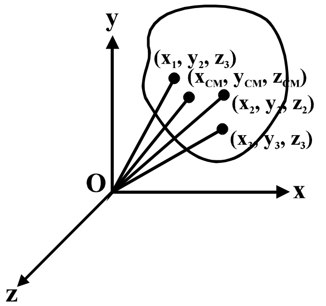
System of Particles and Centre of Mass, Collision

SYSTEM OF PARTICLES AND CENTRE OF MASS, COLLISION

C1 Centre of Mass

A special point of a baseball that is flipped into the air moves in a simple parabolic path, but all other points of the bat follow more complicated curved paths. In fact, that special point moves as though (1) the bat's total mass were concentrated that and (2) the gravitational force on the bat acted only there. That special point is said to be the centre of mass of the bat. In general ; the centre of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there, and all external forces were applied there.

For a system of particles, that the distributed in three dimensions shown in the figure



The center of mass of the system of particles is given by

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i \qquad z_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

The position vector of centre of mass of the system of particles is given by $\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{v} + z_{CM}\hat{k}$

i.e., $$\vec{R}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Centre of Mass of Solid Bodies

Solid bodies are treated as continuous distribution of matter and the centre of mass for these bodies is given by

$$x_{CM} = \frac{\int x\,dm}{\int dm} \qquad y_{CM} = \frac{\int y\,dm}{\int dm} \qquad z_{CM} = \frac{\int z\,dm}{\int dm}$$

Here x, y, z are the centre of mass of the differential elements of the solid bodies and dm is the mass of the differential elements.

<u>Practice Problems :</u>

1. **Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body ?**

2. **Find the centre of mass of (i) quarter circular ring of radius r (ii) half circular ring of radius r (iii) a sector of ring of radius r and forming angle α at the centre (iv) half circular disc of radius r.**

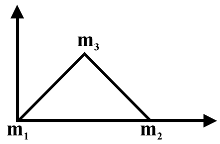
3. **Find the centre of mass (i) masses m, 2m, 3m, 4m are placed at the corners of a square of side length L (ii) masses m, 2m, 3m, 4m, 5m and 6m are placed at the corners of a regular hexagon of side length L.**

4. **Four particles of masses m, 2m, 4m, 4m are placed at (l, l), ($-l$, l), ($-l$, $-l$) and (l, $-l$) respectively. The centre of mass will lie in**

   (a) First quadrant **(b) Second quadrant**

   (c) Third quadrant **(d) Fourth quadrant**

5. **Three particles of masses m_1 = 1.2 kg, m_2 = 2.5 kg and m_3 = 3.4 kg form an equilateral triangle of edge length l = 140 cm as shown in figure.**



The center of mass of this three particle system is

(a) **(0.83 m, 0.58 m)** **(b)** **(0.78 m, 0.58 m)**

(c) **(0.83 m, 0.48 m)** **(d)** **(0.78 m, 0.48 m)**

[Answers : (4) c (5) a]

C2 Newton's Second Law For System of Particles

For a system of n particles, $\mathbf{M\vec{R}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}$

Differentiating with respect to time, $\mathbf{M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}$

where $\mathbf{\vec{V}_{CM}}$ is the velocity of centre of mass of the system of particles.

Differentiating with respect to time, $\mathbf{M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + ...m_n\vec{a}_n}$

where $\mathbf{\vec{a}_{CM}}$ is the acceleration of the centre of mass of the system of particles.

From Newton's second law

$$\mathbf{M\vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n}$$

Among the forces that contribute to the right side of the above equation will be forces that the particles of the system exert on each other (internal forces) and forces exerted on the particles from outside the system (external forces). By Newton's third law, the internal forces cancel out in the sum that appears on the right side of the above equation, what remains is the vector sum of all the external forces that act on the system.

Hence $\mathbf{\vec{F}_{net} = M\vec{a}_{CM}}$.

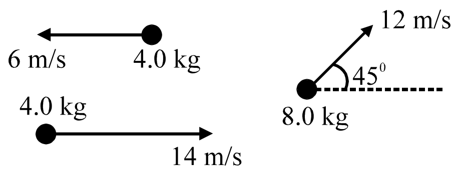
<u>Practice Problems :</u>

1. **If a bomb is exploded during the flight in parabolic path then what is the path of the centre of mass of the fragments after the explosion ?**

2. **Two spheres of masses M and 2M are initially at rest at a distance R apart. Due to mutual force of attraction they approach each other.When they are at separation R/2, the acceleration of their centre of mass would be**

(a) **0** **(b)** **g m/s²** **(c)** **3g m/s²** **(d)** **12g m/s²**

3. **The three particles are moving with constant velocity in the horizontal plane as shown.**



