MOTION IN A STRAIGHT LINE

3.1 Introduction :

Q. What is rectilinear motion ?

Solution : The study of motion of objects along a straight line is known as rectilinear motion.

3.2 Position, Path Length and Displacement :

Q. What is distance ?

Solution : Distance is called the path length.

Q. What is displacement ?

Solution : Displacement is defined as the change in position.

Q. Can the magnitude of displacement equals to the distance traversed by an object ?

Solution : The magnitude of displacement may or may not be equal to the path length traversed by an object.

Q. Can the magnitude of displacement for a moving particle be zero ?

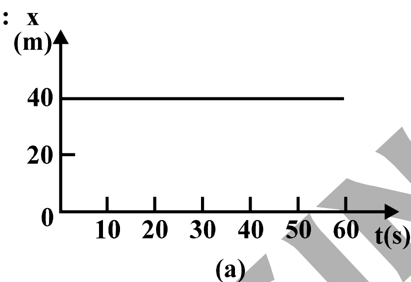
Solution : Yes, the magnitude of the displacement for a course of motion may be zero.

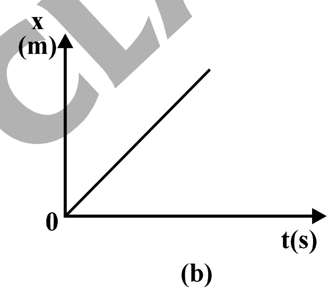
Q. Can the distance for a moving particle be zero ?

Solution : No.

Q. Draw the position time graph of (a) stationary object (b) an object in uniform motion.

Solution :





Q. If the displacement of a body is zero, is the distance covered by it necessarily zero ? Comment with illustrations.

Solution : No, because the distance travelled by an object is the actual path traversed by the object during motion in a given interval of time. The displacement of an object in a given internal of time is given by the vector drawn from the initial position to final position. For illustration : when an object completes a circular motion on a circle of radius r, then its displacement is zero but distance travelled is $2 \pi r$.

3.3 Average velocity and Average speed :

Q. What is Average velocity ?

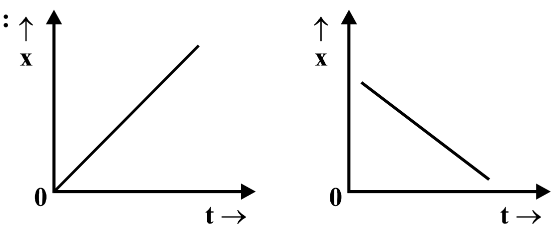
Solution : Average velocity is defined as the change in position or displacement divided by the time interval. It is a vector quantity.

Q. Can average velocity positive, negative or zero for a moving particle ?

Solution : The average velocity can be positive or negative depending upon the sign of the displacement is zero if the displacement is zero.

Q. Draw the position time graph, for an object (a) moving with positive velocity and (b) moving with negative velocity.

Solution :

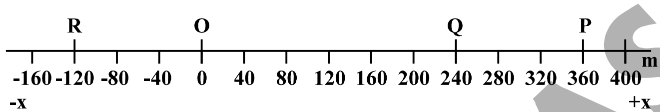


(a) (b)

Q. What is average speed ?

Solution : Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place : $\text{Average speed} = \dfrac{\text{Total path length}}{\text{Total time interval}}$

Q. A car is moving along a straight line, say OP in figure. It moves from O to P in 18 s and returns from P to Q in 6.0 s.



What are the average velocity and average speed of the car in going (a) from O to P ? and (b) from O to P and back to Q ? [NCERT Solved Example 3.1]

Solution : (a) 20 m s^{-1}, 20 m s^{-1} (b) 10 m s^{-1}, 20 m s^{-1}

Q. Explain that a body can have zero average velocity but not zero average speed.

Solution : Average velocity = displacement/(total time taken) and average speed = total distance travelled/(total time taken). If an object completes a circular path of radius r in time t, then its displacement is zero but distance travelled by body is $2\pi r$. Therefore, the average velocity of body = zero but average speed of body = $2\pi r/t$.

