KINEMATICS

CONCEPTS

C1A DISPLACEMENT & DISTANCE

Displacement is defined as the change in position vector of the particle during a time interval whereas distance is defined as the length of actual path. Displacement is a vector quantity whereas distance is a scalar quantity.

C1B VELOCITY AND SPEED

Average Velocity : The change in position vector i.e. displacement divided by time interval during which this change occurs is known as average velocity. For example, a particle changes its position from x_i to x_f along x - axis at time t_i and t_f respectively . Then average velocity along x-axis is given by :

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Instantaneous Velocity : It is given by $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$, $v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}$

Average Speed : The average speed of a particle in a time interval is defined as the distance travelled by the particle divided by the time interval.

Instantanous Speed : The instantaneous speed equals the magnitude of the instantaneous velocity. The instantaneous speed is given by $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$, where Δs is the distance travel during time Δt.

__Practice Problems :__

1. **Which of the following statement is true ?**
   (a) **|displacement| ≤ distance**
   (b) **|Average velocity| ≤ Average speed**
   (c) **distance and average speed never be zero or negative**
   (d) **all the above**

2. **A train travels from one station to another at a speed of v_1 and returns to the first station at the speed of v_2. The average speed and average velocity of the train is respectively**
   (a) $\frac{2v_1v_2}{v_1+v_2}, 0$ (b) $0, \frac{2v_1v_2}{v_1+v_2}$ (c) $0, 0$ (d) $\frac{2v_1v_2}{v_1+v_2}, \frac{2v_1v_2}{v_1+v_2}$

3. **A particle covers one quarter of a circular path of radius R. It takes time T. The average speed and the magnitude of average velocity are given by respectively.**
   (a) $\frac{\pi R}{2T}, \frac{\sqrt{2}R}{T}$ (b) $\frac{\pi R}{2T}, \frac{\pi R}{2T}$ (c) $\frac{\sqrt{2}R}{T}, \frac{\sqrt{2}R}{T}$ (d) $\frac{\sqrt{2}R}{T}, \frac{\pi R}{2T}$

[Answers : (1) d (2) a (3) a]

C2 ACCELERATION

Average Acceleration : Average acceleration is defined as the ratio of change in velocity to the time taken.

$<\vec{a}> = \vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$, where $\vec{v}_f$ and $\vec{v}_i$ are the velocity of the particle at t_f (final time) and t_i (initial time) respectively.

For straight line motion (i.e. along x-axis) $a_{av} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$.

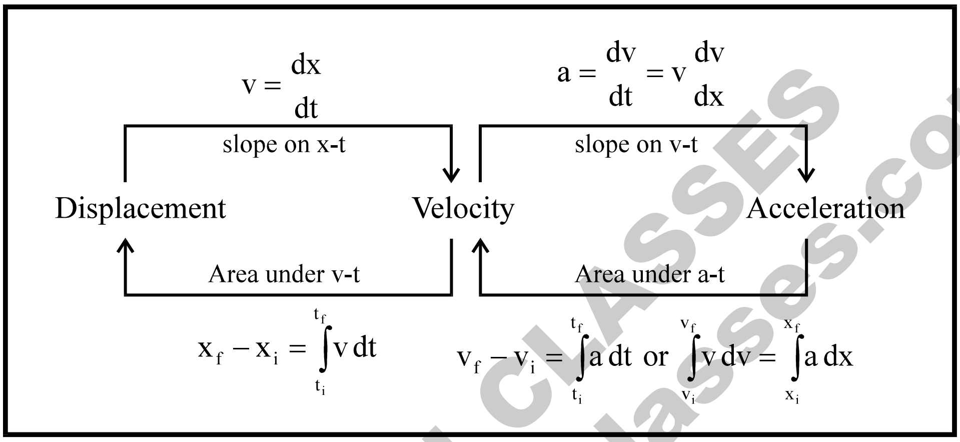
Instantaneous Acceleration : Instantaneous acceleration is defined as $\vec{a} = \frac{d\vec{v}}{dt}$

For straight line motion (i.e. along x-axis) $a_x = \frac{dv_x}{dt}$

Acceleration can also be expressed as $a_x = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = v_x \frac{dv_x}{dx}$

For uniform velocity $\vec{a} = 0$.

