from q_i to q_0.

C12 Total potential energy of several charges : The total potential energy U is the sum of the potential energies of intersection for each pair of charges. We can write this as $U = \dfrac{1}{4\pi \in_0}\sum_{i<j}\dfrac{q_i q_j}{r_{ij}}$

Practice Problems :

1. Four charges +q, –q, +q and –q are placed at the corners A, B, C and D respectively of a square of side a. The potential energy of the system is

(a) $k\dfrac{q^2}{a}(-4+\sqrt{2})$ **(b)** $k\dfrac{q^2}{2a}(-4+\sqrt{2})$

(c) $k\dfrac{4q^2}{a}$ **(d)** $-k\dfrac{4\sqrt{2}q^2}{a}$

[Answers : (1) a]

C13 **Energy is an electric field :** This energy is stored in the electric field generated by the charges. i.e. in the space where the electric field exists and it is found that the energy stored in the field per unit volume is given by $\frac{1}{2}\varepsilon_0 E^2$.

C14 **Electrical potential** Electric potential, a scalar quantity, is the potential energy per unit charge. Mathematically, Potential, $V = \frac{U}{q_0}$ or $U = q_0V$. The unit of potential is volt (V) or J/c.

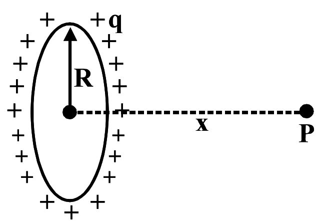
Work done by external source to move a charge (q_0) very slowly from initial point to final point in an electric field $W = q_0(V_f - V_i)$ where V_f is the potential due to charge distribution at point and V_i is the find potential due to charge distribution at initial point.

C15 The potential V due to a point charge q at distance r, $V = \frac{kq}{r}$

C16 Potential due to collection of point charges, $V = k\sum_i \frac{q_i}{r_i}$, where r_i is the distance of charge q_i at the point where potential will be calculated.

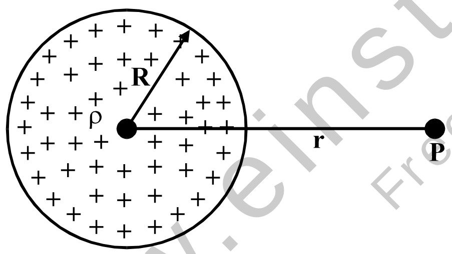
C17 **Electric potential due to special type of charge distribution**

1. **A Ring of Charge**



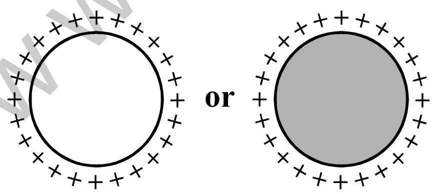
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + x^2}}$$

2. **A sphere of Charge**



$$V = \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3}\ r < R,\quad V = \frac{Q}{4\pi\varepsilon_0 r}\ r \geq R$$

3. **Conducting Sphere (Hollow or Solid)**



$$V = \frac{Q}{4\pi\varepsilon_0 R}\ r < R,\quad V = \frac{Q}{4\pi\varepsilon_0 r}\ r \geq R$$

Practice Problems :

1. **A hollow metal sphere of radius 5 cm is charged so that the potential on its surface is 10 V. Let the potential at the centre of the sphere, at a distance 3 cm from the center and at a distance 10 cm from the center are V_1, V_2, and V_3 respectively. Then $V_1 : V_2 : V_3$ equals to**

   (a) 1 : 1 : 2 **(b)** 2 : 2 : 1 **(c)** 1 : 2 : 2 **(d)** 1 : 3 : 5

   (b)

2. **Two identical thin rings, each of radius R meters, are coaxially placed at a distance R meters apart. If Q_1 coulomb and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q coulomb from the centre of one ring to that of the other is**

   (a) zero **(b)** $q(Q_1 - Q_2)(\sqrt{2} - 1)/(4\sqrt{2}\,\pi\epsilon_0 R)$

   (c) $q\sqrt{2}(Q_1 + Q_2)/4\pi\epsilon_0 R$ **(d)** $q(Q_1 + Q_2)(\sqrt{2} + 1)/(4\sqrt{2}\,\pi\epsilon_0 R)$