3.4 Instantaneous Velocity and Speed :

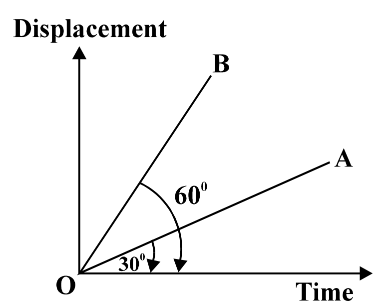
Q. The position of an object moving along x-axis is given by $x = a + bt^2$ where a = 8.5 m, b = 2.5 m s^{-2} and t is measured in seconds. What is the velocity at t = 0 s and t = 2 s. What is the average velocity between t = 2 s and t = 4 s ? [NCERT Solved Example 3.2]

Solution : 0, 10 m/s, 15 m/s

Q. Does the velocity equal to average velocity at all instants in uniform motion ?

Solution : Yes

Q. Two straight lines drawn on the same displacement - time graph make angles 30^0 and 60^0 with time-axis respectively. Which line represent greater velocity ? What is the ratio of the two velocities ?



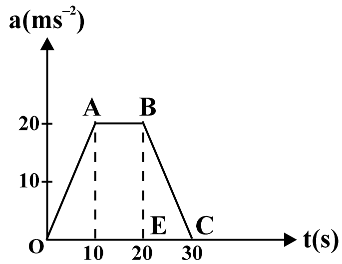
Solution : Line B will represent the greater velocity, 1 : 3

Q. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$ (x in metres and t in seconds). What is its velocity at t = 2s ?

Solution : 54 ms^{-1}

3.5 Acceleration :

Q. Figure shows the time - acceleration graph for a particle in rectillinear motion. Find the average acceleration in first twenty seconds.



Solutin : 15 ms^{-2}

Q. A body starting from rest accelerates uniformly along a straight line, at the rate of 10 ms^{-2} for 5s. It moves for 2 s with uniform velocity of 50 ms^{-1}. Then, it retards uniformly and comes to rest in 3 s. Draw velocity-time graph of the body and hence find the total distance travelled by body.

Solution : 300 m

Q. An object of mass m is covering distance x, in proportional to $t^{3/2}$, where t is the time elapsed. (a) What conclusion might you draw about the acceleration ? Is it increasing, decreasing, zero or constant (b) What will be the force acting on the object ?

Solution : (a) acceleration of the object is decreasing with time. (b) force is decreasing with time.

Q. The displacement x of a particle along a straight line at time t is given by $x = \alpha - \beta t + \gamma t^2$. Find the acceleration of the particle.

Solution : 2γ

Q. An electron starting from rest has a velocity that increases linearly with time that is v = k t, where k = 2 m/s^2. What will be the distance covered in first 3 seconds of its motion ?

Solution : 9m

Q. If the velocity of a particle by $v = \sqrt{180 - 16x}$ m/s , what will be its acceleration ?

Solution : –8 m/s^2

Q. The distance traversed by a moving particle at any instant is half of the product of its velocity and the time of traverse. Show that the acceleration of particle is constant.

Q. The acceleration of a particle, starting from rest, varies with time according to relation;

$a = - r\omega^2 \sin \omega t$. Find the displacement of this particle at a time t.

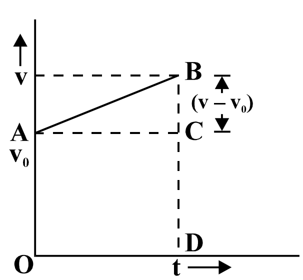
Solution : r sin ωt.

3.6 Kinematical Equation for Uniformly Accelerated Motion :

Q. Obtain the kinematical equation (i) $v = v_0 + at$ (ii) $x = v_0 t + \frac{1}{2}at^2$ (iii) $v^2 = v_0^2 + 2ax$ using the graphs or analytically.

Solution : (i) $v = v_0 + at$

As we know that slope of velocity-time graph of uniformly accelerated motion represents the acceleration of the object.



