Electrostatics

ELECTROSTATICS

C1 Properties of charges :

(i) Two kinds of charges exist in nature, positive and negative with the property that unlike charges attract each other and like charges repel each other.

(ii) Excess of electrons means negative charge and deficiency of charge means positive charge.

(iii) Charge is conserved for an isolated system.

(iv) Charge is quantized i.e. $q = \pm ne$ where n = 1, 2, 3..... and $e = 1.6 \times 10^{-19}$ C

(v) Charge is invariant.

(vi) On charging a neutral body, the mass of the body will change.

<u>Practice Problems</u> :

1. Consider a neutral body A and a charged body B on which –10μC charge exists. From the body B 6.25×10^{12} electrons are extracted and supplied to the body A. Then

(a) the charge on body A will become –1 μC

(b) the charge on body B will become –9 μC

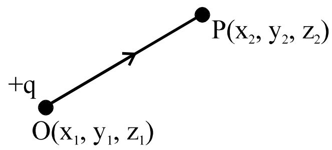
(c) the mass of body A will increase

(d) the mass of body B will be more than the mass when it is neutral

<u>Answers :</u> [(1) a, b, c, d]

C2 Electric field : It is a vector quantity. The magnitude of electric field due to a point charge q at the distance r is given by $\frac{\mathbf{kq}}{\mathbf{r^2}}$. It is defined for a point charge as follows :

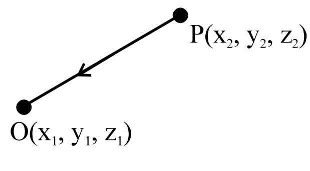
(a) Due to positive charge



$$\vec{E} = \frac{kq}{(OP)^2}\frac{\overrightarrow{OP}}{|\overrightarrow{OP}|}, \quad \overrightarrow{\mathbf{OP}} = (\mathbf{x_2 - x_1})\hat{\mathbf{i}} + (\mathbf{y_2 - y_1})\hat{\mathbf{j}} + (\mathbf{z_2 - z_1})\hat{\mathbf{k}}$$

$$|\overrightarrow{\mathbf{OP}}| = \sqrt{(\mathbf{x_2 - x_1})^2 + (\mathbf{y_2 - y_1})^2 + (\mathbf{z_2 - z_1})^2}$$

(b) Due to negative charge



$$\vec{\mathbf{E}} = \frac{\mathbf{kq}}{(\mathbf{OP})^2}\frac{\overrightarrow{\mathbf{PO}}}{|\overrightarrow{\mathbf{PO}}|}, \quad \overrightarrow{\mathbf{PO}} = -\overrightarrow{\mathbf{OP}}$$

Here $k = \frac{1}{4\pi \in_0 \in_r}$ where $\in_0$ is known as absolute permittivity constant and $\in_r$ is known as relative permittivity or dielectric constant of the medium. The value of $\in_0 = 8.85 \times 10^{-12}$ C^2/Nm^2 and $k = 9 \times 10^9$ Nm^2/C^2

Principle of superposition : The electric field due to a group of charges can be obtained using the superposition principle. That is, the total electric field equals the vector sum of the electric fields of all the charges at some point :

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3....$$

<u>Practice Problems :</u>

1. **A positive charge is placed at the origin. The minimum electric field produced at the point (3, 4, 0) is given by**

(a) $2.6(-3\hat{i}+4\hat{j})\times 10^{-11}\,N/C$ (b) $2.6(-3\hat{i}-4\hat{j})\times 10^{-11}\,N/C$

(c) $2.6(3\hat{i}-4\hat{j})\times 10^{-11}\,N/C$ (d) $2.6(3\hat{i}+4\hat{j})\times 10^{-11}\,N/C$

2. **Five point charges, each of value + q, are placed on five vertices of a regular hexagon of side L meters. The magnitude of the force on a point charge of value – q placed at the centre of the hexagon is**

(a) $5kq^2/L^2$ (b) 0 (c) kq^2/L^2 (d) $3kq^2/L^2$

3. **(n – 1) number of equal charges, each of charge q, are placed at the corners of a regular polygon of side length a. The net electric field at the centre of the polygon is**

