Kinematics

KINEMATICS

The part of mechanics that deals with the description of motion is called kinematics. There are two types of motion :

1. **One dimensional motion or Motion in straight a line :** In this motion the velocity vector and acceleration vector are always along the same line.
2. **Two dimensional motion :** In this motion the velocity vector and acceleration vector will be in the same plane but they are inclined at some angle, this angle may change during the motion or may be constant. For example : circular motion and parabolic motion.

C1 Displacement and Distance

Displacement is defined as the change in position vector of the particle during a time interval whereas distance is defined as the length of actual path. Displacement is a vector quantity whereas distance is a scalar quantity.

Note that : (i) distance ≥ |displacement| (ii) distance and magnitude of displacement are equal during the time interval in which the velocity of the particle should not be zero at any moment along the straight line motion.

C2 Velocity and Speed

Average Velocity : The change in position vector i.e. displacement divided by time interval during which this change occurs is known as average velocity. For example, a particle changes its position from x_i to x_f along x - axis at time t_i and t_f respectively . Then average velocity along x-axis is given by : $v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$. In general, for a particle moving on curved path :

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$. Here $\Delta\vec{r}$ is the displacement during the time interval Δt.

Instantaneous Velocity : The velocity of the particle at a particular point or at a particular instant of time is called the instantaneous velocity of the particle. It is given by

$$\vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$.

For constant velocity, displacement = (velocity) (time)

Average Speed

The average speed of a particle in a time interval is defined as the distance travelled by the particle divided by the time interval.

Note that : (i) Average Speed ≥ |Average Velocity| (ii) average speed & magnitude of average velocity are equal during the time interval in which the velocity of the particle should not be zero at any moment along the straight line motion.

Instantanous Speed : The instantaneous speed equals the magnitude of the instantaneous velocity. The instantaneous speed is given by $v = \lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t} = \frac{ds}{dt}$ where Δs is the distance travel during time Δt. Also the speed is given by $\sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2 + \mathbf{v}_z^2}$.

For constant speed, distance = (speed) (time)

Practice Problems :

1. **Which of the following statement is true ?**
   - (a) |displacement| $\le$ distance
   - (b) |Average velocity| $\le$ Average speed
   - (c) distance and average speed never be zero or negative
   - (d) all the above

2. **A train travels from one station to another at a speed of v_1 and returns to the first station at the speed of v_2. The average speed and average velocity of the train is respectively**

   (a) $\frac{2v_1v_2}{v_1+v_2}, 0$ (b) $0, \frac{2v_1v_2}{v_1+v_2}$ (c) $0, 0$ (d) $\frac{2v_1v_2}{v_1+v_2}, \frac{2v_1v_2}{v_1+v_2}$

3. **A particle covers one quarter of a circular path of radius R. It takes time T. The average speed and the magnitude of average velocity are given by respectively.**

   (a) $\frac{\pi R}{2T}, \frac{\sqrt{2}R}{T}$ (b) $\frac{\pi R}{2T}, \frac{\pi R}{2T}$ (c) $\frac{\sqrt{2}R}{T}, \frac{\sqrt{2}R}{T}$ (d) $\frac{\sqrt{2}R}{T}, \frac{\pi R}{2T}$

4. **A particle starts from one point to another point along the straight path. It covers this path in n equal distance with speed $v_1, v_2.....v_n$. Find the average speed for the complete journey.**

5. **A particle starts from one point to another point along the straight path. It covers this path in n equal time interval with speed $v_1, v_2.....v_n$. Find the average speed for the complete journey.**

6. **A particle is moving along a circular path of radius r. Find magnitude of displacement and distance for (a) one quarter of circle (b) half circle (c) three quarter circle (d) complete one circle (e) 2.5 circle.**

[Answers : (1) d (2) a (3) a (4) $\frac{n}{\sum_{i=1}^{n}\frac{1}{v_i}}$ (5) $\frac{\sum_{i=1}^{n} v_i}{n}$ (6) (a) $\sqrt{2}R, \frac{\pi R}{2}$ (b) 2R, πR (c) $\sqrt{2}R, \frac{3\pi R}{2}$ (d) 0, 2πR (e) 2R, 5πR]

C2 Acceleration

Average Acceleration : Average acceleration is defined as the ratio of change in velocity to the time taken.

$< \vec{a} > = \vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ where $\vec{v}_f$ and $\vec{v}_i$ are the velocity of the particle at t_f (final time) and t_i (initial time) respectively.