The velocity of cm is

(a) $\left\{(3\sqrt{2} - 2)\hat{i} + 3\sqrt{2}\hat{j}\right\}$ **m/s.** **(b)** $\left\{(3\sqrt{2} + 2)\hat{i} + 3\sqrt{2}\hat{j}\right\}$ **m/s.**

(c) $\left\{(2\sqrt{2} + 2)\hat{i} + 3\sqrt{2}\hat{j}\right\}$ **m/s.** **(d)** $\left\{(2\sqrt{2} - 2)\hat{i} + 3\sqrt{2}\hat{j}\right\}$ **m/s.**

4. Two bodies of masses 1.0 kg and 3.0 kg are connected by a spring and rest on a frictionless surface. They are given velocities towards each other such that the 1.0 kg block travels initially at 1.7 m/s towards right. Choose the correct statement from the following :

(a) The velocity of centre of mass is always zero.

(b) The velocity of second block at the initial moment is $\frac{1.7}{3}$ m / sec towards left.

(c) both (a) and (b) are correct

(d) none

[Answers : (2) a (3) b (4) c]

C3 Linear Momentum

For a single particle, we define a quantity $\vec{\mathbf{P}}$ called its linear momentum as $\vec{\mathbf{P}} = \mathbf{m}\vec{\mathbf{v}}$ and can write Newton's second law in terms of this momentum $\vec{\mathbf{F}}_{\text{external}} = \frac{d\vec{\mathbf{P}}}{dt}$

For a system of particles these relations become

$$\vec{\mathbf{P}} = \mathbf{M}\vec{\mathbf{v}}_{\mathbf{CM}} \quad \text{and} \quad \vec{\mathbf{F}}_{\text{external}} = \frac{d\vec{\mathbf{P}}}{dt}$$

Conservation of Linear Momentum

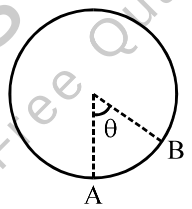
If a system is isolated so that no net external force acts on the system, the linear momentum $\vec{\mathbf{P}}$ of the system remains constant i.e., $\vec{\mathbf{P}} = \text{constant}$.

This can be written as $\vec{\mathbf{P}}_i = \vec{\mathbf{P}}_f$

where the subscripts refer to the value of $\vec{\mathbf{P}}$ at some initial time and at a later time.

<u>Practice Problems :</u>

1. A particle of mass m is moving in a circle of radius r center at O with constant speed v. The magnitude of change in linear momentum in moving from A to B is



(a) 0 **(b)** $2mv\sin\frac{\theta}{2}$ **(c)** $2mv\cos\frac{\theta}{2}$ **(d)** $2mv\tan\frac{\theta}{2}$

2. A 20.0 kg body is moving in the positive x-direction with a speed of 200 m/s when, owing to an internal explosion, if breaks into three parts. One part, with a mass of 10.0 kg moves away from the point of explosion with a speed of 100 m/s in positive y direction. A second fragment, with a mass of 4.0 kg moves in the negative x-direction with a speed of 500 m/s. The energy released in the explosion is

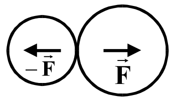
(a) 1.25 MJ (b) 2.5 MJ (c) 3.23 MJ (d) 4.75 MJ

[Answers : (1) b (2) c]

C4 Collision :

In a collision, two bodies exert strong forces on each other for a relatively short time. That's why these forces are known as impulsive. These forces are internal to the forces and direction of these forces along the normal to body system and are significantly larger than any external forces during the collision.

These forces are third law force pair. Their magnitude vary with time during the collision, but at any given instant those magnitudes are equal. Figure shows the third law force pair, $\vec{F}$ **and** $-\vec{F}$, that acts during the collision.



C5 Impulse and Linear Momentum

The forces during the collision will change the linear momentum of both bodies; the amount of change will depend not only on the forces, but also on the time dt during which they act and they are related by

$$d\vec{p} = \vec{F}\,dt \qquad \Rightarrow \qquad \int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} \vec{F}\,dt \qquad (1)$$

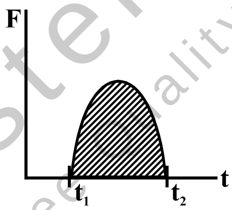
Here left side represent change in linear momentum and the right side, which is a measure of both the strength and the duration of the collision force, this is called the impulse $\vec{\tau}$ of the collision thus, $\vec{\tau} = \int_{t_i}^{t_f} \vec{F}\,dt$

Equation (1) represents the **Impulse – Linear Momentum Theorem**

In component form $\quad p_{fx} - p_{ix} = \Delta p_x = \tau_x, p_{fy} - p_{iy} = \Delta p_y = \tau_y, p_{fz} - p_{iz} = \Delta p_z = \tau_z$

If F_{avg} is the average magnitude of $\vec{F}$ during the collision and Δt is the duration of the collision then for one dimensional motion the equation (1) becomes $\Delta p = \tau = F_{avg}\Delta t$

Equation (1) tells us that of the magnitude of the impulse is equal to the area under as shown in figure.



