# MOTION IN A PLANE

## 4.2 Scalars and Vectors :

**Q. Define Scalars and Vectors ? What is the difference between them ?**

**Solution :** A scalar quantity is a quantity with magnitude only. It is specified completely by a single number, along with the proper unit. Examples are : the distance between two points, mass of an object, the temperature of a body and the time at which a certain event happened. The rules for combining scalars are the rules of ordinary algebra. Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers.

A vector quantity is a quantity that has both a magnitude and a direction and obeys the triangle law of addition or equivalently the parallelogram law of addition. So, a vector is specified by giving its magnitude by a number and its direction. Some physical quantities that are represented by vectors are displacement, velocity, acceleration and force.

**Q. Although time and current has direction but they are scalars. Why ?**

**Solution :** Although time and current has direction but they are scalars because they are added, subtracted, multiplied and divided as the ordinary numbers.

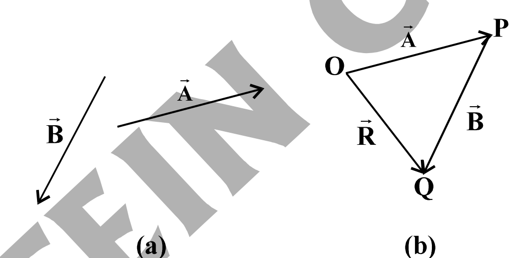
**Q. What is equality of vectors ?**

**Solution :** Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are said to be equal if, and only if, they have the same magnitude and the same direction.

## 4.4 Addition and Subtraction of Vectors – Graphical Method :

**Q. Explain triangle law of vector addition or head-to-tail method of vector addition.**

**Solution :**




Let us consider two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ that lie in a plane as shown in figure. The lengths of the line segment representing these vectors are proportional to the magnitude of the vectors. To find the sum $\vec{\mathbf{A}}+\vec{\mathbf{B}}$, we place vector $\vec{\mathbf{B}}$ so that its tail is at the head of the vector $\vec{\mathbf{A}}$, as in figure. Then, we join the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$. This line OQ represents a vector $\vec{\mathbf{R}}$, that is the sum of the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since, in this procedure of vector addition, vectors are arranged head to tail, this graphical method is called the head-to-tail method. The two vectors and their resultant form three sides of a triangle, so this method is also known as triangle method of vector addition.

**Q. Write down the properties of vector addition.**

**Solution :** (i) Vector addition is commutative : $\vec{\mathbf{A}}+\vec{\mathbf{B}}=\vec{\mathbf{B}}+\vec{\mathbf{A}}$. (ii) The addition of vectors also obeys the associative law.

**Q. What is null vector and what is the direction of null vector ?**

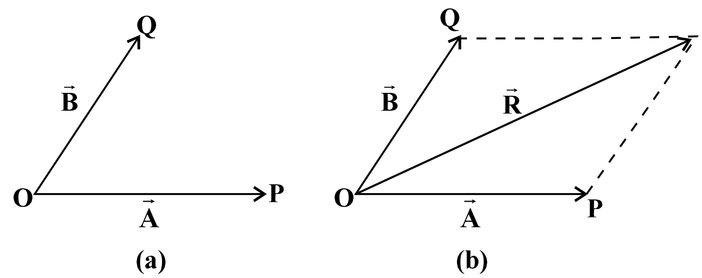
**Solution :** The vector which has magnitude equals to zero is known as null vector or zero vector. Its direction cannot be specified.

**Q. What is the result of adding two equal and opposite vectors ?**

**Solution :** Their sum is null vector

**Q. Discuss parallelogram method of vector addition ?**

**Solution :**




Parallelogram method is used to find the sum of two vectors. Suppose we have two vectors $\vec{A}$ and $\vec{B}$. To add these vectors, we bring their tails to a common origin O as shown in figure. Then we draw a line from the head of $\vec{A}$ parallel to $\vec{B}$ and another line from the head of $\vec{B}$ parallel to $\vec{A}$ to complete a parallelogram OQSP. Now we join the point of the intersection of these two lines to the origin O. The resultant vector $\vec{R}$ is directed from the common origin O along the diagonal (OS) of the parallelogram.