Acceleration = slope of the velocity-time graph AB

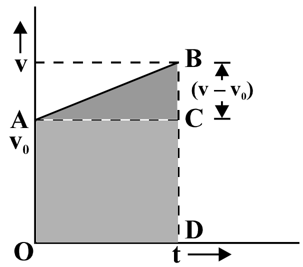
$$\mathbf{a = \frac{BC}{AC} = \frac{BC}{OD} = \frac{v - v_0}{t}} \Rightarrow v - v_0 = at \Rightarrow v - v_0 + at$$

(ii) $\mathbf{x = v_0 t + \frac{1}{2}at^2}$

The area under this curve is

Area between instants 0 and t = Area of triangle ABC + Area of rectangle OACD

$$= \frac{1}{2}(v - v_0)t + v_0 t$$



The area under v-t curve represents the displacement. Therefore, the displacement x of the object is :

$$x = \frac{1}{2}(v - v_0)t + v_0 t \qquad ...(A)$$

But $\qquad v - v_0 = at$

Therefore, $\qquad x = \frac{1}{2}at^2 + v_0 t$ or $x = v_0 t + \frac{1}{2}at^2$

(iii) $v^2 = v_0^2 + 2ax$

Equation (A) can also be written as $x = \frac{v + v_0}{2}t$

Put $t = (v - v_0)/a$ in $x = \frac{v + v_0}{2}t$, we get

$$x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0^2}{2a} \Rightarrow v^2 = v_0^2 + 2ax$$

Q. Obtain equations of motion for constant acceleration using method of calculus. [NCERT Solved Example 3.3]

Q. Find the distance travelled by the uniformly accelerated object moving in one dimension in nth second.

Solution : If the object starts with velocity u and constant acceleration 'a' then distance travelled by the object in nth second, denoted by D_n, is given by

$$D_n = x_n - x_{n-1}$$

where x_n and x_{n-1} are distances travelled by object in n seconds and (n – 1) seconds respectively.

$$x_n = un + \frac{1}{2}an^2$$

$$x_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$D_n = \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

$$= u + an - \frac{a}{2} = u + a\left(n - \frac{1}{2}\right)$$

Q. A ball is thrown vertically upwards with a velocity of 20 m s^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25.0 m from the ground. (a) How high will the ball rise ? (b) How long will it travel before the ball hits the ground ?

Take g = 10 m s^{-2}. [NCERT Solved Example 3.4]

Solution : (a) 20 m (b) 5 s

Q. Free-fall : Discuss the motion of an object under free fall. Neglect air resistance. [NCERT Solved Example 3.5].

OR

Q. Draw the following graphs for the motion of an object under free-fall : (a) Variation of acceleration with time. (b) Variation of velocity with time. (c) Variation of distance with time.

Q. Galileo's law of odd numbers : The distances transversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity (namely, 1 : 3 : 5 : 7......]. Prove it. [NCERT Solved Example 3.6]

Q. Stopping Distance of vehicles : When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. It is an important factor for road safety and depends on the initial velocity (v_0) and the braking capacity, or deceleration, –a that is caused by the breaking. Derive an expression for stopping distance of the vehicle in terms of v_0 and a. [NCERT Solved Example 3.7]

Solution : $\frac{v_0^2}{2a}$

Q. Reaction time : When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appeals on the road, then the time elapsed before he slams the brakes of the car is the reaction time. Reaction time depends on complexity of the situation and on an individual.

You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between you thumb and forefinger. After you catch it, find the distance d travelled by the ruler. In a particular case, d was found to be 21.0 cm. Estimate reaction time. [NCERT Solved Example 3.8]

Solution : 0.2 s

3.7 Relative Velocity :

Q. Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km h^{-1}, and train B moves south with a speed of 90 km h^{-1}. What is the

(a) velocity of B with respect to A ?

(b) velocity of ground with respect to B ? and

(c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground ?

[NCERT Solved Example 3.9]

Solution : (a) 40 m s^{-1} from north to south (b) 25 m s^{-1} (c) 10 m s^{-1}

NCERT EXERCISE

3.1 In which of the following examples of motion, can the body be considered approximately a point object :

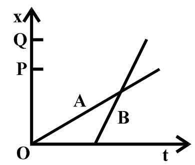
(a) a railway carriage moving without jerks between two stations.

(b) a monkey sitting on top of a man cycling smoothly on a circular track.

(c) a spinning cricket ball than turns sharply on hitting the ground.

(d) a tumbling beaker that has slipped off the edge of a table.