C3 Flow chart to find displacement, velocity & acceleration :



<u>Practice Problems :</u>

1. A particle moves along a straight line such that its displacement at any time t is given by $(t^3 - 3t^2 + 2)$m. The displacement when the acceleration is zero

(a) 0 m (b) 2 m (c) 3 m (d) –2 m

2. The initial velocity of a particle is u and the acceleration at time t is at, a being a constant. Then the velocity v at time t is given by

(a) v = u (b) v = u + at (c) $v = u + at^2$ (d) $v = u + \frac{1}{2}at^2$

3. The displacement x of a particle moving in one dimension under constant acceleration is related to the time t as $t = \sqrt{x} + 3$. The displacement of the particle when its velocity is zero is

(a) zero (b) 3 units (c) $\sqrt{3}$ units (d) 9 units

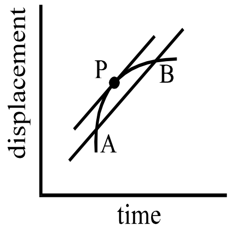
4. The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v in m/s and x is in m. Its acceleration in m/s^2 when passing through the point x = 2m.

(a) 0 (b) 5 (c) 11 (d) 30

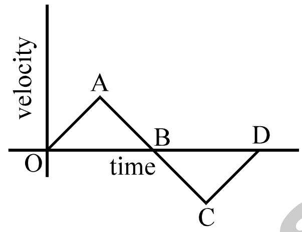
[Answers : (1) a (2) d (3) a (4) d]

C4 GRAPHICAL REPRESENTATION

1. The average velocity between two points A and B is the slope of line AB, whereas the instantaneous velocity of the particle at P is the slope of tangent drawn at this point.



2.

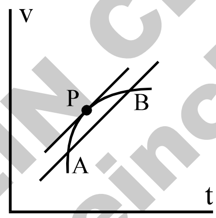


Consider the velocity time graph for a particle moving along the straight line as shown in figure. Let the magnitude of area of the triangle OAB is A_1 and BCD is A_2 then

Distance = $A_1 + A_2$

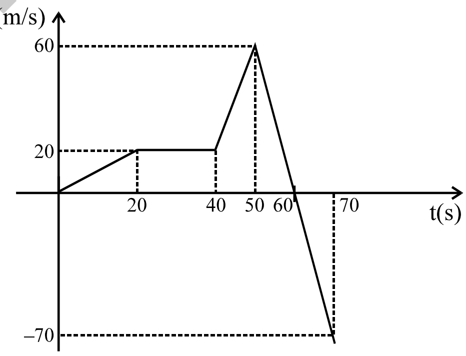
Magnitude of displacement = $|A_1 - A_2|$

3. The average acceleration between two points A and B is the slope of line AB, whereas th e instantaneous acceleration of the particle at P is the slope of tangent drawn at this point.



Practice Problems :

1. The velocity of a car moving along straight road is changing with time as shown in figure



Then :

(a) The maximum acceleration of the car is between 40s to 50s.

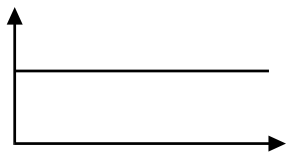
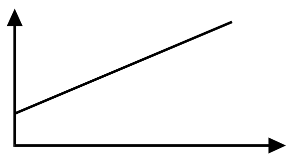
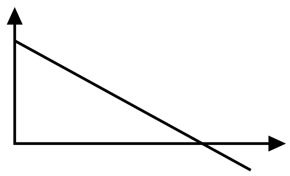
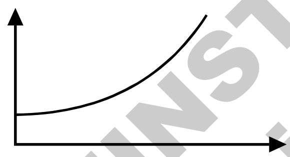
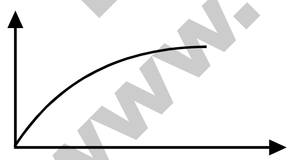
(b) The total distance covered by the car is 650 m

(c) The total displacement covered by the car is 320 m

(d) During the journey there is always non-uniform motion.