**Q. Rain is falling vertically with a speed of 35 m s$^{-1}$. Winds starts blowing after sometime with a speed of 12 m s$^{-1}$ in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella ? [NCERT Solved Example 4.1]**

**Solution :** $19^0$ with the vertical towards the east.

**4.5 Resolution of Vectors :**

**Q. What is unit vector ? Does unit vector have dimensions and units ?**

**Solution :** A unit vector is a vector of unit magnitude and points in a particular direction. It has no dimensions and units. It is used to specify a direction only.

**Q. What is the value of $|\hat{i}|+|\hat{j}|+|\hat{k}|$ and $|\hat{i}+\hat{j}+\hat{k}|$ ?**

**Solution :** 3, √3

**Q. Consider a vector $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$. What is the magnitude of vector $\vec{A}$ ?**

**Solution :** $\sqrt{A_x^2 + A_y^2 + A_z^2}$.

**Q. Find the magnitude and direction of the resultant of two vectors $\vec{A}$ and $\vec{B}$ in terms of their magnitudes and angle θ between them. [NCERT Solved Example 4.2]. What is Law of cosines and Law of sines ?**

**Q. A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of $60^0$ east of south. Find the resultant velocity of the boat. [NCERT Solved Example 4.3]**

**Solution :** The speed of boat is 22 km/h in the direction of $23.4^0$ east of north.

**4.7 Motion in a Plane :**

**Q. The position of a particle is given by $r = 3.0t\,\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$ where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres. (a) Find $\vec{v}(t)$ and $\vec{a}(t)$ of the particle. (b) Find the magnitude and direction of $\vec{v}(t)$ at t = 1.0 s. [NCERT Solved Example 4.4]**

**Solution : (a) $\vec{v} = 3.0\hat{i} + 4.0t\hat{j}, \vec{a} = 4.0\,\hat{j}$ (b) 5m/s at an angle of $53^0$ with x-axis.**

**4.8 Motion in a Plane with Constant Acceleration :**

**Q. A particle starts from origin at t = 0 with a velocity $5.0\hat{i}\,m/s$ and moves in x-y plane under action of a force which produces a constant acceleration of $(3.0\hat{i} + 2.0\hat{j})m/s^2$. (a) What is the y-coordinate of the particle at the instant when its x-coordinate is 84 m ? (b) What is the speed of the particle at this time ? [NCERT Solved Example 4.5]**

**Solution :** (a) 36.0m (b) 26 $ms^{-1}$

**4.9 Relative Velocity in Two Dimension :**

**Q. Rain is falling vertically with a speed of 35 m $s^{-1}$. A woman rides a bicycle with a speed of 12 m $s^{-1}$ in east to west direction. What is the direction in which she should hold her umbrella ? [NCERT Solved Example 4.6]**

**Solution :** The women should hold her umbrella at an angle of about $19^0$ with the vertical towards the west

**4.10 Projectile Motion :**

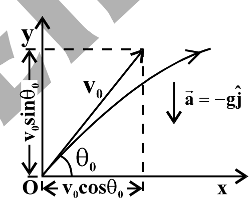
**Q. What is projectile ?**

**Solution :** An object that is in flight after being thrown or projected is called a projectile. Such a projectile might be a football, a cricket ball, a baseball or any other object.