[Answers : (1) b (2) b]

C18 Relation between electric field and electric potential : Relation between field and potential is given by $dV = -\vec{E}.d\vec{r}$ integrating between points a and b, $\int_a^b dV = V_b - V_a = -\int_a^b \vec{E}.d\vec{r}$ where V_a and V_b are the potentials at a and b. In differential form, we have $\vec{E} = -\frac{dV}{dr}\hat{r}$

Practice Problems :

1. **A nonconducting ring of radius 0.5 m carries a total charge of 1.11×10^{-10}C distributed non-uniformity of its circumference producing an electric field $\vec{E}$ everywhere in space. The value of the line integral $\int_{l=\infty}^{l=0} -\vec{E}.\vec{dl}$ (l = 0 at the centre of the ring) in volt is :**

   (a) + 2 **(b)** – 1 **(c)** – 2 **(d)** zero

[Answers : (1) a]

C19 Equipotential Surface : There is another way to demonstrate the graphical representation of field using the concept of Equipotential Surfaces. An equipotential surface is three dimensional surfaces on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface the two are perpendicular.

Note the following points :

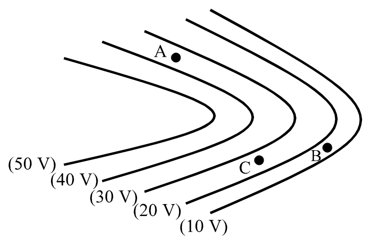
(a) the field is stronger where the equipotential surfaces are closely spaced.

(b) the work done to move a charge on a equipotential surface is zero.

(c) the work done to move a charge q_0 from one equipotential surface (having potential, (V_1) to another equipotential surface (having potential V_2) is q_0 ($V_2 - V_1$).

Practice Problems :

1. **The figure shows lines of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point is**



(a) **A** **(b)** **B** **(c)** **C** **(d)** **equal at all points**

[Answers : (1) b]

C20 **Conductors and insulators :** In conductors charges are free to move throughout the volume of such bodies whereas in case of insulators or dielectrics, the charges remain fixed at the places where they were initially distributed. Hence charge given to a conductor always resides on its surface.

In conductors electric charges are free to move throughout the volume of such bodies. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.

The following are the properties of a conductor in electrostatic equilibrium.

1. The electric field is zero everywhere inside the conductor (E = 0)
2. No volume charges exist inside a conductor
3. The surface of a conductor is an equipotential surface and lines of forces always meet a conducting surface normally.
4. Charge density is inversely proportional to radius of curvature.
5. If there is cavity inside a conductor, the field strength inside the cavity equals zero, whatever is the field outside the conductor.
6. The field intensity near a conducting surface is always $\mathbf{E} = \frac{\sigma}{\varepsilon_0}$, where σ is the local surface charge density at that point.
7. **Redistribution of Charge :** If two conductors are brought into contact, the charges from one of them will flow over to the other until their potentials become equal. The equality of potential implies that charges on each sphere (as shown) is proportional to its radius. i.e., $\frac{q_1}{R_1} = \frac{q_2}{R_2}$. For a uniform surface charge density σ, the total charge $q = 4\pi R^2\sigma$, so the above equation becomes $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$.
8. **Potential of concentric conducting spheres : Superposition Principle :** Let us consider two concentric spheres of radii r_1 and r_2 with uniformly distributed charges q_1 and q_2. Using the principle of superposition, the potential of the small and large sphere may be written as $V_1 = \frac{k_1 q_1}{r_1} + \frac{kq_2}{r_2}$, $V_2 = \frac{kq_1}{r_2} + \frac{kq_2}{r_2}$

<u>Practice Problems :</u>

1. **A charge Q is distributed over two concentric hollow spheres of radii r and R(R > r) such that the surface densities are equal. The potential at the common centre is $1/4\pi\in_0$ times**