For straight line motion (i.e. along x-axis) $a_{av} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$.

Instantaneous Acceleration : Instantaneous acceleration is defined as $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta\vec{t}} = \frac{d\vec{v}}{dt}$

For straight line motion (i.e. along x-axis) $a_x = \frac{dv_x}{dt}$

Acceleration can also be expressed as $a_x = \frac{dv_x}{dx}.\frac{dx}{dt} = v_x\frac{dv_x}{dx}$. For uniform velocity $\vec{a} = 0$.

Uniform acceleration means that the acceleration of the particle is constant and in this case $<\vec{a}> = \vec{a}$. If acceleration is in same direction to the velocity then speed of the particle increases. If acceleration is in opposite direction to the vleocity then speed decreases. This situation is called retardation. Note that negative acceleration does not mean that motion is retardation.

C3 Flow chart to find displacement, velocity & acceleration :

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = v\frac{dv}{dx}$$

$$\text{Displacement} \xrightarrow{\text{slope on x-t}} \text{Velocity} \xrightarrow{\text{slope on v-t}} \text{Acceleration}$$

$$\text{Displacement} \xleftarrow{\text{Area under v-t}} \text{Velocity} \xleftarrow{\text{Area under a-t}} \text{Acceleration}$$

$$x_f - x_i = \int_{t_i}^{t_f} v\,dt \qquad v_f - v_i = \int_{t_i}^{t_f} a\,dt \text{ or } \int_{v_i}^{v_f} v\,dv = \int_{x_i}^{x_f} a\,dx$$

<u>Practice Problems :</u>

1. A particle moves along a straight line such that its displacement at any time t is given by $(t^3 - 3t^2 + 2)$m. The displacement when the acceleration is zero

(a) 0 m (b) 2 m (c) 3 m (d) –2 m

2. The initial velocity of a particle is u and the acceleration at time t is at, a being a constant. Then the velocity v at time t is given by

(a) $v = u$ (b) $v = u + at$ (c) $v = u + at^2$ (d) $v = u + \frac{1}{2}at^2$

3. The displacement x of a particle moving in one dimension under constant acceleration is related to the time t as $t = \sqrt{x} + 3$. The displacement of the particle when its velocity is zero is

(a) zero (b) 3 units (c) $\sqrt{3}$ units (d) 9 units

4. The velocity of a particle moving on the x-axis is given by $v = x^2 + x$ where v in m/s and x is in m. Its acceleration in m/s^2 when passing through the point x = 2m.

(a) 0 (b) 5 (c) 11 (d) 30

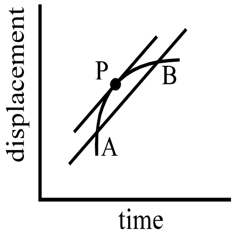
5. A particle is moving along the x-axis such that its velocity, $v = a\sqrt{x}$ where a is a constant quantity. Prove that the acceleration of the particle is constant.

6. A particle is moving along a straight path such that acceleration $a = -\alpha v$, where α is the constant and v is the instant velocity. If initial velocity is u then (i) find velocity at any time t (ii) velocity after covering the distance x (iii) draw velocity-time graph and velocity-distance graph. Also find the maximum distance covered.

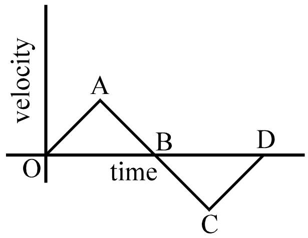
[Answers : (1) a (2) d (3) a (4) d (6) (i) $v = ue^{-\alpha t}$ (ii) $v = u - \alpha x$ (iii) max. distance = u/α]

C4 GRAPHICAL REPRESENTATION

1. The average velocity between two points A and B is the slope of line AB, whereas the instantaneous velocity of the particle at P is the slope of tangent drawn at this point.



2.