**Q. A particle or an object (projectile) is projected with speed $v_0$ at an angle of $\theta_0$ with the horizontal in the vertical plane. (a) Find the position of the particle at any time t ? (b) Find the velocity of the particle at any time t ? (c) What is the velocity at maximum height ? (d) Find the equation of path and hence prove that the path of the projectile is a parabola ? (e) Find the time to reach maximum height ? (f) Calculate the total time of flight ? (g) Find the maximum height of projectile ? (h) Define horizontal range and find this ? Also write down the assumptions.**

**Solution :** The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.

Assumptions : we shall assume that the air resistance has negligible effect on the motion of the projectile. Suppose that the projectile is launched with velocity $v_0$ that makes an angle $\theta_0$ with the x-axis as shown.




After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward :

$$\vec{a} = -g\hat{j} \text{ or } a_x = 0,\ a_y = -g$$

The components of initial velocity $\vec{v}_0$ are : $v_{ox} = v_0\cos\theta_0$, $v_{oy} = v_0\sin\theta_0$

If we take the initial position to be the origin of the reference frame as shown, we have : $x_0 = 0$, $y_0 = 0$

(a) position of projectile at any time 't' :

from $x = x_0 + v_{ox}t + \frac{1}{2}a_x t^2$, we have $x = v_{ox}t = (v_0 \cos \theta_0)\, t$ ...(i)

from $y = y_0 + v_{oy}t + \frac{1}{2}a_y t^2$, we have $y = (v_0 \sin \theta_0)\, t - (½)\, g\, t^2$ ...(ii)

(b) velocity at time t can be obtained from $v_x = v_{ox} + a_x t$, $v_y = v_{oy} + a_y t$

$v_x = v_{ox} = v_0 \cos \theta_0$

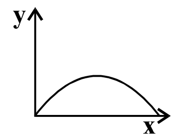
$v_y = v \sin \theta_0 - g\, t$

One of the components of velocity, i.e., x-component remains constant throughout the motion and only the y- component changes, like an object in free fall in vertical direction.

(c) Note that at the point of maximum height, $v_y = 0$ and therefore velocity at the maximum height equals to $v_0 \cos \theta_0$.

(d) equation of path of a projectile

This can be seen by eliminating the time between the expressions for x and y as given in equation (i) and (ii), we obtain : $\mathbf{y = (\tan\theta_0)x - \dfrac{g}{2(v_0 \cos\theta_0)^2} x^2}$




Now, since g, $\theta_0$ and $v_0$ are constants, is of the form $y = ax + bx^2$, in which a and b are constants. This is the equation of a parabola, i.e. the path of the projectile is a parabola.

(e) time of maximum height

Let this time be denoted by $t_n$. Since at this point, $v_y = 0$, we have from equation,

$$v_y = v_0 \sin\theta_0 - gt_m = 0 \qquad \text{Or,} \qquad t_m = v_0 \sin\theta_0/g$$

(f) total time $T_f$ during which the projectile is in flight can be obtained by putting y = 0 in equation (ii), we get $T_f = 2(v_0 \sin \theta_0)/g$

$T_f$ is known as the time of flight of the projectile. We note that $T_f = 2\, t_m$, which is expected because of the symmetry of the parabolic path.

(g) maximum height of a projectile

The maximum height $h_m$ reached by the projectile can be calculated by substituting $t = t_m$ in equation (ii)

$$\mathbf{y = h_m = (v_0 \sin\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{g}{2}\left(\frac{v_0 \sin\theta_0}{g}\right)^2} \qquad \text{Or,} \qquad \mathbf{h_m = \frac{(v_0 \sin\theta_0)^2}{2g}}$$

(h) horizontal range of projectile

The horizontal distance travelled by a projectile from its initial position (x = y = 0) to the position where it passes y = 0 during its fall is called the horizontal range, R. It is the distance travelled during the time of flight $T_f$. Therefore, the range R is

$$R = (v_0 \cos \theta_0)(T_f) = (v_0 \cos \theta_0)(2v_0 \sin \theta_0)/g \qquad \text{Or,} \qquad \mathbf{R = \frac{v_0^2 \sin 2\theta_0}{g}}$$

**Q. At what angle of projection, the projectile will have maximum range ? Also calculate the maximum range ?**

**Solution :** For a given projection velocity $v_0$. R is maximum when $\sin 2\theta_0$ is maximum, i.e., when $\theta_0 = 45^0$.

