SIMPLE HARMONIC MOTION

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C1 Periodic and Oscillatory Motions :

The motion which repeats itself after cetain time of interval is known as periodic motion. For e.g. motion of a satellite around the planet, uniform circular motion etc. To and fro periodic motions about a fixed point is known as oscillatory motion. Every oscillatory motion is periodic but the converse is not true.

C2 Period and Frequency :

The smallest interval of time after which the motion is repeated is called its period, denoted by T. The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the frequency of the periodic motion denoted by f. The relation between f and T is given by, f = 1/T. The unit of frequency is s^{-1} or Hz (hertz).

C3 Displacement in Periodic Motion :

Let the displacement is given by x = f(t), where t is the time. In periodic motion of period T.

f(t) = f(t + T)

Any displacement represented by cosine function or sine function is periodic. Any periodic function can be expressed as a superposition of sine and cosine functions of different time periods with suitable coefficients.

C4 Simple Harmonic Motion :

If a particle moves to and fro about a fixed point (equilibrium position) under the application of a force or torque (called restoring force or torque) which is directly proportional to the displacement (linear or angular), directed towards the fixed point, is called simple harmonic motion.

C5 Displacement, Velocity and Acceleration in SHM

The necessary and sufficient condition for the motion to be simple harmonic (linear) is that the force should be directly proportional to the displacement i.e. $F \propto x$

$$F = -kx \quad \text{or} \quad m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} + \omega^2 x = 0 \text{ with } \omega^2 = \frac{k}{m}$$

The solution of the above differential equation representing SHM is given by :

$$x = A \sin(\omega t + \phi)$$

where A is the amplitude of the motion, ω is the angular (circular) frequency = 2π/T (T = time period) and ϕ is phase constant.

For rotational SHM the analogues equation is $\theta = \theta_0 \sin(\omega t + \phi)$, where θ is the angular displacement and θ_0 is the maximum angular displacement.

Velocity is given by : $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$ **or** $v = A\omega\sqrt{1 - \frac{x^2}{A^2}} = \omega\sqrt{A^2 - x^2}$

The maximum and minimum velocity of the particle are $v_{max} = A\omega$ and $V_{min} = 0$.

Acceleration is : $a = \frac{d^2x}{dt^2} = -\omega^2 x$. Thus $|a_{max}| = \omega^2 A$ and $|a_{min}| = 0$

Practice Problems :

1. **The equation of S.H.M. of a particle is** $\frac{d^2y}{dt^2} + ky = 0$**, where k is a positive constant. The time period of motion is given by**

(a) $\frac{2\pi}{\sqrt{k}}$ (b) $\frac{2\pi}{k}$ (c) $\frac{k}{2\pi}$ (d) $\frac{\sqrt{k}}{2\pi}$

2. **A particle is executing S.H.M. of period 4 s. Then the time taken by it to move from the extreme position to half the amplitude is**

(a) 1/3 s **(b)** 2/3 s **(c)** 3/4 s **(d)** 4/3 s

3. **A particle is vibrating in S.H.M. If its velocities are v_1 and v_2 when the displacements from the mean position are y_1 and y_2, respectively, then its time period is**

(a) $2\pi\sqrt{\frac{y_1^2+y_2^2}{v_1^2+v_2^2}}$ **(b)** $2\pi\sqrt{\frac{v_1^2+v_2^2}{y_1^2+y_2^2}}$ **(c)** $2\pi\sqrt{\frac{v_2^2-v_1^2}{y_1^2-y_2^2}}$ **(d)** $2\pi\sqrt{\frac{y_1^2-y_2^2}{v_2^2-v_1^2}}$

4. **A particle is executing S.H.M. Then the graph of velocity as a function of displacement is**

(a) straight line **(b)** circle **(c)** ellipse **(d)** hyperbola

5. **A particle is executing S.H.M. Then the graph of acceleration as a function of displacement is**

(a) straight line **(b)** circle **(c)** ellipse **(d)** hyperbola

[Answers : (1) a (2) b (3) d (4) c (5) a]

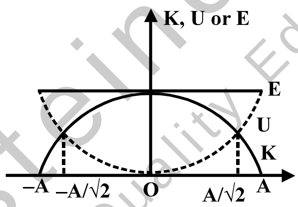
C6 Energy in SHM

(i) Kinetic Energy (K) $= \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

(ii) Potential Energy (U) $= \frac{1}{2}m\omega^2x^2$

(iii) Total energy (E) $= \frac{1}{2}m\omega^2A^2$ = constant

Graph for the variation of K, U and E with the position 'x' is given by :



<u>**Practice Problems :**</u>

1. **A body executes S.H.M. with an amplitude A. Its energy is half kinetic and half potential when the displacement is**

(a) A/3 **(b)** A/2 **(c)** A/√2 **(d)** A/2√2

2. **When the potential energy of a particle executing simple harmonic motion is one-fourth of its maximum value during the oscillation, the displacement of the particle from the equilibrium position in terms of its amplitude a is**

(a) a/4 **(b)** a/3 **(c)** a/2 **(d)** 2a/3

[Answers : (1) c (2) c]

C7 Simple Harmonic Motion and Uniform Circular Motion :

Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the circular motion occurs.