C6 Coefficient of Restitution and types of collision

Collisions can be classified according to the relation of velocities of the two bodies. When two bodies collide, their relative velocity after impact is in a constant ratio to their relative velocity before impact. This constant ratio is known as coefficient of restitution of two bodies, given by

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

The velocities are taken along the direction of impulsive force. The value of e is varying from zero to one. On the basis of the value of e, we have three types of collision :

(a) Elastic Collision

For elastic collision, e = 1 i.e, $v_1 - v_2 = u_2 - u_1$. In an elastic collision between two bodies, the initial and final kinetic energies are equal.

(b) Perfectly Inelastic Collision

For perfectly inelastic collision , e = 0 i.e, the two bodies have the same final velocity after collision i.e., the two bodies will stick together and then move. In an inelastic two bodies collision the final total kinetic energy is less than the initial total kinetic energy.

(c) Inelastic Collision

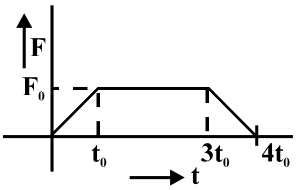
For inelastic collision, the value of e is given by $0 < e < 1$.

In this case the two bodies do not have the same final velocity after collision. In an inelastic two body collision the final total kinetic energy is less than the initial kinetic energy.

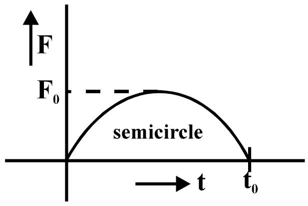
<u>Practice Problems :</u>

1. Find the kinetic energy of the particle of mass 'm' at the end time if initial speed is zero :

(a)



(b)



2. An impulse is supplied to a moving object with the force at an angle of θ with the velocity vector. The angle between the impulse vector and the change in momentum vector is

(a) θ **(b)** 0 **(c)** θ/2 **(d)** 2θ

3. A disc of mass 10 g is kept floating horizontally by throwing 10 marbles per second against it from below. The marble strike the disc normally and rebound downward with the same speed. If the mass of each marble is 5 g, the velocity with which the marble are striking the disc is ($g = 9.8\ m/s^2$)

(a) 0.98 m/s **(b)** 9.8 m/s **(c)** 1.96 m/s **(d)** 19.6 m/s

4. A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that if is turned back with velocity of 20 m/s. The force of blow acts for 0.01 s. The average force exerted by the bat on the ball is

(a) 120 N **(b)** 240 N **(c)** 480 N **(d)** 960 N

5. A ball A, moving with a speed u, collides directly with another similar ball B moving with a speed v in the opposite direction. A comes to rest after the collision. If the coefficient of restitution is e then u/v is

(a) $\frac{1+e}{1-e}$ **(b)** $\frac{1-e}{1+e}$ **(c)** $\frac{e}{1-e}$ **(d)** $\frac{e}{1+e}$

6. A ball is dropped from a height of 1 m. If the coefficient of restitution between the surface and the ball is 0.6, the ball rebounds to a height of

(a) 0.6 m **(b)** 0.4 m **(c)** 0.16 m **(d)** 0.36 m

7. A ball of mass m collides head-on and elastically with a ball of mass nm, initially at rest. The fraction of the incident energy transferred to the heavier ball is

(a) $\frac{n}{n+1}$ **(b)** $\frac{n}{(n+1)^2}$ **(c)** $\frac{2n}{(n+1)^2}$ **(d)** $\frac{4n}{(n+1)^2}$

8. A body of mass m_1, moving with a velocity u_1, collides head-on with a body of mass m_2, moving with a velocity u_2. The two bodies stick together after the collision. The loss of kinetic energy during the collision is

(a) $\frac{m_1 m_2 (u_1 - u_2)^2}{2(m_1 + m_2)}$ **(b)** $\frac{2m_2 m_1 (u_2 - u_1)^2}{(m_1 + m_2)}$

(c) $\frac{m_2 m_1 (u_2 + u_1)^2}{2(m_1 - m_2)}$ **(d)** $\frac{2m_2 m_1 (u_2 - u_1)^2}{(m_1 - m_2)}$

[Answers : (2) b (3) a (4) c (5) a (6) d (7) d (8) a]