The maximum horizontal range is, therefore, $\mathbf{R_m = \dfrac{v_0^2}{g}}$

**Q. Galileo, in his book <u>Two new sciences</u>, stated that "for elevations which exceeds or fall short of $45^0$ by equal amounts, the ranges are equal". Prove this statement. [NCERT Solved Example 4.7]**

**Q. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m $s^{-1}$. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take g = 9.8 m $s^{-2}$). [NCERT Solved Example 4.8]**

**Solution :** 10 s, 99 m/s

**Q. A cricket ball is thrown at a speed of 28 m s$^{-1}$ in a direction 30$^0$ above the horizontal. Calculate (a) the maximum height. (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level. [NCERT Solved Example 4.9]**

**Solution :** (a) 10 m (b) 2.9 s (c) 69 m

## 4.11 Uniform Circular Motion :

**Q. What is uniform circular motion ?**

**Solution :** When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion.

**Q. Is there any acceleration for an object in uniform circular motion ? If yes what is the direction ?**

**Solution :** Suppose an object is moving with uniform speed v in a circle of radius R. Since the velocity of the object is changing continuously in direction, the object undergoes acceleration.

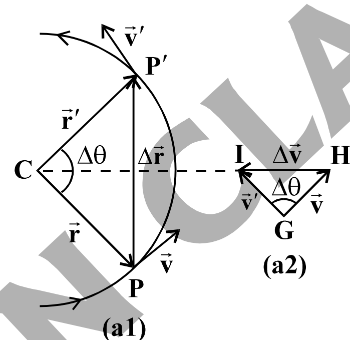
The direction of acceleration is always directed towards the centre of circle.

**Q. What is the angle between velocity and acceleration at any moment in uniform circular motion ?**

**Solution :** 90$^0$

**Q. Prove that the acceleration of an object in uniform circular motion is always directed towards the centre of the circle ?**

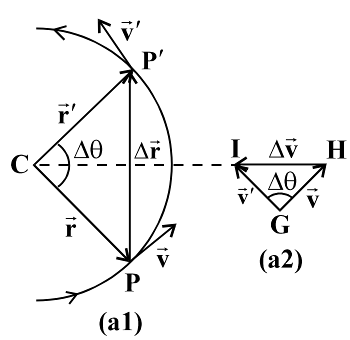
**Solution :**




By definition, velocity at a point is along the tangent at that point in the direction of motion. The velocity vectors $\vec{v}$ and $\vec{v}'$ are as shown. $\Delta\vec{v}$ is obtained in figure using the triangle law of vector addition. Since the path is circular, $\vec{v}$ is perpendicular to $\vec{r}$ and so is $\vec{v}'$ to $\vec{r}'$. Therefore, $\Delta\vec{v}$ is perpendicular to $\Delta\vec{r}$. Since average acceleration is along $\Delta\vec{v}\left(\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}\right)$, the average acceleration $\vec{a}$ is perpendicular to $\Delta\vec{r}$. If we place $\Delta\vec{v}$ on the line that bisects the angle between $\vec{r}$ and $\vec{r}'$, we see that it is directed towards the centre of the circle. For smaller time interval $\Delta\vec{v}$ and hence $\vec{a}$ is again directed towards the centre. Thus, we find that the acceleration of an object in uniform circular motion is always directed towards the centre of the circle.

**Q. An object is moving with constant speed 'v' in uniform circular motion of radius 'r'. Derive the expression for mangnitude and the direction of the acceleration. Hence define the centripetal acceleration.**

**Solution :**



Let $\vec{\mathbf{r}}$ and $\vec{\mathbf{r}}'$ be the position vector and $\vec{\mathbf{v}}$ and $\vec{\mathbf{v}}'$ the velocities of the object when it is at point P and $\mathbf{P}'$ as shown in figure.