(a) $\dfrac{4kq\sin^2\frac{\pi}{n}}{a^2}$ (b) $\dfrac{4kq\cos^2\frac{\pi}{n}}{a^2}$ (c) $\dfrac{4kq\tan^2\frac{\pi}{n}}{a^2}$ (d) $\dfrac{4kq\cot^2\frac{\pi}{n}}{a^2}$

4. **Two charges of equal values (each has the value q) are placed distance d apart. The maximum value of electric field on the perpendicular bisector of the line joining them is**

(a) $\dfrac{16kq}{3^{3/2}d^2}$ (b) $\dfrac{8kq}{3^{3/2}d^2}$ (c) $\dfrac{4kq}{3^{3/2}d^2}$ (d) $\dfrac{2kq}{3^{3/2}d^2}$

[Answers : (1) d (2) c (3) a (4) a]

C3 Electric field for continuous charge distribution.

Apply the following steps to calculate the electric field at the concerned point due to continuous charge distribution.

(a) Take the differential element and calculate the charge on this element.

For line charge distribution, $dq = \lambda dl$, where λ is the charge per unit length.

For surface charged distribution = σdA, where σ is the charge per unit area.

For volume charge distribution $dq = \rho dV$, where ρ is the charge per unit volume.

(b) Calculate the electric field at the concerned point due to differential element $(d\vec{E})$

(c) Take the components of $d\vec{E}$, if possible

(d) Integrate with proper limits.

<u>Practice Problem :</u>

1. **Find the electric field due to following configuration : (i) due to uniformly charged rod of charged per unit length λ and length L at a distance x from one of the end of the rod along the length of the rod (ii) due to uniformly charged rod of charged per unit length λ and length L at a distance x from mid-point of the rod along the perpendicular bisector (iii) due to a ring of radius r with uniform charge distribution Q on the axis at the distance x from the center (iv) due to uniformly charged disc of radius r with surface charge density σ on the axis of the disc at the distance x from the center (v) due to a long thin wire of uniform linear charge density λ at perpendicular bisector at the distance r from the wire.**

C4 Coulomb Force: $\vec{F} = q_0\vec{E}$, where q_0 is the test charge placed in the external electric field $\vec{E}$. Column Force is central, inverse square and conservative field. Coulamb's force is valid for distances from 10^{-13} cm to several kilometers. i.e. up to infinity.

C5 Motion of charged particles in electric field

When a particle of charge q and mass m is placed in an electric field $\vec{E}$ then the force on the charge particles is $q\vec{E}$ and according to Newton's second law $\vec{F} = q\vec{E} = m\vec{a}$ $\Rightarrow \vec{a} = \frac{q\vec{E}}{m}$ where $\vec{a}$ is the acceleration of the particle. If the particle is released from rest in uniform $\vec{E}$ or projected with certain speed along the direction of uniform $\vec{E}$ or opposite to the direction of uniform $\vec{E}$ then the path is straight line otherwise the path is parabolic in uniform $\vec{E}$.

Practice Problems :

1. A liquid drop of mass m and carrying n electrons can be balanced by applying an electric field. The direction and magnitude of the electric field is

(a) upward, $\frac{mg}{ne}$ **(b)** downward, $\frac{mg}{ne}$

(c) upward, $\frac{2mg}{ne}$ **(d)** downward, $\frac{2mg}{ne}$

2. Which of the following path of a charge particle is possible in uniform electric field ?

(a) Straight path **(b)** Parabolic path

(c) Circular path **(d)** Hyperbolic path

[Answers : (1) b (2) a, b]

C6 Electric Field Lines : A convenient specialized pictorial representation for visualizing electric field patterns is created by drawing lines showing the direction of the electric field vector at any point. These lines, called electric field lines, are related to the electric field in any region of space in the following manner :

- The electric field vector E is tangent to the electric field line at each point.
- The number of electric field lines per unit area through a surface that is perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, E is large where the field lines are close together and small where they are far apart.

For a positive point charge, the lines are directed radially outward and for a negative point charge, the lines are directed radially inward. The number of lines that originate from or terminate on a charge is proportional to the magnitude of the charge.

