Electro Magnetic Induction & Alternating Current

ELECTRO MAGNETIC INDUCTION AND ALTERNATING CURRENT

C1 Magnetic Flux

Like electric flux, magnetic flux, ϕ_B, through a surface $d\vec{S}$ is defined as $\phi_B = \int_S \vec{B}.d\vec{S}$. If $\vec{B}$ is uniform then

B = $\vec{B}.\vec{S}$ and it represents total lines of induction crossing through a given surface S.

C2 Magnetic Induction and Faraday's Laws

If the magnetic flux through a circuit or closed loop changes, an emf and a current are induced in the circuit. This phenomenon is known as electromagnetic induction and the law which governs this phenomenon is known as **Faraday's Law.** This law states that the magnitude of induced emf in a circuit is equal to the time rate of change of the magnetic flux. Mathematically, $|e| = \frac{d\phi}{dt}$. As $\phi = \vec{B}.\vec{A} = BA\cos\theta$. Hence if there is any change in magnetic field (B) or area (A) or orientation (θ) then there is induced emf. If some situation, more than one of these may contribute in induced emf, in this case magnitude of induced emf is written as

$$|e| = \frac{d}{dt}(BA\cos\theta) = (A\cos\theta)\frac{dB}{dt} + (B\cos\theta)\frac{dA}{dt} - BA\sin\theta\frac{d\theta}{dt}$$

This induced emf creates an induced current in the circuit whose magnitude is given as

$$I = \frac{\text{induced emf}}{\text{net resistance of circuit}} = \frac{|e|}{R}$$. Also the charge flown = $\frac{\Delta\phi}{R}$.

<u>**Practice Problems :**</u>

1. A circular coil (constant radius) of total length L having number of turns N is rotated about the diameter in a uniform magnetic field B with an angular velocity ω. Initially the magnetic field is perpendicular to the plane of the coil. The maximum value of the emf induced in it is

(a) $\frac{BL^2\omega}{2\pi N}$ **(b)** $\frac{NBL^2\omega}{2\pi}$

(c) $\frac{BL^2\omega}{4\pi N}$ **(d)** $\frac{NBL^2\omega}{4\pi}$

2. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(a) 2AB/R **(b)** AB/R **(c)** $\frac{AB}{4R}$ **(d)** $\frac{AB}{3R}$

[Answers : (1) c (2) b]

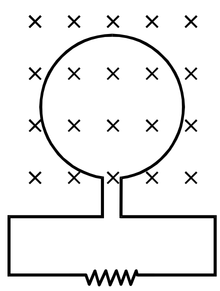
C3 Lenz's Law

The direction of induced emf is governed by Lenz's Law. This law states that an induced emf is always in the direction that opposes the change of magnetic flux that induced it. Incorporating this law into Faraday's Law, the induced emf is given by $e = -\frac{d\phi}{dt}$. The negative sign indicates that the induced emf opposes the change of the flux.

Note the Lenz's Law is based on conservation of energy principle.

Practice Problems :

1. **In the figure the flux through the loop perpendicular to the plane of the coil and directed into the paper is varying according to the relation $\Phi = 6t^2 + 7t + 1$ where Φ is in milliweber and t is in seconds.**



Choose the correct statement :

(a) **At time t = 2s, the current flowing through R is 10mA from left to right**

(b) **At time t = 2s, the current flowing through R is 10mA from right to left**

(c) **The current through R is always increasing linearly**

(d) **both (a) and (c) are correct**

2. **A rectangular coil (having resistance per unit length 10/3 Ω/m) of 100 turns and size 0.1 m × 0.05 m is placed perpendicular to a magnetic field of 0.1 T. If the field drops to 0.05 T in 0.05 s then**

(a) **the magnitude of average induced current is 4mA**

(b) **the total charge flown in the coil is 5μC**

(c) **the total charge flown in the coil isindependent of time during which the field will change**