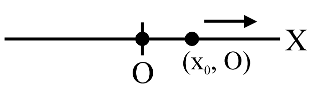
[Answers : (1) a]

C5 **Some typical graph :** In the following graphs time is on the horizontal axis whereas displacement or velocity on the vertical axis

GRAPHS	Displacement time	Velocity time
	1. Particle is at rest 2. Velocity is zero	1. Uniform velocity 2. Acceleration is zero
	1. Uniform positive velocity 2. Acceleration is zero	Uniform positive acceleration
	1. Uniform negative velocity 2. Acceleration is zero	Uniform retardation & then uniform negative acceleration
	Uniform positive acceleration if the graph is parabolic	Positive increasing acceleration
	Uniform retardation if the graph is parabolic	Decreasing acceleration

C6 **MOTION WITH CONSTANT ACCELERATION, ALONG STRAIGHT LINE OR RECTELINEAR MOTION**

For a uniformly accelerated motion along a straight line (sav x-axis) the following equations can be used.



$x = x_0 + u_x(t - t_0) + ½\, a_x(t - t_0)^2$

$v_x = u_x + a_x\,(t - t_0)$

$$x = x_0 + \frac{u_x + v_x}{2}(t - t_0)$$

$v_x^{\,2} = u_x^{\,2} + 2a_x\,(x - x_0)$

The symbols used above have following meaning;

$x_0 \to$ Initial position of the particle on x-axis at initial time t_0.

$u_x \to$ Initial velocity of the particle along x-axis.

$v_x \to$ Velocity of the particle at any position x and any time t.

$a_x \to$ Constant acceleration of the particle along x-axis.

NOTE :

we must decide at the beginning of a problem where the origin of co-ordinates is and which direction is positive. The choices of frame of reference are usually a matter of convenience.

Practice Problems :

1. A particle starts with velocity u along a straight line path with constant acceleration. It ends its journey with velocity v. The velocity of the particle at the mid point of the journey is

(a) $\frac{v+u}{2}$ **(b)** $\sqrt{\frac{v^2+u^2}{2}}$ **(c)** $\frac{2vu}{v+u}$ **(d)** $\sqrt{\frac{2v^2u^2}{v^2+u^2}}$

[Answers : (1) b]

C7 Vertical Motion Under Gravity

If a body is moving vertically downwards or upwards, it experiences a downward acceleration due to the gravitational force of the earth. This is called acceleration due to gravity and is denoted by the symbol g. Strictly speaking g is not a constant, but varies form place to place on the surface of the earth and also with height. However the variation of g is so small that it can be neglected and g can be considered a constant unless very large heights are involved. Therefore, we can use the above equations of motion for constant acceleration.

For solving problems of vertical motion under gravity, either the upward or the downward direction is taken as positive. If the upward direction is taken as positive, then g becomes negative and vice-versa. The signs of other quantities like initial velocity, initial position will be decided according to the frame of reference.

Practice Problems :

1. A stone is dropped from the top of a 30 m high cliff. At the same instant another stone is projected vertically upwards from the ground with a speed of 30 m/s. The two stones will cross each other after a time t and the height it which they cross each other is h then ($g = 10\ m/s^2$)

(a) t = 2s, h = 25 m **(b)** t = 1s, h = 25 m

(c) t = 1s, h = 15 m **(d)** t = 2s, h = 15 m

2. A particle, dropped from a height h, travels a distance 9h/25 in the last second. If $g = 9.8\ m/s^2$, then h is

(a) 100 m **(b)** 122.5 m **(c)** 145 m **(d)** 167.5 m

[Answers : (1) b (2) b]

C8A MOTION IN A PLANE OR 2D MOTION

If a particle is moving in a plane, its motion can be split into two rectilinear motions along two perpendicular directions. These two motions can be treated independently of each other and then the results can be combined according to the rules of vector addition & requirement of the problem.