Practice Problems :

1. Draw the pattern of electric lines of forces for the following :

(a) positively charged spherical conductor

(b) a negative charge placed at the centre of a spherical conducting shell

(c) a positive charge placed at any point inside a hollow conducting sphere

---

C7 Electric flux : Electric flux is proportional to the number of electric field lines that penetrate a surface. The electric flux through a surface is defined by the expression $\phi = \int \vec{E}.d\vec{A}$ If $\vec{E}$ is surface uniform then $\phi = \vec{E}.\vec{A} = EA\cos\theta$ where θ is the angle that the electric field makes with the normal to the surface.

C8 **Gauss's Law :** Gauss's Law says that the net electric flux Φ_E through any closed gaussian surface is equal to the net charge inside the surface divided by $\in_0$: $\Phi_E = \oint E.dA = \frac{q_{in}}{\in_0}$. Using Gauss's law, one can calculate the electric field due to various symmetric charge distributions.

C9 **Application of Gauss's Law to symmetric charge distributions**

Gauss's Law is useful in determining electric fields when the charge distribution has a high degree of symmetry. The surface should always be chosen to take advantage of the symmetry of the charge distribution, so that we can remove E from the integral and solve for it. The goal in this type of calculation is to determine a surface that satisfies one or more of the following conditions :

1. The value of the electric field can be argued by symmetry to be constant over the surface.
2. The dot product can be expressed as a simple algebraic product E dA because **E** and d**A** are parallel.
3. The dot product is zero because **E** and d**A** are perpendicular.
4. The field can be argued to be zero everywhere on the surface.

<u>Practice Problems :</u>

1. The electric charge is placed at the centre of a cube of side a. The electric flux through one of its faces will be

(a) $\frac{q}{6\in_0}$ **(b)** $\frac{q}{\in_0}$ **(c)** $\frac{q}{4\pi\in_0}$ **(d)** $\frac{2q}{\in_0}$

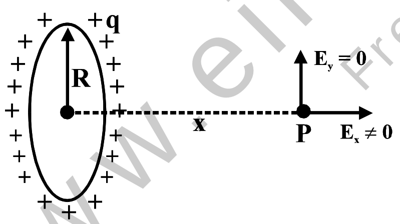
2. Electric charge 'q' is placed at the corner of cube. The electric flux through the opposite face of the cube is

(a) $\frac{q}{8\in_0}$ **(b)** $\frac{q}{12\in_0}$ **(c)** $\frac{q}{16\in_0}$ **(d)** $\frac{q}{24\in_0}$

[Answers : (1) a (2) d]

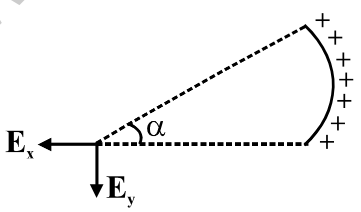
C10 **Electric Field due to special type of configuration :**

1. A Ring of Charge



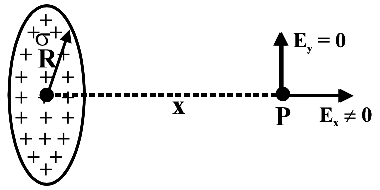
$$E_y = 0,\ E_x = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$$

2. At the Centre due to sector of a ring



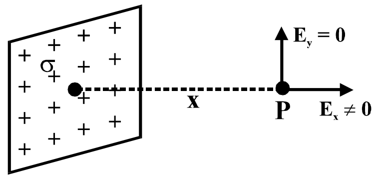
$$E_x = \frac{\lambda}{4\pi\varepsilon_0 R}\sin\alpha\ ,\ E_y = \frac{\lambda}{4\pi\varepsilon_0 R}(1-\cos\alpha)$$

3. A Disc of Charge



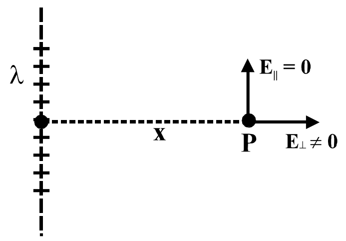
$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

4. Infinite Sheet of Charge



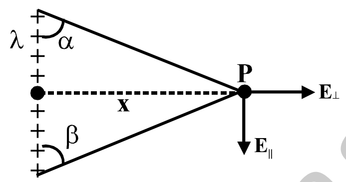
$E_x = \frac{\sigma}{2\varepsilon_0}$, **where σ is the surface charge density**