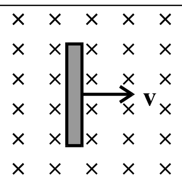
(d) **both (a) and (c) are correct**

3. **A solenoid has 2000 turns wound over a length of 0.3 m. Its cross-sectional area is 1.2×10^{-10} m². Around its central section a coil of 300 turns is wound. If an initial current of 2A flowing in the solenoid is reversed in 0.25 s, the emf induced in the coil will be**

(a) **6.0×10^{-4} V** (b) **6.0×10^{-2} V** (c) **4.8×10^{-4} V** (d) **4.8×10^{-2} V**

[Answers : (1) d (2) c (3) d]

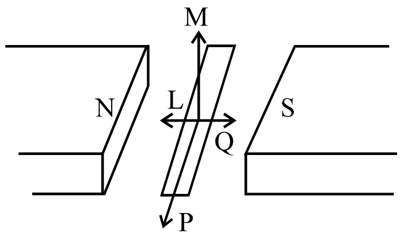
C4 Motional Electromotive Force



If a conductor with length L moves with speed v in a uniform magnetic field with magnitude B, and if the length and velocity are both perpendicular to the field, the induced emf is e = vBL. More general, when a conductor moves in a magnitude field $\vec{B}$, the induced emf in the direction is given by $\mathbf{e} = \int_b^a (\vec{v} \times \vec{B}).d\vec{l}$

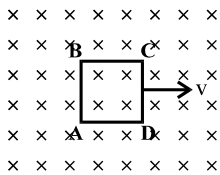
Practice Problems :

1. **An electric potential difference will be induced between the ends of the conductor shown in the diagram when it moves in the direction**



(a) **P** **(b)** **Q** **(c)** **L** **(d)** **M**

2. **A conducting square loop ABCD of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere, then**



(a) **The current induced in the loop is zero**

(b) **There is no induced emf in the rod BC and AD**

(c) **There is an induced emf BLv in each rod AB and CD**

(d) **All the above statements are correct**

[Answers : (1) d (2) d]

C5 Induded Electric Field :

When an emf is induced by a changing magnetic flux through a stationary closed path, there is an induced electric fleld $\vec{E}$ of non-electrostatic origin such that

$$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$$

Properties of Induced Electric Field

1. It is not a Coulomb field.
2. The lines of induced field form closed loop. Therefore, it is called a circuital field or vortex field.
3. This field is nonconservative and cannot be associated with a potential.

Practice Problems :

1. **Consider a cylindrical space of radius R in which a time varying magnetic field is confined. Find the dependence of induced electric field on the distance r from the centre inside the space and outside the space ?**

[Answers : (1) inside E is directly proportional to r and outside it is inversely proportional to r]

C6 Self inductance and Inductors

Any circuit that carries a varying current will have an emf induced in it by the variation in its own magnetic field. Such an emf is called a self-induced emf. Self-induced emf's can occur in any circuit, since there will always be some magnetic flux through the closed loop of a current-carrying circuit. But the effect is greatly enhanced if the circuit contains a coil with N turns of wire. As a result of the current i, there is an average magnetic flux ϕ_B, through each turn of the coil. Here we defined the self inductance L of the circuit as follows $L = \frac{N\phi_B}{I}$

The SI unit of inductance is the henry (H).

Self inductance of the solenoid

The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is $\frac{L}{l} = \mu_0 n^2 A$

Self induced emf

The self-induced emf, using Faraday's law, is given by $e = -L\frac{dI}{dt}$

<u>Practice Problems :</u>

1. **The current in a coil changes from 0 to 2A in 0.05 s. If the induced emf is 80 V, the self-inductance of the coil is**

   (a) 1 H (b) 0.5 H (c) 1.5 H (d) 2 H

2. **A torodial solenoid with an air core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Ignoring the field variation across the cross-section of the toroid, the self-inductance of the toroid is**

   (a) 4.6 mH (b) 6.9 mH (c) 2.3 mH (d) 9.2 mH

3. **A coil is wound on a frame of rectangular cross-section. If all the linear dimensions of the frame are increased by a factor 2 and the number of turns per unit length of the coil remains the same, self-inductance of the coil increases by a factor of**

   (a) 4 (b) 8 (c) 12 (d) 16

[Answers : (1) d (2) c (3) b]

C6 Energy Stored in an Inductor

If an inductor L carries a current i. the inductor's magnetic field stores an energy given by $U = \frac{1}{2}Li^2$

C7 LR Circuits :

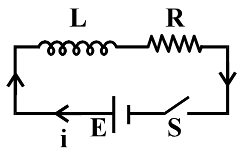
Applying Kirchoff's voltage law across an inductor.