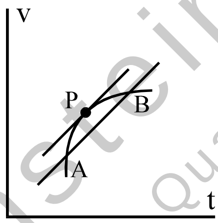


Consider the velocity time graph for a particle moving along the straight line as shown in figure. Let the magnitude of area of the triangle OAB is A_1 and BCD is A_2 then

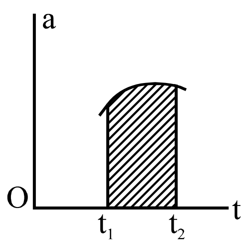
Distance = $A_1 + A_2$

Magnitude of displacement = $|A_1 - A_2|$

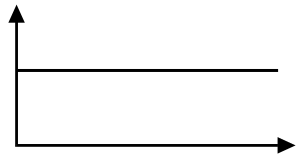
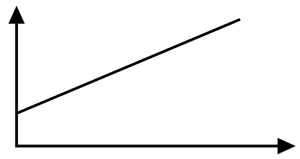
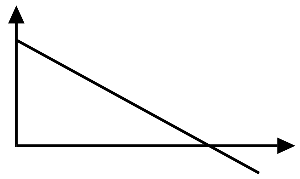
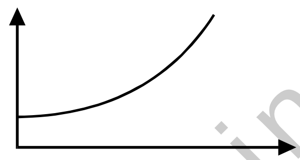
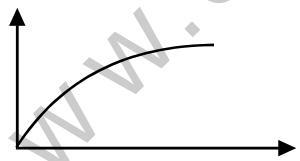
3. The average acceleration between two points A and B is the slope of line AB, whereas th e instantaneous acceleration of the particle at P is the slope of tangent drawn at this point.



4. On an acceleration (a) versus time (t) graph, the change in velocity in velocity is the area bounded as shown in figure :

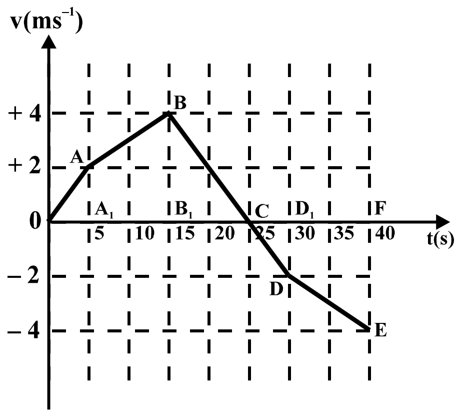


C5 **Some typical graph :** In the following graphs time is on the horizontal axis whereas displacement or velocity on the vertical axis

GRAPHS	Displacement time	Velocity time
	1. Particle is at rest 2. Velocity is zero	1. Uniform velocity 2. Acceleration is zero
	1. Uniform positive velocity 2. Acceleration is zero	Uniform positive acceleration
	1. Uniform negative velocity 2. Acceleration is zero	Uniform retardation & then uniform negative acceleration
	Uniform positive acceleration if the graph is parabolic	Positive increasing acceleration
	Uniform retardation if the graph is parabolic	Decreasing acceleration

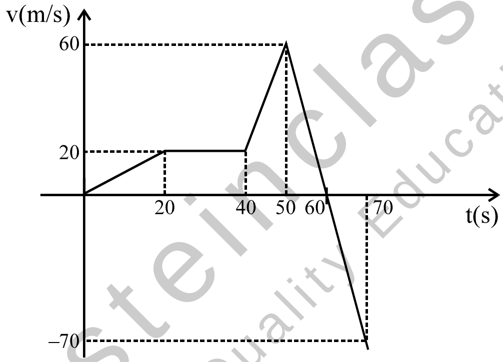
Practice Problems :

1. **The velocity-time graph for straight line motion is shown in figure.**



Find (a) total distance (b) total displacement (c) average velocity between 5s to 40s (d) total average speed (e) total average velocity (f) average acceleration between 15s to 25s (g) acceleration at t = 0 & 10s(h) draw the acceleration-time graph, distance-time graph and displacement-time graph.

2. **The velocity of a car moving along straight road is changing with time as shown in figure**



Then :

(a) The maximum acceleration of the car is between 40s to 50s.

(b) The total distance covered by the car is 650 m

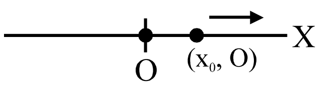
(c) The total displacement covered by the car is 320 m

(d) During the journey there is always non-uniform motion.

[Answers : (1) (a) 90 m (b) 20 m (c) 85/35 m/sec (d) 9/4 m/sec (e) 1/2 m/sec (f) –0.4 m/s^2 (g) 2/5 m/s^2, 1/5 m/s^2 (2) a]

C6 MOTION WITH CONSTANT ACCELERATION, ALONG STRAIGHT LINE OR RECTELINEAR MOTION

For a uniformly accelerated motion along a straight line (sav x-axis) the following equations can be used.