5. Infinitely Long Line of Charge



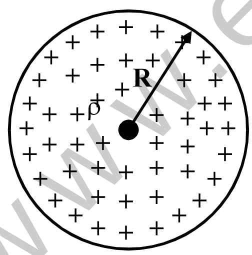
$E_{\parallel} = 0, E_{\perp} = \frac{\lambda}{2\pi\varepsilon_0 x}$, **where λ is the linear charge density**

6. Finite Line of Charge



$$E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0 x}[\cos\alpha + \cos\beta],\ E_{\parallel} = \frac{\lambda}{4\pi\varepsilon_0 x}\left|\sin\beta - \sin\alpha\right|$$

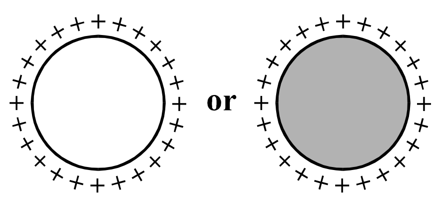
7. A Spherical Volume Charge Distribution (non-conducting solid sphere)



$$E = \frac{\rho r}{3\varepsilon_0}\ r < R,\ E = \frac{q}{4\pi\varepsilon_0 r^2}\ r \geq R\left(q = \rho\frac{4}{3}\pi R^3\right)$$

ρ is volume charge density

8. Spherical Conductor (Hollow or Solid)



E = 0 r < R, $E = \frac{q}{4\pi\varepsilon_0 r^2}\ r \geq R$

Practice Problems :

1. **A sphere of radius R has a uniform volume charge distribution. At a distance x from its centre, for x < R, the electric field is directly proportional to**

   (a) $1/x^2$ **(b)** $1/x$ **(c)** x **(d)** x^2

2. **Consider two non-conducting solid spheres of uniform volume charge distribution with some part overlaped with each other. One of the sphere has positive charge and another has negative charge but the magnitude of volume charge density are equal. The electric field in the overlaped part is**

   (a) uniform everywhere
   (b) non-uniform
   (c) directed from centre to centre of the spheres
   (d) none

3. **The electric field inside the cylinderical cavity of a non-conducting uniform volume cylinderical charged distribution is**

   (a) zero everywhere **(b)** uniform everywhere
   (c) non-uniform everywhere **(d)** none

4. **A thin tunnel is made along the diameter of uniform spherical volume negative charged distribution of charged density ρ. A positive charge q and mass m is released inside the tunnel. If the charge performes SHM then the time period of this charge particle is**

   (a) $2\pi\sqrt{\dfrac{3m\varepsilon_0}{q\rho}}$ **(b)** $2\pi\sqrt{\dfrac{3m\varepsilon_0}{2q\rho}}$ **(c)** $\pi\sqrt{\dfrac{3m\varepsilon_0}{2q\rho}}$ **(d)** $\pi\sqrt{\dfrac{m\varepsilon_0}{2q\rho}}$

[Answers : (1) c (2) a, c (3) a, b (4) a]

C11 Electric Potential Energy : The electric force caused by any collection of charges at rest is a conservative force. The work W done by the electric force on a charged particle moving in an electric field can be represented by a potential-energy function U : $W_{a \to b} = U_a - U_b$.

C12 Electric Potential Energy of two point charges : The potential energy for two point charges q and q_0 separated by a distance r is $U = \dfrac{1}{4\pi \in_0} \dfrac{qq_0}{r}$

C13 Electric potential energy of a point charge in the electric field of several charges : The potential energy for a charge q_0 in the electric field of a collection of charges q_i is given by

$$U = \frac{q_0}{4\pi \in_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ...\right) = \frac{q_0}{4\pi \in_0}\sum_i \frac{q_i}{r_i}$$ where r_i is the distance from q_i to q_0.