(a) If the direction of assumed current coincides with the direction of motion, the voltage across the inductor falls and is given by $-L\frac{dI}{dt}$.

(b) If the direction of assumed current is opposite to the diretion of motion the voltage across the inductor rises and is given by $+L\frac{dI}{dt}$.

Growth of Current in RL circuit :

Let us connect a coil of self-induction L with a resistance R across a cell of emf E as shown in figure. If the switch S is thrown in contact at t = 0, current i in the circuit tends to grow. Hence an emf is induced across the coil in such a direction as to oppose this current.



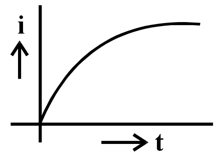
By Kirchoff's voltage law, we have

$$\mathbf{E - iR - L\frac{di}{dt} = 0} \quad \Rightarrow \quad \mathbf{\frac{di}{E - iR} = \frac{dt}{L}} \quad \Rightarrow \quad \mathbf{\int_0^i \frac{-Rdi}{E - iR} = \int_0^t \frac{-Rdt}{L}}$$

$$\mathbf{[\log(E - iR)]_0^i = -\frac{Rt}{L} \Rightarrow i(t) = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)}$$

Here $\frac{L}{R}$ is known as time constant of the circuit.

The current grown in the circuit exponentially as shown in figure.



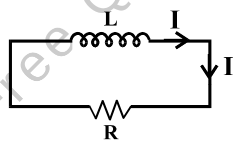
Note the following points :

1. At t = 0, i = 0, we can say at t = 0, the inductor behaves like a breaking wire.

2. In steady state : At $t \to \infty$, $i = \frac{E}{R}$, we can say at $t \to \infty$, the inductor behaves like a connecting wire.

3. The rate at which the source or battery will supply energy = Ei, rate at which the energy is dissipated in resistor = i^2R and the rate at which the energy stored in the inductor = $\mathbf{i\left(L\frac{di}{dt}\right)}$. From conservation of energy $\mathbf{Ei = i^2R + i\left(L\frac{di}{dt}\right)}$.

Decay of current in LR circuit : At t = 0, the current passing through the inductor is I_0 and it is connected across a resistor as shown in figure :

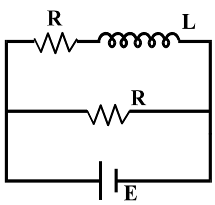


Using KVL, $\mathbf{iR + L\frac{dI}{dt} = 0 \Rightarrow \frac{dI}{I} = -\frac{R}{L}dt \Rightarrow \int_0^i \frac{dI}{I} = -\frac{R}{L}\int_0^t dt \Rightarrow I = I_0e^{-t/\tau_L}}$

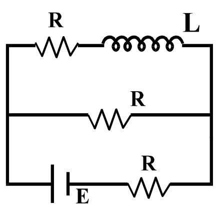
<u>**Practice Problems :**</u>

1. **In the following circuit initially there is no current through the inductor. Find the current passing through the battery at any time t. Also find the current through the battery at t = 0 and t = ∞.**

(a)



(b)



2. A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. **If it is connected to a battery, how long will the current take to reach half its final equilibrium value ?**

3. **A solenoid having an inductance of 6.30 μH is connected in series with a 1.20 kΩ resistor. (a) If a 14.0 V battery is switched across the pair, how long will it take for the current through the resistor to reach 80.0% of its final value ? (b) What is the current through the resistor at time $t = 1.0\tau_L$?**

4. **At time t = 0, a 45.0 V potential difference is suddenly applied to a coil with L = 50.0 mH and R = 180 Ω. At what rate is the current increasing at t = 1.20 ms ?**

[Answers : (2) 0.10 s (3) (a) 8.45 ns; (b) 7.37 mA (4) 12.0 A/s]

C8 Energy Density of a Magnetic Field

If B is the magnitude of a magnetic field at any point (in an inductor or anywhere else), the density of stored magnetic energy at that point is $u_B = \frac{B^2}{2\mu_0}$.