3.2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in figure. Choose the correct entries in the brackets below :



(a) (A/B) lives closer to the school than (B/A)

(b) (A/B) starts from the school earlier than (B/A)

(c) (A/B) walks faster than (B/A)

(d) A and B reach home at the (same/different) time

(e) (A/B) overtakes (B/A) on the road (once/twice).

3.3 A woman starts from her home at 9.00 am. walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x-t graph of her motion.

3.4 A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

3.5 A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on the ground ?

3.6 A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop ?

3.7 Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The drive of B decides to overtake A and accelerates by 1 m s^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them ?

3.8 On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident ?

3.9 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?

3.10 A player throws a ball upwards with an initial speed of 29.4 m s^{-1}.

(a) What is the direction of acceleration during the upward motion of the ball ?

(b) What are the velocity and acceleration of the ball at the highest point of its motion ?

(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(d) To what height does the ball rise and after how long does the ball returns to the player's hands ? (Take $g = 9.8 \text{ m s}^{-2}$ and neglect air resistance)

3.11 Read each statement below carefully and state with reasons and examples, it it is true or false ;

A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant

(b) with zero speed many have non-zero velocity

(c) with constant speed must have zero acceleration

(d) with positive value of acceleration must be speeding up

3.12 A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

3.13 Explain clearly, with examples, the distinction between :

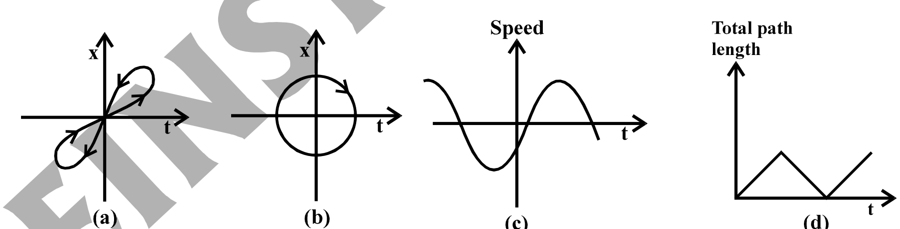
(a) manitude of displacement (sometimes called distance) over an internal of time, and the total length of path covered by a particle over the same interval;

(b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true ? [For simplicity, consider one-dimensional motion only].

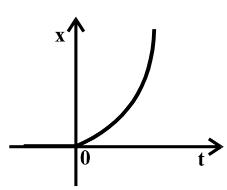
3.14 A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Find the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the magnitude of average velocity and average speed of the man over the interval of time (i) 0 to 30 min. (ii) 0 to 50 min. (iii) 0 to 40 min. ?

3.15 In Exercises 3.13 and 3.14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why ?

3.16 Look at the graphs (a) to (d) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle ?

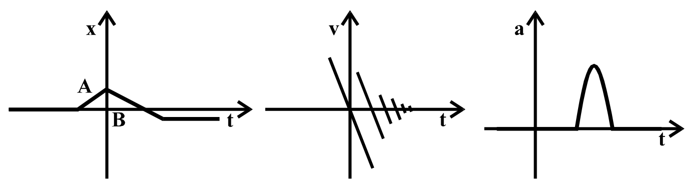


3.17 Figure shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t > 0 ? If not, suggest a suitable physical context for this graph.



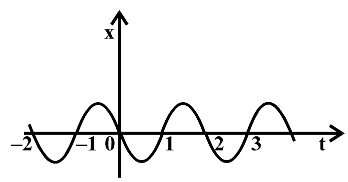
3.18 A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car ? (Note : Obtain that speed which is relevant for damaging the thief's car).

3.19 **Suggest a suitable physical situation for each of the following graphs**

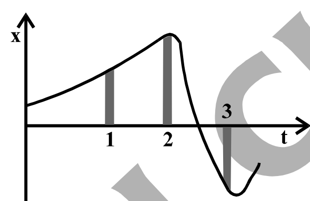


(a) (b) (c)

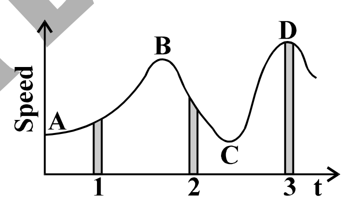
3.20 **Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the sign of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, –1.2 s.**



3.21 **Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity of each interval.**



3.22 **Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ? In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, given the sign of v and a in the three intervals.**



What are the acceleration at the point A, B, C and D ?