C14 Total potential energy of several charges : The total potential energy U is the sum of the potential energies of intersection for each pair of charges. We can write this as $U = \dfrac{1}{4\pi \in_0}\sum_{i<j} \dfrac{q_i q_j}{r_{ij}}$

Practice Problems :

1. **Four charges +q, –q, +q and –q are placed at the corners A, B, C and D respectively of a square of side a. The potential energy of the system is**

   (a) $k\dfrac{q^2}{a}(-4+\sqrt{2})$ **(b)** $k\dfrac{q^2}{2a}(-4+\sqrt{2})$
   (c) $k\dfrac{4q^2}{a}$ **(d)** $-k\dfrac{4\sqrt{2}q^2}{a}$

2. **A positive charge q is placed at the centre of the circle of radius r. Another negative charge q_0 and mass m is moving with certain speed around this circular path. Then**

(a) **the speed of the charged particle is** $\sqrt{\frac{kqq_0}{mr}}$

(b) **the total energy of the charged particle is** $\frac{-kqq_0}{2r}$

(c) **the linear momentum of charged particle is variable**

(d) **the angular momentum of charged particle about the centre of the circle remains constant**

3. The closest distance of approach of an α**-particle (released from very large distance) of energy E from a fixed nucleus of atomic number Z.**

(a) $\frac{2kZe^2}{E}$ (b) $\frac{kZe^2}{E}$ (c) $\frac{kZe^2}{2E}$ (d) $\frac{4kZe^2}{E}$

[Answers : (1) a (2) a, b, c, d (3) a]

C15 Energy is an electric field : This energy is stored in the electric field generated by the charges. i.e. in the space where the electric field exists and it is found that the energy stored in the field per unit volume is given by $\frac{1}{2}\varepsilon_0 E^2$.

Practice Problems :

1. **Consider a spherical charged conductor of total charge q and radius r. Then**

(a) **the energy contained inside the conductor is zero**

(b) **the total energy stored in the space is** $\frac{kq^2}{2r}$

(c) **fraction of the energy stored in the spherical shell of inner radius r and outer radius 2r is 0.5**

(d) **all the above**

2. **Consider a spherical uniform volume charged distribution of total charge q and radius r. Then**

(a) **the energy contained inside the sphere is** $\frac{kq^2}{10r}$

(b) **the fraction of energy stored outside the sphere is** $\frac{5}{6}$

(c) **fraction of the energy stored in the spherical shell of inner radius r and outer radius 2r is 0.5**

(d) **all the above**

[Answers : (1) d (2) a, b]

C16 Electrical potential Electric potential, a scalar quantity, is the potential energy per unit charge. Mathematically, Potential, $V = \frac{U}{q_0}$ or $U = q_0V$. The unit of potential is volt (V) or J/c.

Work done by external source to move a charge (q_0) very slowly from initial point to final point in an electric field $W = q_0(V_f - V_i)$ where V_f is the potential due to charge distribution at point and V_i is the find potential due to charge distribution at initial point.

C17 The potential V due to a point charge q at distance r, $V = \frac{kq}{r}$

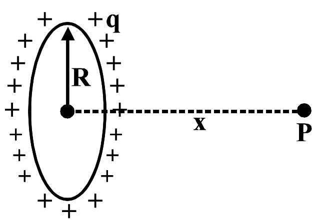
C18 Potential due to collection of point charges, $V = k\sum_i \frac{q_i}{r_i}$, where r_i is the distance of charge q_i at the point where potential will be calculated.

C19 **Potential due to continuous charge distribution**

For a given continuous charge distribution, define a differential element of charge, find its potential at the concerned point, and by using the following equation find the potential of all the charge elements, $V = \int \frac{kdq}{r}$. Carry out integration, using appropriate limits to include the entire charge distribution. In the integral be careful about which geometric quantities vary and which are constant.

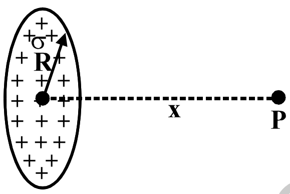
Electric potential due to special type of charge distribution

1. **A Ring of Charge**



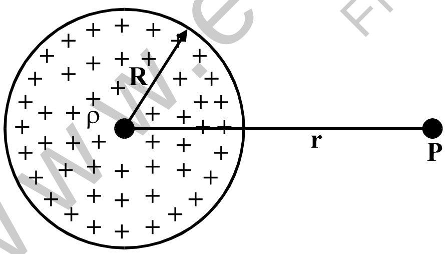
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2 + x^2}}$$