C9 Mutual Induction

When a changing current i_1 in one circuit causes a changing magnetic flux in a second circuit, an emf e_2 is induced in the second circuit; likewise, a changing current i_2 in the second circuit induced an emf e_1 in the first circuit. This is called mutual induction.

$$e_2 = -M\frac{di_1}{dt} \text{ and } e_1 = -M\frac{di_2}{dt}$$

The constant M, called the mutual inductance, depends on the geometry of the two coils and on the material between them. If the circuits are coils of wire with N_1 and N_2 turns, respectively, the mutual inductance can be expressed in terms of the average flux ϕ_{B2} through each turn of coil 2 that is caused by the current i_1 in coil 1 or in terms of the average flux ϕ_{B1} through each turn of coil 1 that is caused by the current i_2 in coil 2 :

$$M = \frac{N_2\phi_{B2}}{i_1} = \frac{N_1\phi_{B1}}{i_2}$$

The SI unit of mutual inductance is the henry, abbreviated H. Equivalent units are

1 H = 1 Wb/A = 1V.s/A = 1Ω.s.

Mutual inductance of two solenoids one surrounding the other is given by $\mu_0 n_p n_s A l$ where n_p and n_s are number of terms per unit length for primary and secondary coils and A is the cross-sectional area of primary coil and l is the length of the primary coil.

C10 LC Circuit

An L-C circuit, which contains inductance L and capacitance C, undergoes electrical oscillations with angular frequency ω :

$$\omega = \sqrt{\frac{1}{LC}}$$

Such a circuit is analogous to a mechanical harmonic oscillator, with inductance L analogous to mass m, the reciprocal of capacitance 1/C to force constant k, charge q to displacement x, and current i to velocity v.

Practice Problems :

1. **A capacitor of capacitance 1 μ F is charged upto 10V and then connected across an ideal inductor of 10 mH. Choose the correct statement :**
   - **(a)** **The angular frequency of LC oscillation is 10^4 rad/s**
   - **(b)** **At any moment total energy is 50μJ**
   - **(c)** **The current in the circuit changes with time sinusoidally**
   - **(d)** **All are correct**

2. **A capacitor of 1 μ F initially charged to 10 V is connected across an ideal inductor of 0.1 mH. The maximum current in the circuit is**

(a) 0.5 A (b) 1 A (c) 1.5 A (d) 2 A

[Answers : (1) d (2) b)

C11 Back EMF in D.C. Motor : A motor is the reverse of generator – it converts electrical energy into mechanical energy. When currents is passed through a coil placed in a magnetic field, it rotates. As the coil rotates, the magnetic flux linked with changes, giving rise to an induced emf. This emf opposes the applied emf (ε) and is, therefore, called back emf (e). If R is the resistance of the coil, the current through it is given by $\mathbf{I = \frac{\varepsilon - e}{R}}$.

Practice Problems :

1. **In a dc motor, if E is the applied emf and e is the back emf, then the efficiency is**

(a) $\frac{E-e}{E}$ (b) $\frac{e}{E}$ (c) $\left(\frac{E-e}{E}\right)^2$ (d) $\left(\frac{e}{E}\right)^2$

[Answers : (1) b]

C12 Eddy Currents

When a metallic body is moved in a magnetic field in such a way that the flux through it changes or is placed in a changing magnetic field, induced currents circulate throughout the volume of the body. These are called eddy currents.

C13 Alternating Current

An alternator or ac source produces an emf that varies sinusoidally with time.

Production of A.C.