ADDITIONAL EXERCISES

3.23 **A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the nth second (n = 1, 2, 3 ....) versus n. What do you expect this plot to be during accelerated motion : a straight line or a parabola ?**

3.24 **A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands ? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?**

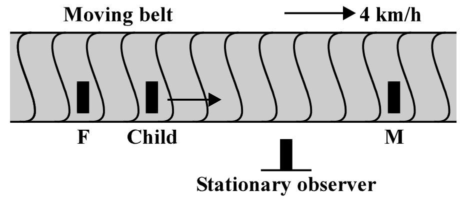
3.25 **On a long horizontally moving belt, a child runs to and from with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the**

(a) **speed of the child running in the direction of motion of the belt ?**

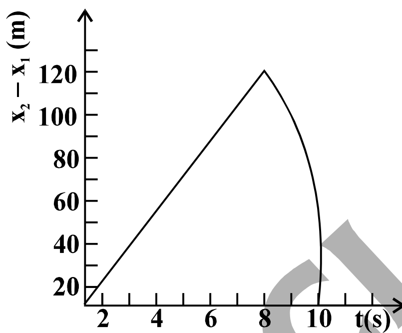
(b) **speed of the child running opposite to the direction of motion of the belt ?**

(c) time taken by the child in (a) and (b) ?

Which of the answers alter if motion is viewed by one of the parents ?

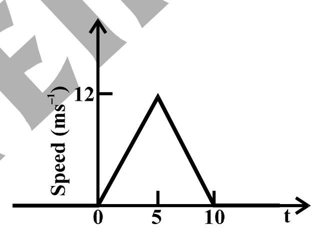


3.26 **Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m s^{-1} and 30 m s^{-1}. Verify that the graph correctly represents the time variation of the relative position of the second strone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m s^{-2}.**



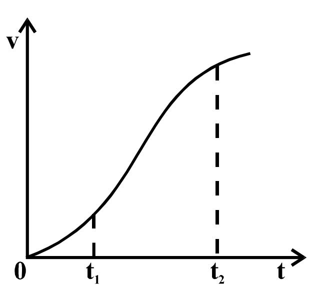
Give the equations for the linear and curved parts of the plot.

3.27 **The speed-time graph of a particle moving along a fixed direction is shown in figure. Obtain the distance traversed by the particle between (a) t = 0 s to 10 s (b) t = 2 s to 6 s.**



What is the average speed of the particle over the intervals in (a) and (b) ?

3.28 **The velocity-time graph of a particle in one-dimensional motion is shown in figure**



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2 :

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + (½)\, a\, (t_2 - t_1)^2$

(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(c) $v_{average} = (x(t_2) - x(t_1))/(t_2 - t_1)$

(d) $a_{average} = (v(t_2) - v(t_1))/(t_2 - t_1)$

(e) $x(t_2) = x(t_1) + v_{average}(t_2 - t_1) + (½)\, a_{average}\, (t_2 - t_1)^2$

(f) $x(t_2) - x(t_1)$ = area under the v-t curve bounded by the t-axis and the dotted line shown.

ANSWERS

3.1 (a), (b)

3.2 (a) A....B, (b) A....B, (c) B....A, (d) Same, (e) B....A....once

3.4 37 s

3.5 1000 km/h

3.6 3.06 m/s^2, 11.4 s

3.7 1250 m (Hint : view the motion of B relative to A)

3.8 1 m s^{-2} (Hint : view the motion of B and C relative to A)

3.9 T = 9 min, speed = 40 km/h. Hint : $vT / (v - 20) = 18$; $v T / (v + 20) = 6$

3.10 (a) Vertically downwards; (b) zero velocity, acceleration of 9.8 m s^{-2} downwards; (c) $x > 0$ (upward and downward motion); $v < 0$ (upward), $v > 0$ (downward), $a > 0$ throughout; (d) 44.1 m, 6s