2. **A Disc of Charge**



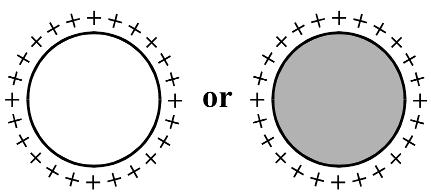
$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2 + x^2} - x\right]$$

3. **A sphere of Charge**



$$V = \frac{3Q}{8\pi\varepsilon_0 R} - \frac{Qr^2}{8\pi\varepsilon_0 R^3}\ r < R,\ V = \frac{Q}{4\pi\varepsilon_0 r}\ r \geq R$$

4. **Conducting Sphere (Hollow or Solid)**



$$V = \frac{Q}{4\pi\varepsilon_0 R}\ r < R,\ V = \frac{Q}{4\pi\varepsilon_0 r}\ r \geq R$$

Practice Problems :

1. **An α-particle and a proton are accelerated through the same potential difference. The ratio of speed acquired by the two particles.**

(a) $1 : \sqrt{2}$ **(b)** $1 : 2$ **(c)** $\sqrt{2} : 1$ **(d)** $2 : 1$

2. **Two point charges +4μC and –6μC are separated by a distance of 20 cm in air. The number of points on the line joining the two charges at which the electric potential zero**

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

3. **The work done to carry a charge particle of +10nC from infinity to the center of the square of side $\sqrt{2}$ m, which carries at its four corners charges of +2 nC, + 1nC, –2 nC and –3 nC respectively.**

(a) –60 nJ **(b)** –120 nJ **(c)** –180 nJ **(d)** –200 nJ

4. **A cube of side b has a charge q at each of its vertices. The potential due to this charge array at the centre of the cube.is**

(a) $4q/\sqrt{3}\,\pi\in_0 b$ **(b)** $2q/\sqrt{3}\,\pi\in_0 b$ **(c)** $q/\sqrt{3}\,\pi\in_0 b$ **(d)** $6q/\sqrt{3}\,\pi\in_0 b$

[Answers : (1) a (2) b (3) c (4) a

C20 Relation between electric field and electric potential : Relation between field and potential is given by

$\mathbf{dV = -\vec{E}.d\vec{r}}$ integrating between points a and b, $\int_a^b dV = V_b - V_a = -\int_a^b \vec{E}.d\vec{r}$ where V_a and V_b are the potentials at a and b. In differential form, we have $\vec{E} = -\dfrac{dV}{dr}\hat{r}$

Practice Problems :

1. **Find the electrostatic potential due to :**

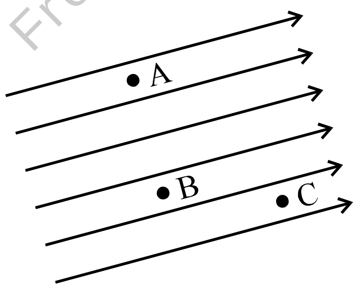
(a) **uniformly charged hollow sphere of radius R inside and outside**

(b) **spherical conductor of radius R inside and outside**

(c) **non-conducting sphere of radius R with uniform volume charge distribution inside and outside**

In each case the total charge is q. Also draw the variation of potential with distance in each case.

2. **Consider points A, B and C in a uniform electrostatic field as shown. Then the potential is minimum at**

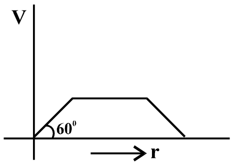


(a) A **(b)** B **(c)** C **(d)** none

3. **In a space the electric potential is changing with distance r according to $V = kr^2$. The electric field has the variation with distance as**

(a) linear **(b)** constant **(c)** parabolic **(d)** hyperbolic

4. The electrostatic potential V is changing with distance r according to the following graph. Then



(a) The electric field with distance r remains constant

(b) The electric field with distance r continuously increasing

(c) For some distance the electric field will be constant, for some distance it will be zero and then it will become constant

(d) The electric field will change its direction

5. A metal sphere with a charge Q is surrounded by an uncharged concentric thin spherical shell. The potential difference between them is V. If the shell is now given an additional charge Q, what is the new potential difference between them ?

(a) V (b) V/2 (c) zero (d) 2V

[Answers : (2) c (3) a (4) c, d (5) a]

C21 Equipotential Surface : There is another way to demonstrate the graphical representation of field using the concept of Equipotential Surfaces. An equipotential surface is three dimensional surfaces on which the potential has the same value at every point. At a point where a field line crosses an equipotential surface the two are perpendicular.