Production of A.C. is based on Faraday's law of electromagnetic induction. Suppose a coil of N turns, and area A is rotated in a uniform magnetic field B with angular velocity ω. As the coil rotates, the flux through it changes and therefore an emf is induced in it, given by $\varepsilon = \varepsilon_0 \sin \omega t$ where $\varepsilon_0 = NBA\omega$.

A sinusoidal voltage or current can be represented by a phasor, a vector that rotates counterclockwise with constant angular velocity ω equal to the angular frequency of the sinusoidal quantity. Its projection on the horizontal axis at any instant represent the instantaneous value of the quantity.

C14 Average and root mean square value of a.c.

For a sinusoidal current the average and rms (root-mean-square) currents are related to the current amplitude I_0 by

$$\mathbf{I_{av} = \frac{2}{\pi} I_0 = 0.637 I_0}, \quad \mathbf{I_{rms} = \frac{I_0}{\sqrt{2}}}.$$

In the same way, the rms value of the snusoidal voltage is related to the voltage amplitude V_0 by

$$\mathbf{V_{rms} = \frac{V_0}{\sqrt{2}}}$$

The voltage v in an ac circuit is represented by $v = v_0 \sin\omega t$ and current in a.c. circuit is represented by $i = i_0 \sin(\omega t + \phi)$ where ϕ is the phase angle between the current and voltage.

C15 A.C. Circuit

Pure resistive a.c. circuit

The voltage across a resistor R is in phase with the current, and the voltage and current amplitude are related by $V_R = IR$

Pure inductive circuit

The voltage across an inductor L leads the current by 90^0, the voltage and current amplitude are related by

$$V_L = IX_L,$$

where $X_L = \omega L$ is the inductive reactance of the inductor.

Pure capacitive circuit

The voltage across a capacitor C lags the current by 90^0; the voltage and current amplitudes are related by

$$V_C = IX_C,$$

where $X_C = 1/\omega C$ is the capacitive reactance of the capacitor.

LCR series circuit

In an ac circuit the voltage and current amplitudes are related by

$$V = IZ,$$

where Z is the impedance of the circuit. In an L-C-R series circuit,

$$\mathbf{Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - (1/\omega C)]^2}},$$

and the phase angle ϕ of the voltage relative to the current is

$$\tan\phi = \frac{\omega L - 1/\omega C}{R}$$

<u>**Practice Problems :**</u>

1. **A 40Ω electric heater is connected to 200 V, 50 Hz main supply. The peak value of the electric current flowing in the circuit is approximately**

   (a) 2.5 A (b) 5.0 A (c) 7 A (d) 10 A

2. **An alternating voltage V = 200√2 sin 100 t, where V in volt and t seconds, is connected to a series combination of 1 μF capacitor and 10 kΩ resistor through an ac ammeter. The reading of the ammeter will be**

   (a) √2 mA (b) 10√2 mA (c) 2 mA (d) 20 mA

3. **Choose the correct statement :**

   (a) the current leads the voltage in phase if an ac source is connected across a capacitor

   (b) the current lags behind the voltage in phase if an ac source is connected across an inductor

   (c) the current and voltage are in same phase if an ac source is connected across a resistor.

   (d) all are correct

[Answers : (1) c (2) b (3) d]

C16 Power in A.C. circuit

The average power input P_{av} to an ac circuit is

$$P_{av} = \frac{1}{2} VI\cos\phi = V_{rms} I_{rms} \cos\phi$$

where ϕ is the phase angle of voltage with respect to current. The quantity cos ϕ is called the power factor.

<u>**Practice Problems :**</u>

1. **If a current $I = I_0 \sin(\omega t - \pi/2)$ flows in a circuit across which an alternating potential $E = E_0 \sin \omega t$ has been applied, then the power consumed in the circuit depends on**

   (a) E_0 (b) I_0 (c) both (d) none

2. **In circuit 1, an alternating current of 2 A flows for 10 minutes. In another similar circuit 2, a direct current of 2 A flows for the same time. If the heat produced in circuit 1 is X then the heat produced in circuit 2 is**

   (a) 0.5 X (b) 1.5 X (c) X (d) 2X