$x = x_0 + u_x(t - t_0) + \frac{1}{2} a_x(t - t_0)^2$

$v_x = u_x + a_x (t - t_0)$

$$x = x_0 + \frac{u_x + v_x}{2}(t - t_0)$$

$v_x^2 = u_x^2 + 2a_x (x - x_0)$

The symbols used above have following meaning;

$x_0 \rightarrow$ Initial position of the particle on x-axis at initial time t_0.

$u_x \rightarrow$ Initial velocity of the particle along x-axis.

$v_x \rightarrow$ Velocity of the particle at any position x and any time t.

$a_x \rightarrow$ Constant acceleration of the particle along x-axis.

NOTE :

we must decide at the beginning of a problem where the origin of co-ordinates is and which direction is positive. The choices of frame of reference are usually a matter of convenience.

Practice Problems :

1. A particle starts with velocity u along a straight line path with constant acceleration. It ends its journey with velocity v. The velocity of the particle at the mid point of the journey is

(a) $\frac{v+u}{2}$ (b) $\sqrt{\frac{v^2+u^2}{2}}$ (c) $\frac{2vu}{v+u}$ (d) $\sqrt{\frac{2v^2u^2}{v^2+u^2}}$

2. A body travels 200 cm in the first two seconds and 220 cm in the next four seconds. The velocity at the end of the seventh second from the start is

(a) 10 cm/s (b) 12 cm/s (c) 14 cm/s (d) 16 cm/s

3. The speed of a train is reduced from 60 km/h to 15 km/h while it travels a distance of 450 m. If the retardation is uniform, how much further it will travel before coming to rest ?

(a) 10 m (b) 20 m (c) 30 m (d) 40 m

4. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time to the driver. If he is driving a car at a speed of 54 km/h and the brakes causes a deceleration of 6.0 m/s², the distance travelled by the car after he sees the need to put the brakes on.

(a) 22 m (b) 18 m (c) 15 m (d) 13 m

5. A body, starting from rest, moves in a straight line with a constant acceleration a for a time interval t during which it travels a distance s_1. It continues to move with the same acceleration for the next time interval t during which it travels a distance s_2. The relation between s_1 and s_2 is

(a) $s_2 = s_1$ (b) $s_2 = 2s_1$ (c) $s_2 = 3s_1$ (d) $s_2 = 4s_1$

6. A body moving in a straight line with constant acceleration of 10 ms^{-2} covers a distance of 40 m in the 4th second. How much distance will it cover in the 6th second ?

(a) 50 m (b) 60 m (c) 70 m (d) 80 m

7. A car, starting from rest, is accelerated at a constant rate α until it attains a speed v. It is then retarded at a constant rate β until it comes to rest. The average speed of the car during its entire journey is

(a) zero (b) $\frac{\alpha v}{2\beta}$ (c) $\frac{\beta v}{2\alpha}$ (d) $\frac{v}{2}$

[Answers : (1) b (2) a (3) c (4) a (5) c (6) b (7) d]

C7 Vertical Motion Under Gravity

If a body is moving vertically downwards or upwards, it experiences a downward acceleration due to the gravitational force of the earth. This is called acceleration due to gravity and is denoted by the symbol g. Strictly speaking g is not a constant, but varies form place to place on the surface of the earth and also with height. However the variation of g is so small that it can be neglected and g can be considered a constant unless very large heights are involved. Therefore, we can use the above equations of motion for constant acceleration.

For solving problems of vertical motion under gravity, either the upward or the downward direction is taken as positive. If the upward direction is taken as positive, then g becomes negative and vice-versa. The signs of other quantities like initial velocity, initial position will be decided according to the frame of reference.

Practice Problems :

1. **A stone is dropped from the top of a 30 m high cliff. At the same instant another stone is projected vertically upwards from the ground with a speed of 30 m/s. The two stones will cross each other after** a time t and the height it which they cross each other is h then ($g = 10\ m/s^2$)

(a) **t = 2s, h = 25 m** (b) **t = 1s, h = 25 m**

(c) **t = 1s, h = 15 m** (d) **t = 2s, h = 15 m**

2. **A particle, dropped from a height h, travels a distance 9h/25 in the last second. If $g = 9.8\ m/s^2$, then h is**

(a) **100 m** (b) **122.5 m** (c) **145 m** (d) **167.5 m**

3. **A stone is dropped from a height h, simultaneously, another stone is thrown up from the ground which reaches a height 4h. The two stones cross each other after time**

(a) $\sqrt{\frac{h}{2g}}$ (b) $\sqrt{\frac{h}{8g}}$ (c) $\sqrt{8hg}$ (d) $\sqrt{2hg}$

[Answers : (1) b (2) b (3) b]

C8A MOTION IN A PLANE OR 2D MOTION

If a particle is moving in a plane, its motion can be split into two rectilinear motions along two perpendicular directions. These two motions can be treated independently of each other and then the results can be combined according to the rules of vector addition & requirement of the problem.