Note the following points :

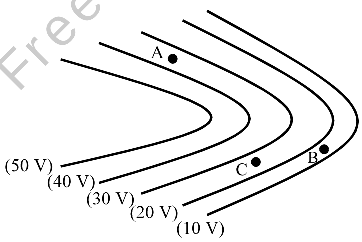
(a) the field is stronger where the equipotential surfaces are closely spaced.

(b) the work done to move a charge on a equipotential surface is zero.

(c) the work done to move a charge q_0 from one equipotential surface (having potential, (V_1) to another equipotential surface (having potential V_2) is q_0 ($V_2 - V_1$).

<u>Practice Problems :</u>

1. The figure shows lines of constant potential in a region in which an electric field is present. The values of the potential are written in brackets. Of the points A, B and C, the magnitude of the electric field is greatest at the point is



(a) A (b) B (c) C (d) equal at all points

[Answers : (1) b]

C22 Conductors and insulators : In conductors charges are free to move throughout the volume of such bodies whereas in case of insulators or dielectrics, the charges remain fixed at the places where they were initially distributed. Hence charge given to a conductor always resides on its surface.

In conductors electric charges are free to move throughout the volume of such bodies. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.

The following are the properties of a conductor in electrostatic equilibrium.

1. The electric field is zero everywhere inside the conductor (E = 0)
2. No volume charges exist inside a conductor

3. The surface of a conductor is an equipotential surface and lines of forces always meet a conducting surface normally.
4. Charge density is inversely proportional to radius of curvature.
5. If there is cavity inside a conductor, the field strength inside the cavity equals zero, whatever is the field outside the conductor.
6. The field intensity near a conducting surface is always $\mathbf{E} = \frac{\sigma}{\varepsilon_0}$, where σ is the local surface charge density at that point.
7. **Redistribution of Charge :** If two conductors are brought into contact, the charges from one of them will flow over to the other until their potentials become equal. The equality of potential implies that charges on each sphere (as shown) is proportional to its radius. i.e., $\frac{q_1}{R_1} = \frac{q_2}{R_2}$. For a uniform surface charge density σ, the total charge $q = 4\pi R^2\sigma$, so the above equation becomes $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$.
8. **Potential of concentric conducting spheres : Superposition Principle :** Let us consider two concentric spheres of radii r_1 and r_2 with uniformly distributed charges q_1 and q_2. Using the principle of superposition, the potential of the small and large sphere may be written as $V_1 = \frac{k_1q_1}{r_1} + \frac{kq_2}{r_2}$, $V_2 = \frac{kq_1}{r_2} + \frac{kq_2}{r_2}$
9. **Force acting on a conducting surfae :** The force d**F** acting on a small element of area **dS** of the conductor where the local charge density is σ, is given by **dF** = $\frac{\sigma^2}{2\varepsilon_0}$ **dS**

<u>**Practice Problems :**</u>

1. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres ? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

2. Consider two concentric spheres of radii r and 2r with same surface charged density σ. Find

(a) total charge on each sphere

(b) potential at the common centre

(c) potential at any point inside the first sphere

(d) potential of first sphere

(e) potential at any point between the two spheres at the distance r' from the center

(f) potential of outer sphere

(g) potential at any point outside at the distance r' from the center

3. How the charge can be transferred from one sphere to another sphere completely ?

4. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surface of the shell ?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape ? Explain.

[Answers : (1) b/a (2) (a) $4\pi\sigma r^2$, $16\pi\sigma r^2$ (b) $12\pi k\sigma r$ (c) $12\pi k\sigma r$ (d) $12\pi k\sigma r$ (e) $\frac{4\pi k\sigma r}{r'} + 8\pi k\sigma r$ (f) $10\pi k\sigma r$ (g) $\frac{20\pi k\sigma r^2}{r'}$ (4) (a) $-q/(4\pi r_1^2)$, $(Q + q)/(4\pi r_2^2)$]

C23 **Electric Dipole :** An electric dipole is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge –q) separated by a distance d. The characteristic of a dipole is its dipole moment defined as $\vec{p} = q\vec{d}$, the direction is from negative charge to positive charge.