(a) $Q\left[\frac{r+R}{r^2+R^2}\right]$ **(b)** $\frac{Q}{2}\left(\frac{r+R}{r^2+R^2}\right)$ **(c)** $2Q\left(\frac{r+R}{r^2+R^2}\right)$ **(d)** **zero**

(a)

2. **Two charges conducting spheres of radii R_1 and R_2, separated by a large distance, are connected by a long wire. The ratio of the electric fields on the surface of the two spheres is**

(a) $\frac{R_1}{R_2}$ **(b)** $\frac{R_2}{R_1}$ **(c)** $\frac{R_1^2}{R_2^2}$ **(d)** $\frac{R_2^2}{R_1^2}$

Answers : (1) a (b) b]

C21 Electric Dipole : An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge –q) separated by a distance d. The characteristic of a dipole is its dipole moment defined as $\vec{p} = q\vec{d}$, the direction is from negative charge to positive charge.

Practice Problems :

1. Electric charges q, q and –2q are placed at the three corners of an equilateral triangle of side *l*. The magnitude of the electric dipole moment of the system is

(a) q*l*   (b) 2 q*l*   (c) √3 q*l*   (d) 4 q*l*

[Answers : (1) c]

C22 Electric field due to a dipole

(i) Along the Axis : The electric field intensity along the axis always point in the direction of the dipole. It magnitude is given by $E_{\|} = \frac{2kp}{x^3}$.

(ii) Along the Bisector : The direction of electric field along the bisector is always opposite to the dipole moment. Math emetically, $E_{\perp} = -\frac{kp}{y^3}$

Practice Problems :

1. A given charge situated at a certain distance from a short electric dipole in the end-on position experiences a force F_1. If the distance of the charge from the dipole is doubled then the force acting on the charge is F_2. Then F_1/F_2 equals to

(a) 2   (b) 1/2   (c) 1/4   (d) 8

2. If E_a be the electric field intensity due to a short dipole at a point on the axis and E_r be that on the right bisector at the same distance from the dipole, then

(a) $E_a = E_r$ (b) $E_a = 2E_r$ (c) $E_r = 2E_a$ (d) $E_a = \sqrt{2}\, E_r$

[Answers : (1) d (2) b]

C23 Electric potential due to a dipole

(i) Along the Axis : The electric potential along the x-axis is given by $V_{\|} = \frac{2kp}{x^2}$

(ii) Along the Bisector : The electric potential along the bisector is always zero.

C24 Dipole in an external electric field :

1. The net force experienced by a dipole in an external uniform electric field is zero.

2. When an electric dipole of dipole moment $\vec{p}$ is placed in an electric field $\vec{E}$, the field exerts a torque $\vec{\tau}$ on the dipole : $\vec{\tau} = \vec{p} \times \vec{E}$.

3. The dipole has a potential energy U associated with its orientation in the field : $U = -\vec{p}.\vec{E}$. This potential energy is defined to be zero when $\vec{p}$ is perpendicular to $\vec{E}$; it is least (U = –pE) when $\vec{p}$ is aligned with $\vec{E}$, and most (U = pE) when $\vec{p}$ is directed opposite $\vec{E}$. Hence a dipole is in stable equilibrium when $\vec{p}$ is aligned with $\vec{E}$.

4. The net force on the dipole in the non-uniform field is non-zero and calculated by $\mathbf{F = -\frac{dU}{dx}}$.

Practice Problems :

1. **The potential energy of an electric dipole in a uniform electric field is U. The magnitude of the torque acting on the dipole due to the field is N. Then**

   (a) U is minimum and N is zero when the dipole is parallel to the field.

   (b) U is zero and N is zero when the dipole is perpendicular to the field.

   (c) U is minimum and N is maximum when the dipole is perpendicular to the field

   (d) U is minimum and N is zero when the dipole is anti-parallel to the field.

2. **A dipole is placed in the field of infinite sheet of uniform charge density. Which of the following quantity must be zero ?**

   (a) Force **(b) Potential Energy**

   (c) Torque **(d) All the above**

[Answers : (1) a (2) a]

C12 Van De Graaff Generator

It is used to generate the potential in the order of 10^6 volt which is used to accelerates the charged particles.