Now, if the acceleration is constant, then the motions along the two axes are governed by the following two sets of equations :

X-direction

$x = x_0 + u_x(t - t_0) + ½ a_x(t - t_0)^2$

$v_x = u_x + a_x (t - t_0)$

$x = x_0 + \frac{u_x + v_x}{2}(t - t_0)$

$v_x^2 = u_x^2 + 2a_x (x - x_0)$

Y-direction

$y = y_0 + u_y(t - t_0) + ½ a_y(t - t_0)^2$

$v_y = u_y + a_y (t - t_0)$

$y = y_0 + \frac{u_y + v_y}{2}(t - t_0)$

$v_y^2 = u_y^2 + 2a_y (y - y_0)$

C8B Horizontal projection

Suppose a body is projected horizontally from a certain height h with a speed u then

time of flight = $T = \sqrt{\frac{2h}{g}}$ and the horizontal range = $R = uT = u\sqrt{\frac{2h}{g}}$

C8C Oblique Projection

Suppose a body is projected with initial velocity u at an angle θ with the horizontal.

(i) The equation of the trajectory of the projectile is $y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2$

which represents a parabola.

(ii) **Maximum Height** $H = \frac{u^2\sin^2\theta}{2g}$ (iii) **Time of Flight** $T = \frac{2u\sin\theta}{g}$

(iv) **Horizontal Range** $R = \frac{u^2\sin 2\theta}{g}$

Two important points to be noted concerning horizontal range R :

(i) For a given velocity of projection, R is maximum when $\theta = 45^0$.

(ii) For a given velocity, there are two angles of projection for which the range is the same.

If one of these angles is θ, the other is $\frac{\pi}{2} - \theta$.

Practice Problems :

1. The x and y coordinates of a particle at any time t are given by $x = 3t + 4t^2$ and $y = 4t$ where x and y are in m and t in s. Then

(a) The initial speed of the particle is 5 m/s.

(b) The acceleration of the particle is constant.

(c) The path of the particle is parabolic.

(d) All are correct

2. A particle is projected with speed u at an angle of θ with the horizontal. Another particle of different mass is projected with same speed from the same point. Both the particles has same horizontal range. Let the time of flight and maximum height attained by the first particle and second particle are t_1, h_1 and t_2, h_2 respectively. Then t_1/t_2 and h_1/h_2 are given by respectively

(a) tanθ, tan²θ (b) cotθ, cot²θ (c) cotθ, tan²θ (d) tanθ, cot²θ

3. Let the maximum height attained by the projectile is n times the horizontal range. Then the angle of projection with the horizontal is given by

(a) $\tan^{-1}n$ (b) $\tan^{-1}2n$ (c) $\tan^{-1}3n$ (d) $\tan^{-1}4n$

4. Two projectiles are projected from the same point with the same speed but at different angles of projection. Neglect the air resistance. They land at the same point on the ground. Which of the following angle of projections is possible ?

(a) $\frac{\pi}{4}+\theta, \frac{\pi}{4}-\theta$ (b) $\frac{\pi}{3}+\theta, \frac{\pi}{6}-\theta$

(c) $\theta, \frac{\pi}{2}-\theta$ (d) **all are possible**

5. If $y = ax - bx^2$ is the path of a projectile, then which of the following is correct

(a) Range = a/b **(b) Maximum height = $a^2/4b$**

(c) Angle of projection = $\tan^{-1}a$ **(d) all are correct**

[Answers : (1) d (2) a (3) d (4) d (5) d]

C9 RELATIVE MOTION

If x_{AB} is is position of A with respect to B then $x_{AB} = x_A - x_B$ where x_A and x_B are the position of A and B with respect to some common frame of reference. In the similar way for relative velocity $v_{AB} = v_A - v_B$.