Practice Problems :

1. **Electric charges q, q and –2q are placed at the corners of an isoseceles right angle triangle of total perimeter (2 + √2)*l*. Equal charges are placed at the corners of largest length. The magnitude of the electric dipole moment of the system is**

(a) **q*l*** **(b)** **√2 q*l*** **(c)** **√3 q*l*** **(d)** **4 q*l***

[Answers : (1) b]

C24 **Electric field due to a dipole**

(i) **Along the Axis :** The electric field intensity along the axis always point in the direction of the dipole. It magnitude is given by $E_{\parallel} = \frac{2kp}{x^3}$.

(ii) **Along the Bisector :** The direction of electric field along the bisector is always opposite to the dipole moment. Math emetically, $E_{\perp} = -\frac{kp}{y^3}$

C25 **Electric potential due to a dipole**

(i) **Along the Axis :** The electric potential along the x-axis is given by $V_{\parallel} = \frac{2kp}{x^2}$

(ii) **Along the Bisector :** The electric potential along the bisector is always zero.

Practice Problems :

1. **A given charge situated at a certain distance from a short electric dipole in the end-on position experiences a force F_1. If the distance of the charge from the dipole is doubled then the force acting on the charge is F_2. Then F_1/F_2 equals to**

(a) **2** **(b)** **1/2** **(c)** **1/4** **(d)** **8**

2. **If E_a be the electric field intensity due to a short dipole at a point on the axis and E_r be that on the right bisector at the same distance from the dipole, then**

(a) $E_a = E_r$ **(b)** $E_a = 2E_r$ **(c)** $E_r = 2E_a$ **(d)** $E_a = \sqrt{2}\,E_r$

3. **A negative charge –q and mass m is released at the certain distance d from a short dipole of dipole moment p on the axis. Find**

(a) **initial acceleration** **(b)** **speed when the distance will become half**

4. **A charged particle of charge –q and mass m is projected with certain speed u from a point on the perpendicular bisector of the dipole of dipole moment p. After certain time it passes through the point lying on the axis of the dipole at the distance x from the dipole. Find the speed at this moment.**

[Answers : (1) d (2) b (3) (a) $\frac{2kqp}{md^3}$ (b) $\sqrt{\frac{6kqp}{md^2}}$ (4) $\sqrt{u^2 + \frac{2kqp}{mx^2}}$]

C26 **Dipole in an external electric field :**

1. The net force experienced by a dipole in an external uniform electric field is zero.

2. When an electric dipole of dipole moment $\vec{p}$ is placed in an electric field $\vec{E}$, the field exerts a torque $\vec{\tau}$ on the dipole : $\vec{\tau} = \vec{p} \times \vec{E}$.

3. The dipole has a potential energy U associated with its orientation in the field : $U = -\vec{p}.\vec{E}$. This potential energy is defined to be zero when $\vec{p}$ is perpendicular to $\vec{E}$; it is least (U = –pE) when $\vec{p}$ is aligned with $\vec{E}$, and most (U = pE) when $\vec{p}$ is directed opposite $\vec{E}$. Hence a dipole is in stable equilibrium when $\vec{p}$ is aligned with $\vec{E}$.

4. The net force on the dipole in the non-uniform field is non-zero and calculated by $\mathbf{F = -\frac{dU}{dx}}$.

<u>**Practice Problems :**</u>

1. The potential energy of an electric dipole in a uniform electric field is U. The magnitude of the torque acting on the dipole due to the field is N. Then

(a) U is minimum and N is zero when the dipole is parallel to the field.

(b) U is zero and N is zero when the dipole is perpendicular to the field.

(c) U is minimum and N is maximum when the dipole is perpendicular to the field

(d) U is minimum and N is zero when the dipole is anti-parallel to the field.

2. A dipole is placed in the field of infinite sheet of uniform charge density. Which of the following quantity must be zero ?

(a) Force **(b) Potential Energy**

(c) Torque **(d) All the above**

3. A short dipole is placed along the axis of a circular uniformly charged ring very near to the centre. Then the force acting on the dipole is

(a) constant

(b) directly proportional to distance of the dipole from centre

(c) zero

(d) inversely proportional to distance of the dipole from centre

[Answers : (1) a (2) a (3) a]