3.11 (a) True (b) False (c) True (d) False

3.14 (i) 5km/h, 5km/h (ii) 0, 6 km/h (iii) 15/8 km/h, 45/8 km/h

3.15 Because, for an arbitrary small interval of time, the magnitude of displacement is equal to the length of the path.

3.16 All the four graphs are impossible. (a) a particle cannot have two different positions at the same time; (b) a particle cannot have velocity in opposite directions at the same time; (c) speed is always non-negative; (d) total path length of a particle can never decrease with time. (Note, the arrows on the graphs are meaningless

3.17 No, wrong. x-t plot does not show the trajectory of a particle. Context: A body is dropped from a tower (x = 0) at t = 0

3.18 105 m s^{-1}

3.19 (a) A ball at rest on a smooth floor is kicked, it rebounds from a wall with reduced speed and moves to the opposite wall which stops it; (b) A ball thrown up withsome initial velocity rebounding from the floor with reduced speed after each hit; (c) A uniformly moving cricket ball turned back by hitting it with a bat for a very short time-interval

3.20 $x < 0, v < 0, a > 0$; $x > 0, v > 0, a < 0$; $x < 0, v > 0, a > 0$

3.21 Greatest in 3, least in 2; $v > 0$ in 1 and 2, $v < 0$ in 3

3.22 Acceleration magnitude greatest in 2; speed greatest in 3; $v > 0$ in 1, 2 and 3; $a > 0$ in 1 and 3, $a < 0$ in 2; $a = 0$ at A, B, C, D.

3.23 A straight line inclined with the time-axis for uniformly accelerated motion; parallel to the time-axis for uniform motion

3.24 10 s, 10 s

3.25 (a) 13 km h^{-1}; (b) 5 km h^{-1}; (c) 20 s in either direction. viewed by any one of the parents, the speed of the child is 9 km h^{-1} in either direction; answer to (c) is unaltered

3.26 $x_2 - x_1 = 15t$ (linear part); $x_2 - x_1 = 200 + 30t - 5\, t^2$(curved part)

3.27 (a) 60 m, 6 m s^{-1}; (b) 36 m, 9 m s^{-1}

3.28 (c), (d), (f)

ADDITIONAL QUESTIONS AND PROBLEMS

Q. Can an object have constant speed but variable velocity ?

A. Yes, it can be so in case of uniform circular motion of an object.

Q. If a, b, c be the distances moved by a particle moving with a constant acceleration during the *l*th, *m*th and *n*th second of its motion respectively, show that

$$a(m - n) + b(n - l) + c(l - m) = 0$$

Q. Can the direction of velocity of a body change, when acceleration is constant ?

A. Yes, when the body moves vertically upwards, after reaching the highest point, the body starts falling down, i.e., the direction of velocity is reversed, whereas the acceleration is constant.

Q. A ball is thrown up in air. What is the acceleration and velocity at the instant it reaches its highest point ?

A. Acceleration is 9.8 ms^{-2} acting downwards and velocity is zero.

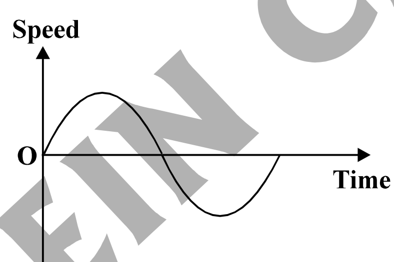
Q. A stone is thrown vertically upwards from the surface of earth. What is the direction of the velocity and acceleration of the stone (a) on its way up (b) on it way down.

A. (a) Velocity is vertically upwards and acceleration is vertically downwards (b) velocity is vertically downwards and acceleration is also vertically downwards.

Q. Is it possible to have a constant rate of change of velocity when velocity changes both in magnitude and direction ?

A. Yes, in projectile motion.

Q. What is wrong with the speed time graph as shown in figure.



A. Speed being scalar can not have negative value.

Q. If the acceleration of the particle is constant in magnitude but not in direction, what type of path does the body follow ?

A. Circular path

Q. Which of the two ; velocity or acceleration decides the direction of motion of a body ? Explain with the help of an example.

A. It is the velocity with decides the direction of motion of a body. The acceleration simply tells the rate of change of velocity. For example, when a body is thrown vertically upwards, its direction of velocity is upwards, that is why the body goes upwards, where as its acceleration is downwards.