INITIAL STEP EXERCISE

1. A motor car is going due (towards) north at a speed of v. It makes a 90^0 left turn without changing the speed. The change in the velocity of the car is about

(a) $\sqrt{2}$v towards west

(b) $\sqrt{2}$v towards south-west

(c) $\sqrt{2}$v towards north-west

(d) zero

2. Water drops fall at regular intervals from a roof. At an instant when a drop is about to leave the roof, the separations between successive drops below the roof are in the ratio

(a) 1 : 2 : 3 : 4 **(b) 1 : 4 : 9 : 16**

(c) 1 : 3 : 5 : 7 **(d) 1 : 5 : 13 : 21**

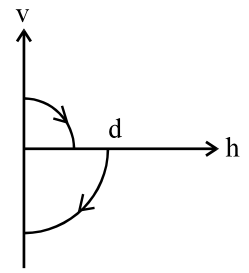
3. A point moves in x-y plane according to the law x = 4 sin 6t and y = 4(1 – cos 6t). The distance traversed by the particle in 4 seconds is (x and y are in meters)

(a) 96 m **(b) 48 m**

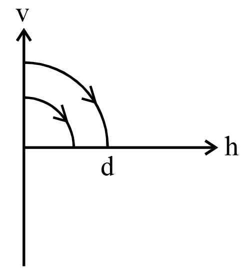
(c) 24 m **(d) 108 m**

4. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as figure

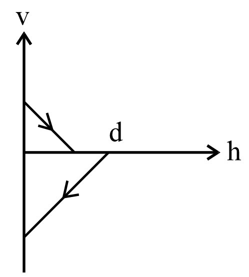
(a)

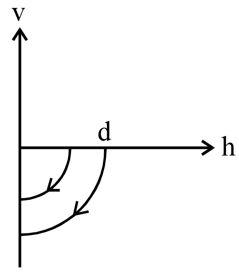


(b)



(c)



(d) 

5. An object of mass m is projected with a momentum p at such an angle that its maximum height (H) is 1/4th of its horizontal range (R). The ratio of maximum kinetic energy to minimum kinetic energy in its path will be

(a) 8 : 1 (b) 2 : 1

(c) 4 : 3 (d) 3 : 2

6. The acceleration vector of a particle is a constant. The trajectory of the particle is a/an

(a) parabola (b) ellipse

(c) hyperbola (d) circle

7. A hot air balloon is ascending at the rate of 10m/s and is 40m above the ground when a ball is dropped over the side. The average speed and average velocity of the ball over the whole time of flight are respectively

(a) 5.5 m/s, 0

(b) 9.5 m/s, 9.5 m/s

(c) 12.5 m/s, 10m/s

(d) 16.5 m/s, 12.5 m/s

8. A body is in straight line motion with an acceleration given by a = 32 – 4v. At t = 0 the velocity of the particle is 4 unit. The velocity when t = *l*n 2 is

(a) 15/2 (b) 17/2

(c) 23/4 (d) 31/4

9. A vector $\vec{a}$ is turned through θ about its initial point. The magnitude of change in vector $\vec{a}$ is

(a) 0 (b) $2|\vec{a}|\theta$

(c) $2|\vec{a}|\sin\theta/2$ (d) $2|\vec{a}|\cos\theta/2$

10. A particle is projected from the top most point of the minar which is at the height of 40 m from its base. The velocity of the particle is 20 m/s at an angle of 60^0 with the vertical. The particle lands the ground at a distance of x from the base of the minar. The value of x is

(a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m

(c) $40\sqrt{3}$ m (d) $25\sqrt{3}$ m

11. Rain is falling with a speed of 4 m/s in a direction making an angle of 30^0 with vertical towards south. What should be the magnitude and direction of velocity of cyclist to hold his umbrella exactly vertical, so that rain does not wet him