The magnitude of $\vec{\mathbf{a}}$ is, by definition, given by $|\vec{\mathbf{a}}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{\mathbf{v}}|}{\Delta t}$.

Let the angle between position vectors $\vec{\mathbf{r}}$ and $\vec{\mathbf{r}}'$ be $\Delta\theta$. Since the velocity vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{v}}'$ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$. Therefore, the triangle $\mathbf{CPP}'$ formed by the position vectors and the triangle GHI formed by the velocity vectors $\vec{\mathbf{v}}, \vec{\mathbf{v}}'$ **and** $\Delta\vec{\mathbf{v}}$ are similar. Therefore, the ratio of the base-length to side-length for one of the triangles is equal to that of the other triangle. That is :

$$\frac{|\Delta\vec{\mathbf{v}}|}{\mathbf{v}} = \frac{|\Delta\vec{\mathbf{r}}|}{\mathbf{R}} \quad \text{or,} \quad |\Delta\vec{\mathbf{v}}| = \mathbf{v}\frac{|\Delta\vec{\mathbf{r}}|}{\mathbf{R}}$$

Therefore,

$$|\vec{\mathbf{a}}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{\mathbf{v}}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\mathbf{v}|\Delta\vec{\mathbf{r}}|}{\mathbf{R}\Delta t} = \frac{\mathbf{v}}{\mathbf{R}} \lim_{\Delta t \to 0} \frac{|\Delta\vec{\mathbf{r}}|}{\Delta t}$$

$$\because \quad \lim_{\Delta t \to 0} \frac{|\Delta\vec{\mathbf{r}}|}{\Delta t} = \mathbf{v}$$

Therefore, the centripetal acceleration $a_c$ is :

$$\mathbf{a_c} = \left(\frac{\mathbf{v}}{\mathbf{R}}\right)\mathbf{v} = \mathbf{v^2 / R}$$

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude $v^2/R$ and is always directed towards the centre. This is why this acceleration is called centripetal acceleration.

**Q. Is centripetal acceleration vector constant ? Explain.**

**Solution :** Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes – pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

**Q. Define angular speed. Relate linear speed 'v' with angular speed.**

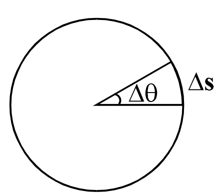
**Solution :** We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement :

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \text{or} \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between linear speed (v) and angular speed (ω) :

Now, if the distance travelled by the object during the time Δt is Δs, then $\mathbf{v} = \frac{\Delta s}{\Delta t}$

$\because$ Δs = R Δθ from figure




Therefore : $\mathbf{v} = \mathbf{R}\frac{\Delta\theta}{\Delta t} = \mathbf{R}\omega$

**Q. Express centripetal acceleration in terms of angular speed.**

**Solution :** We can express centripetal acceleration $a_c$ in terms of angular speed :

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

**Q. What is the time period of revolution and frequency of revolution ?**

**Solution :** The time taken by an object to make one revolution is known as its time period T and the number of revolution made in one second is called its frequency $\nu$ (=1/T).

**Q. Express linear speed in terms of radius of circle R and time period of revolution in uniform circular motion.**

**Solution :** $v = 2\pi R/T$

**Q. Express centripetal acceleration in terms of frequency of revolution.**

Solution : In terms of frequency $\nu$, we have $\omega = 2\pi\nu \Rightarrow v = 2\pi R\nu \Rightarrow a_c = \frac{v^2}{R} = 4\pi^2\nu^2 R$

**Q. An insect trapped in a circular groove of radius 12 cm moves along the groove steadily and completes 7 revolutions in 100 s. (a) What is the angular speed, and the linear speed of the motion ? (b) Is the acceleration vector a constant vector ? What is its magnitude ?**