Now, if the acceleration is constant, then the motions along the two axes are governed by the following two sets of equations :

X-direction	**Y-direction**
$x = x_0 + u_x(t - t_0) + \frac{1}{2} a_x(t - t_0)^2$	$y = y_0 + u_y(t - t_0) + \frac{1}{2} a_y(t - t_0)^2$
$v_x = u_x + a_x (t - t_0)$	$v_y = u_y + a_y (t - t_0)$
$x = x_0 + \frac{u_x + v_x}{2}(t - t_0)$	$y = y_0 + \frac{u_y + v_y}{2}(t - t_0)$
$v_x^2 = u_x^2 + 2a_x (x - x_0)$	$v_y^2 = u_y^2 + 2a_y (y - y_0)$

C8B Horizontal projection

Suppose a body is projected horizontally from a certain height h with a speed u then

time of flight = $T = \sqrt{\frac{2h}{g}}$ and the horizontal range = $R = uT = u\sqrt{\frac{2h}{g}}$

C8C Oblique Projection

Suppose a body is projected with initial velocity u at an angle θ with the horizontal.

(i) The equation of the trajectory of the projectile is $y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2$

which represents a parabola.

(ii) **Maximum Height** $H = \frac{u^2\sin^2\theta}{2g}$ (iii) **Time of Flight** $T = \frac{2u\sin\theta}{g}$

(iv) **Horizontal Range** $R = \frac{u^2\sin 2\theta}{g}$

Two important points to be noted concerning horizontal range R :

(i) For a given velocity of projection, R is maximum when $\theta = 45^0$.

(ii) For a given velocity, there are two angles of projection for which the range is the same.

If one of these angles is θ, the other is $\frac{\pi}{2}-\theta$.

Practice Problems :

1. The x and y coordinates of a particle at any time t are given by $x = 3t + 4t^2$ and $y = 4t$ where x and y are in m and t in s. Then

(a) The initial speed of the particle is 5 m/s.

(b) The acceleration of the particle is constant.

(c) The path of the particle is parabolic.

(d) All are correct

2. A particle is projected with speed u at an angle of θ with the horizontal. Another particle of different mass is projected with same speed from the same point. Both the particles has same horizontal range. Let the time of flight and maximum height attained by the first particle and second particle are t_1, h_1 and t_2, h_2 respectively. Then t_1/t_2 and h_1/h_2 are given by respectively

(a) $\tan\theta, \tan^2\theta$ **(b)** $\cot\theta, \cot^2\theta$ **(c)** $\cot\theta, \tan^2\theta$ **(d)** $\tan\theta, \cot^2\theta$

3. Let the maximum height attained by the projectile is n times the horizontal range. Then the angle of projection with the horizontal is given by

(a) $\tan^{-1}n$ **(b)** $\tan^{-1}2n$ **(c)** $\tan^{-1}3n$ **(d)** $\tan^{-1}4n$

4. Two projectiles are projected from the same point with the same speed but at different angles of projection. Neglect the air resistance. They land at the same point on the ground. Which of the following angle of projections is possible ?

(a) $\frac{\pi}{4}+\theta, \frac{\pi}{4}-\theta$ **(b)** $\frac{\pi}{3}+\theta, \frac{\pi}{6}-\theta$

(c) $\theta, \frac{\pi}{2}-\theta$ **(d)** **all are possible**

5. If $y = ax - bx^2$ is the path of a projectile, then which of the following is correct

(a) Range = a/b **(b) Maximum height = $a^2/4b$**

(c) Angle of projection = $\tan^{-1}a$ **(d) all are correct**

[Answers : (1) d (2) a (3) d (4) d (5) d]

C9 RELATIVE MOTION

The position, velocity and acceleration of a particle are relative terms and are defined with respect to certain frame of reference. This frame of reference may be stationary, moving with constant velocity or have some acceleration.