(a) 2 m/s towards north

(b) 4 m/s towards south

(c) 2 m/s towards south

(d) 4 m/s towards north

12. A stone is projected from the ground with a velocity of 50 m/s at an angle 30^0. It crosses the wall after 4s. The distance beyond the wall at which the stone strikes the ground is

(a) 25 m (b) $25\sqrt{3}$ m

(c) 50 m (d) $25/\sqrt{3}$ m

13. The deceleration experienced by a moving motor-boat, after its engine is cut off is given by kv^3, where k is a constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is

(a) $\frac{v_0}{\sqrt{2v_0^2kt+1}}$ (b) v_0e^{-kt}

(c) $\frac{v_0}{\sqrt{2v_0^2kt-1}}$ (d) v_0e^{kt}

14. The deceleration experienced by a moving motor-boat, after its engine is cut off is given by kv, where k is a constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is

(a) $\frac{v_0}{\sqrt{2v_0^2kt+1}}$ (b) v_0e^{-kt}

(c) $\frac{v_0}{\sqrt{2v_0^2kt-1}}$ (d) v_0e^{kt}

15. The deceleration experienced by a moving motor-boat, after its engine is cut off is given by kv, where k is a constant. If v_0 is the magnitude of the velocity at cut-off, the maximum distance convered by the boat is

(a) $\frac{v_0}{k}$ (b) $\frac{v_0}{2k}$

(c) $\frac{2v_0}{k}$ (d) $\frac{v_0}{4k}$

16. A stone is dropped from a height h, simultaneously, another stone is thrown up from the ground which reaches a height 4h. The two stones cross each other after time

(a) $\sqrt{\frac{h}{2g}}$ (b) $\sqrt{\frac{h}{8g}}$

(c) $\sqrt{8hg}$ (d) $\sqrt{2hg}$

17. A body is projected at time t = 0 from a certain point on a planet's surface with a certain velocity at a certain angle with the planet's surface (assumed horizontal). The horizontal and vertical displacements x and y (in metres) respectively vary with time t (in seconds) as

$$x = 10\sqrt{3}\,t$$
$$y = 10t - t^2$$

What is the magnitude and direction of the velocity with which the body is projected ?

(a) 20 ms^{-1} at an angle of 30^0 with the horizontal

(b) 20 ms^{-1} at an angle of 60^0 with the horizontal

(c) 10 ms^{-1} at an angle of 30^0 with the horizontal

(d) 10 ms^{-1} at an angle of 60^0 with the horizontal

FINAL STEP EXERCISE

1. A bird flies for 4 sec with a velocity of (t – 2) m/s in a straight line, where t = time in seconds. The average speed and average velocity of the bird are

(a) 0, 0 (b) 0, 1m/s

(c) 1m/s, 0 (d) 1m/s, 1m/s

2. A river is flowing from west to east at a speed of u. A man on the south bank of the river, capable of swimming at v with respect to river. The width of the river is *l*. Choose the correct statement.

(a) If the man wants to swim across the river in the shortest time, he should swim due north.

(b) If the man wants to swim across the river in the shortest distance, he should swim due north.

(c) If the man wants to swim across the river in the shortest distance, he should swim $\sin^{-1}\left(\frac{u}{v}\right)$ north of west.

(d) (a) and (c) are correct.

3. A projectile has a maximum range of 500 m. If the projectile is now thrown up an inclined plane of 30^0 with the same velocity, the distance covered by it along the inclined plane will be about

(a) 250 m (b) 500 m

(c) 750 m (d) 1000 m

4. Three particles starts from the origin at the same time, one with a velocity u_1 along the x-axis, the second along the y-axis with a velocity u_2 and the third along the x = y line. The velocity of the third so that the three may always lie on the same line is

(a) $\frac{u_1 + u_2}{2}$ (b) $\sqrt{u_1 u_2}$

(c) $\frac{u_1 u_2}{u_1 + u_2}$ (d) $\frac{\sqrt{2}u_1 u_2}{u_1 + u_2}$

5. Choose the correct statement from the following for a projectile projected from the ground at certain angle with the horizontal.