C8 Method of finding time period of a SHM :

(i) Force method will be used for linear SHM in which $F \propto -x \Rightarrow F = -kx$ and hence time period

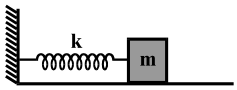
$T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mass of particle executing SHM and k is a constant quantity.

(ii) Torque method will be used for angular SHM in which $\tau \propto -\theta \Rightarrow \tau = -k\theta$ and hence time period $T = 2\pi\sqrt{\frac{I}{k}}$ where I is the moment of the inertia and k is a constant quantity.

(iii) Energy method in which total energy of the system executing SHM is constant.

C9 Spring - Block System :

The time period of SHM for a block of mass 'm' connected by a spring of spring constant 'k' as shown in figure is given by $T = 2\pi\sqrt{\frac{m}{k}}$



Practice Problems :

1. The bodies M and N of equal masses are suspended from two separate massless springs of constants k_1 and k_2, respectively. If the two oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of M to that of N is

(a) k_1/k_2 (b) k_2/k_1 (c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$

2. The vertical extension in a light spring by a weight of 1 kg, in equilibrium, is 9.8 cm. The period of oscillation of the spring, in seconds, will be

(a) $\frac{2\pi}{10}$ (b) $\frac{2\pi}{100}$ (c) 20π (d) 200π

3. A massless spring, having force constant k, oscillates with a frequency n when a mass m is suspended from it. The spring is cut into two equal halves and a mass 2m is suspended from it. The frequency of oscillation will now be

(a) n (b) $n\sqrt{2}$ (c) $n/\sqrt{2}$ (d) 2n

[Answers : (1) d (2) a (3) a]

C10 Simple Pendulum :

The time period of a simple pendulum of length L is given by $2\pi\sqrt{\frac{L}{g}}$.

If length of the pendulum is large, then time period is given by : $T = 2\pi\sqrt{\frac{l}{g\left(\frac{l}{L}+\frac{l}{R}\right)}}$ where R = Radius of the earth.

For the pendulum of infinite length i.e., $L \to \infty$, $T = 2\pi\sqrt{\frac{R}{g}} \approx 84.6$ minutes

A second pendulum is the simple pendulum having a time period of 2s.

Practice Problems :

1. The time period of a simple pendulum is T. If its length is increased by 2%, the new time period becomes

(a) 0.98 T (b) 1.02 T (c) 0.99 T (d) 1.01 T

2. The length of a simple pendulum is increased by 44%. The percentage increase in its time period will be

(a) 44% (b) 22% (c) 20% (d) 11%

3. A girl is swinging on a swing in the sitting position. How will the period of swing be affected if she stands up ?

(a) The period will now be shorter

(b) The period will now be longer

(c) The period will remain unchanged

(d) The period may become longer or shorter depending upon the height of the girl.

4. For a simple pendulum the graph between length and time period will be a

(a) hyperbola (b) parabola (c) straight line (d) none of these

[Answers : (1) d (2) c (3) a (4) b]

C11 Physical Pendulum :

The time period of a physical pendulum is given by : $\mathbf{T = 2\pi\sqrt{\frac{I}{Mdg}}}$

I : moment of inertia about the rotational axis passing through point of suspension

d : distance of the center of mass from the point of suspension

M : total mass of the body

Practice Problems :

1. A rod of mass m and length L is suspended from one of the end point of the rod in vertical plane. The time period of the rod for small oscillation is given by

(a) $2\pi\sqrt{\frac{2L}{3g}}$ **(b)** $2\pi\sqrt{\frac{L}{g}}$ **(c)** $2\pi\sqrt{\frac{3L}{g}}$ **(d)** $2\pi\sqrt{\frac{3L}{4g}}$

[Answers : (1) a]

C12 Damped Simple Harmonic Motion :

The motion of a simple pendulum, swinging in air, dies out eventually. This is because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually. The pendulum is said to execute damped oscillations. The mechanical energy of this oscillator will decrease with time due to presence of damping force which is given by $F_d = -bv$, where v is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by $x(t) = A\,e^{-bt/2m}\cos(\omega' t + \phi)$, where ω', the angular frequency of the damped oscillator, is given by $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$. If the damping constant is small then $\omega' = \omega$, where ω is the angular frequency of the undamped oscillator. The mechanical energy E of the damped oscillator is given by $\mathbf{E(t) = \frac{1}{2}kA^2e^{-bt/m}}$.

C13 Force Oscillations and Resonance :

If an external force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ω_d. The amplitude of oscillations is the greatest when $\omega_d = \omega$, a condition called resonance.