**Solution :** (a) 0.44 rad/s, 5.3 cm/s (b) No, $a_c$ = 2.3 cm/s$^2$

# NCERT EXERCISE

**4.1** **State, for each of the following physical quantities, if it is a scalar or a vector : volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.**

**4.2** **Pick out the two scalar quantities in the following list :**

**force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.**

**4.3** **Pick out the only vector quantity in the following list :**

**Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.**

**4.4** **State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful :**

**(a) adding any two scalars, (b) adding a scalar to a vector of the same dimensions, (c) multiplying any vector by any scalar, (d) multiplying any two scalars, (e) adding any two vectors, (f) adding a component of a vector to the same vector.**

**4.5** **Read each statement below carefully and state with reasons, if it is true or false :**

**(a) The magnitude of a vector is always a scalar, (b) each component of a vector is always a scalar, (c) the total path length is always equal to the magnitude of the displacement vector of a particle. (d) the average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time, (e) Three vectors not lying in a plane can never add up to give a null vector.**

**4.6** **Establish the following vector inequalities geometrically or otherwise :**

(a) $|\vec{a}+\vec{b}| \le |\vec{a}| + |\vec{b}|$

(b) $|\vec{a}+\vec{b}| \ge ||\vec{a}| - |\vec{b}||$

(c) $|\vec{a}-\vec{b}| \le |\vec{a}| + |\vec{b}|$

(d) $|\vec{a}-\vec{b}| \ge ||\vec{a}| - |\vec{b}||$

**4.7** **Given $\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$, which of the following statements are correct :**

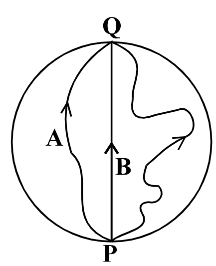
(a) **$\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ must each be a null vector**

(b) **The magnitude of $(\vec{a}+\vec{c})$ equals the magnitude of $(\vec{b}+\vec{d})$**

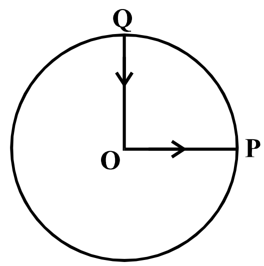
(c) **The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}, \vec{c}$ and $\vec{d}$.**

(d) **$\vec{b}+\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear ?**

**4.8** **Three girls skatting on a circular ice ground of radius 200 m start from a point P on the edge of the ground and react a point Q diaetrically opposite to P following different paths as shown. What is the magnitude of the displacement vector for each ? For which girl is this equal to the actual length of path skate ?**



**4.9** **A cyclist starts along the straight path from the centre of a circular park of radius 1 km, reaches the edge of the park, then cycles along the circumference (for one quarter circle) and returns to the centre along the straight path. If the round trip takes 10 minutes, what is the (a) net displacement, (b) average velocity and (c) average speed of the cyclist ?**




**4.10** On an open ground, a motorist follows a track that turns to his left by an angle of $60^0$ **after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.**

**Ans. : Displacement of magnitude 1 km and direction $60^0$ with the initial direction; total path length = 1.5 km (third turn); null displacement vector; path length = 3 km (sixth turn); 866 m, $30^0$, 4 km (eight turn)**

**4.11** **A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity ? Are the two equal ?**

**4.12** **Rain is falling vertically with a speed of 30 m $s^{-1}$. A woman rides a bicycle with a speed of 10 m $s^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella ?**

**4.13** **A man can swim with a speed of 4 km/h in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km/h and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank ?**

**4.14** **In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at the speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat ?**

**4.15** **The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m $s^{-1}$ can go without hitting the ceiling of the hall ?**

**4.16** **A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball ?**

**4.17** **A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone ?**

**4.18** **An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.**

**4.19** **Read each statement below carefully and state, with reasons, if it is true or false :**

**(a)** **The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre**

**(b)** **The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point**

**(c)** **The acceleration vector of a particle in uniform circular motion averaged over one cycle in a null vector**

**4.20** The position of a particle is given by $\vec{r} = 3.0t\hat{i} - 2.0t^2\hat{j} + 4.0\hat{k}$m where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle ? (b) What is the magnitude and direction of velocity of the particle at t = 2 s ?