If x_{AB} is is position of A with respect to B then $x_{AB} = x_A - x_B$ where x_A and x_B are the position of A and B with respect to some common frame of reference. In the similar way for relative velocity $v_{AB} = v_A - v_B$. In vector form

$$\overrightarrow{r_{AB}} = \overrightarrow{r_A} - \overrightarrow{r_B}$$
$$\overrightarrow{v_{AB}} = \overrightarrow{v_A} - \overrightarrow{v_B}$$
$$\overrightarrow{a_{AB}} = \overrightarrow{a_A} - \overrightarrow{a_B}$$

where $\overrightarrow{r_{AB}}$ is the position of A with respect to B, $\overrightarrow{r_A}$ **and** $\overrightarrow{r_B}$ are the position of A and position of B with respect to some common frame of reference, $\overrightarrow{v_{AB}}$ is the velocity of A with respect to B, $\overrightarrow{v_A}$ **and** $\overrightarrow{v_B}$ are the velocity of A and velocity of B with respect to some common frame of reference, $\overrightarrow{a_{AB}}$ is the acceleration of A with respect to B, $\overrightarrow{a_A}$ **and** $\overrightarrow{a_B}$ are the acceleration of A and velocity of B with respect to some common frame of reference.

The equations of motion for constant relative acceleration are written as :

$x_{rel} = (x_0)_{rel} + u_{rel}(t - t_0) + ½\, a_{rel}(t - t_0)^2$

$v_{rel} = u_{rel} + a_{rel}\,(t - t_0)$

$$x = (x_0)_{rel} + \frac{u_{rel} + v_{rel}}{2}(t - t_0)$$

$v_{rel}^{\,2} = u_{rel}^{\,2} + 2a_{rel}\,(x_{rel} - (x_0)_{rel})$ In general, $v_{rel} = \frac{dx_{rel}}{dt},\ a_{rel} = \frac{dv_{rel}}{dt}$

<u>Practice Problems :</u>

1. Rain is falling with a speed of 4 m/s in a direction making an angle of 30^0 with vertical towards south. What should be the magnitude and direction of velocity of cyclist to hold his umbrella exactly vertical, so that rain does not wet him

(a) 2 m/s towards north (b) 4 m/s towards south

(c) 2 m/s towards south (d) 4 m/s towards north

2. A motorboat covers the distance between two stations on the river t_1 = 8h and t_2 = 12h downstream and upstream respectively. The time taken by the boat to cover this distance in still water is

(a) 9.6 h (b) 4.3 h (c) 2.2 h (d) 6.7 h

3. A lift moves with an acceleration a. A passenger in the lift drops a book. The acceleration of the book with respect to the lift floor if the lift is going up and if the lift is going down is respectively

(a) g + a, g – a (b) g – a, g + a (c) both (d) none

4. A railway carriage moves over a straight level track with an acceleration a. A passenger in the carriage drops a stone. The acceleration of the stone with respect to the carriage and the Earth are respectively

(a) $\sqrt{a^2 + g^2}$, g (b) g, $\sqrt{a^2 + g^2}$ (c) $\sqrt{a^2 - g^2}$, g (d) g, $\sqrt{a^2 - g^2}$

5. A balloon starts rising from the ground with an acceleration of 1.25 m/s^2. After 8 s, a stone is released from the balloon. The stone will

(a) cover a distance of 40 m (b) have a displacement of 50 m

(c) reach the ground in 4 sec (d) begin to move down after being released

6. A boat which has a speed of 5 km/h in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in km/h is

(a) 1 (b) 3 (c) 4 (d) √41

[Answers : (1) c (2) a (3) a (4) a (5) c (6) b]

C10 Circular Motion :

When an object follows a circular path at constant speed, the motion of the object is called uniform circular motion. The magnitude of its acceleration is $a_c = v^2/R$. The direction of a_c is always towards the centre of the circle. The angular speed ω, is the rate of change of angular distance. It is related to velocity v by v = ωR. The acceleration is $a_c = \omega^2 R$. If T is the time period of revolution of the object in circular motion and f is its frequency, we have ω = 2πf, v = 2πfR, $a_c = 4\pi^2 f^2 R$

<u>Practice Problems :</u>

1. A point moves in x-y plane according to the law x = 4 sin 6t and y = 4(1 – cos 6t). The distance traversed by the particle in 4 seconds is (x and y are in meters)

(a) 96 m (b) 48 m (c) 24 m (d) 108 m

2. The modulus of the acceleration vector is constant. The trajectory of the particle is a/an

(a) parabola (b) ellipse (c) hyperbola (d) circle

[Answers : (1) a (2) d]