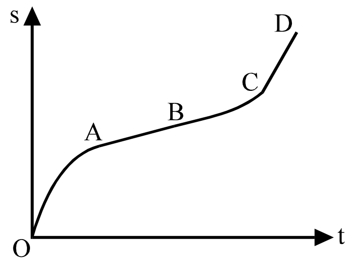
(a) The angle between the velocity vector and acceleration vector at the highest point is $\pi/2$.

(b) The minimum speed at the highest point equals to the initial horizontal speed.

(c) The maximum horizontal range for the projectile is at the angle of projection of $\pi/4$.

(d) All are correct.

6. The graph between the displacement x and time t for a particle moving in a straight line is shown in the diagram. During the intervals OA, AB, BC and CD the acceleration of the particle is



	OA	AB	BC	CD
(a)	+	0	+	+
(b)	–	0	+	0
(c)	+	0	–	+
(d)	–	0	–	0

7. The displacement (x) of a particle depends on time (t) as

$$x = \alpha t^2 - \beta t^3.$$

(a) The particle will return to its starting point after time α/β

(b) The particle will come to rest after time $2\alpha/3\beta$.

(c) The initial velocity of the particle was zero but its initial acceleration was not zero.

(d) all are correct

8. A particle starts from the origin of coordinates at time t = 0 and moves in the xy plane with a constant acceleration α in the y-direction. Its equation of motion is $y = \beta x^2$. Its velocity component in the x-direction is

(a) variable (b) $\sqrt{\frac{2\alpha}{\beta}}$

(c) $\frac{\alpha}{2\beta}$ (d) $\sqrt{\frac{\alpha}{2\beta}}$

9. A projectile is projected with speed u at an angle θ with the horizontal. The time after which the velocity vector of the particle become perpendicular to the initial velocity of projection

(a) $\frac{u}{g\sin\theta}$ (b) $\frac{u}{g\cos\theta}$

(c) $\frac{u\sin\theta}{g}$ (d) $\frac{2u\sin\theta}{g}$

10. A car accelerated from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. Let the total time is T for the journey. The maximum velocity attained and total distance covered is respectively

(a) $\frac{\alpha\beta}{\alpha+\beta}T$ and $\frac{\alpha\beta T^2}{2(\alpha+\beta)}$

(b) $\frac{\alpha\beta}{\alpha+\beta}T$ and $\frac{\alpha\beta T^2}{(\alpha+\beta)}$

(c) $\frac{\alpha\beta}{2(\alpha+\beta)}T$ and $\frac{\alpha\beta T^2}{2(\alpha+\beta)}$

(d) $\frac{\alpha\beta}{2(\alpha+\beta)}T$ and $\frac{\alpha\beta T^2}{(\alpha+\beta)}$

11. A body is projected vertically upwards with velocity 'u'. If t_1 and t_2 be the times at which it is at height h above the point of projection while ascending and descending respectively, then

(a) $h = gt_1t_2, u = g(t_1 + t_2)$

(b) $h = \frac{1}{2}gt_1t_2, u = g(t_1 + t_2)$

(c) $h = gt_1t_2, u = \frac{1}{2}g(t_1 + t_2)$

(d) $h = \frac{1}{2}gt_1t_2, u = \frac{1}{2}g(t_1 + t_2)$

ANSWERS (INITIAL STEP EXERCISE)

1.	b	11.	c
2.	c	12.	b
3.	a	13.	a
4.	a	14.	b
5.	b	15.	a
6.	a	16.	b
7.	c	17.	a
8.	d		
9.	c		
10.	c		

ANSWERS (FINAL STEP EXERCISE)

1.	c	6.	b
2.	d	7.	d
3.	a	8.	d
4.	d	9.	a
5.	d	10.	a
		11.	d