Q. A man standing on the edge of a cliff throws a strone straight up with initial speed u and then throws another stone straight down with same initial speed and from the same position. Find the ratio of the speeds, the stones would have attained when they hit ground at the base of the cliff.

A. 1 : 1

Q. Can an object be accelerated without speeding up or slowing down ?

A. Yes, it will be so if an object is moving with constant speed in a circle. In this case, the object is accelerating but its speed neither decrease nor increases.

Q. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward direction with the greater speed ?

A. Let u be the initial velocity of projection of body and v be the velocity of the same body while passing downwards through point of projection. The displacement of body s = 0. Using the relation $v^2 = u^2 + 2\,as$, and u = u, v = ? ; a = –g, s = 0, we have $v^2 = u^2 + 2(-g) \times 0 = u^2$ or v = u.

It means the final speed is independent of mass of the body. Hence, both the bodies will acquire the same speed while passing through point of projection.

Q. Differntiate the following with respect to x.

(i) $4x^3 - 3x^2 + \frac{4}{x^2} - 8$ (ii) $5x^4 + 4x^{3/4} - 3x^2 + 2x$

(iii) $(4x + 2)(5x^2 + 4)$ (iv) $(2x^3 + 3)(2x^{-3} + 1)$

(v) sin 3x (vi) $\cos^2 x$

A. (i) $12x^2 - 6x - \frac{8}{x^3}$ (ii) $20x^3 + 3x^{-1/4} - 6x + 2$ (iii) $60x^2 + 20x + 16$ (iv) $6x^2 - \frac{18}{x^4}$ (v) 3 cos 3x (vi) – sin 2x

Q. A particle starts from rest with a uniform acceleration. Its displacement after t seconds is given in metres by the relation $x = 5 + 6t + 7t^2$. Calculate the magnitude of its (i) initial velocity (ii) velocity at t = 3 s (iii) uniform acceleration and (iv) displacement at t = 5 s.

A. (i) 6 m/s (ii) 48 m/s (iii) 14 m/s^2 (iv) 210 m

Q. If $x = at^3$ and $y = bt^2$, find $\frac{dy}{dx}$.

A. $\frac{2b}{3at}$

Q. If the displacement of the particle at an instant is given by $y = r \sin(\omega t - \theta)$ where r is amplitude of oscillation, ω is the angular velocity and –θ is the initial phase of the particle, then find the particle velocity and particle acceleration.

A. $r\omega \cos(\omega t - \theta)$; $-\omega^2 y$

Q. Integrate the following with respect to x

(i) $\sqrt[3]{x^5}$ (ii) $\sqrt{x} - \frac{1}{\sqrt{x}}$ (iii) $\sqrt{(3x-4)^3}$ (iv) $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

(v) $\left(x + \frac{1}{x}\right)^3$ (vi) $\left(3x^2 + \frac{1}{x^2} - 2x\right)$

A. (i) $\frac{3}{8}x^{8/3} + C$ (ii) $\frac{2}{3}x^{3/2} - 2x^{1/2} + C$ (iii) $\frac{2}{15}(3x-4)^{5/2} + C$ (iv) $\frac{x^2}{2} + \log_e x + 2x + C$

(v) $\frac{x^4}{4} + \frac{3x^2}{2} + 3\log_e x - \frac{1}{2x^2} + C$ (vi) $x^3 - \frac{1}{x} - x^2 + C$

Q. Evaluate the following integrals

(i) $\int_{15}^{30} \cos(4x-3)dx$ (ii) $\int_{0}^{30} \sin 7x\, dx$

(iii) $\int_{0}^{30} \cos 5x\, dx$ (iv) $\int_{0}^{10} \sec^2(3x+6)dx$

A. (i) $\frac{1}{4}[\sin 117^0 - \sin 57^0]$ (ii) 0.2666 (iii) $\frac{1}{10}$ (iv) $\frac{1}{3}(\tan 36^0 - \tan 6^0)$

Q. The relation between time t and distance x is $t = \alpha x^2 + \beta x$ where α and β are constants. Show that retardation is $2\alpha v^3$, where v is the instantaneous velocity.