**4.21** A particle starts from the origin at t = 0 s with a velocity of $10.0\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})$ m $s^{-2}$. (a) At what time is the x-coordinate of the particle 16 m ? What is the y-coordinate of the particle at that time ? (b) What is the speed of the particle at the time ?

**4.22** $\hat{i}$ and $\hat{j}$ are unit vectors along x- and y- axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ ? What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ ? [You may use graphical method]

**4.23** For any arbitrary motion in space, which of the following relations are true

(a) $v_{average} = (1/2)\,(v(t_1) + v(t_2))$

(b) $v_{average} = [r(t_2) - r(t_1)]/(t_2 - t_1)$

(c) $v(t) = v(0) + at$

(d) $r(t) = r(0) + v(0)t + (1/2)\,at^2$

(e) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval $t_1$ to $t_2$)

**4.24** Read each statement below carefully and state, with reasons and examples, if it is true or false :

A scalar quantity is one that

(a) is conserved in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observes with different orientations of axes

**4.25** An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is $30^0$, what is the speed of the aircraft ?

**ADDITIONAL EXERCISES**

**4.26** A vector has magnitude and direction. Does it have a location in space ? Can it vary with time ? Will two equal vectors $\vec{a}$ and $\vec{b}$ at different locations in space necessarily have identical physical effects ? Give examples in support of your answer.

**4.27** A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector ? The rotation of a body can be specified by the direction of the axis of rotation, and the angle of rotation about the axes. Does that make any rotation a vector ?

**4.28** Can you associate vectors with (a) the length of a wire bent into a loop, (b) a plane area, (c) a sphere ? Explain.

**4.29** A bullet fired at an angle of $30^0$ with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away ? Assume the muzzle speed to the fixed, and neglect air resistance.

**4.30** A fighter plane flying horizontally at an altitute of 1.5 km with speed 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m $s^{-1}$ to hit the plane ? At what minimum altitude should the pilot fly the plane to avoid being hit ? (Take g = 10 m $s^{-2}$)

**4.31** A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies breakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn ?

**4.32** (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by $\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$.

(b) Show that the projection angle $\theta_0$ for a projectile launched from the origin is given by $\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$, where the symbols have their usual meaning.

## ANSWERS



4.1 Volume, mass, speed, density, number of moles, angular frequency are scalars; the rest are vectors

4.2 Work, current

4.3 Impulse

4.4 Only (c) and (d) are permissible

4.5 (a) T, (b) F, (c) F, (d) T, (e) T

4.6 Hint : The sum (difference) of any two sides of a triangle is never less (greater) than the third side. Equality holds for collinear vectors.

4.7 All statements except (a) are correct

4.8 400 m for each; B

4.9 (a) 0 (b) 0 (c) 21.4 km/h

4.10 Displacement of magnitude 1 km and direction $60^0$ with the initial direction; total path length = 1.5 km (third turn); null displacement vector; path length = 3 km (sixth turn); 866 m, $30^0$, 4 km (eighth turn)

4.11 (a) 49.3 km $h^{-1}$; (b) 21.4 km $h^{-1}$, No, the average speed equals average velocity magnitude only for a straight path.

4.12 About $18^0$ with the vertical, towards the south

4.13 15 min, 750 m

4.14 East (approximately)

4.15 150.5 m

4.16 50 m

4.17 9.9 m $s^{-2}$, along the radius at every point towards the centre

4.18 6.4 g

4.19 (a) False (b) True (c) True

4.20 (a) $v(t) = (3.0\hat{i} - 4.0t\,\hat{j})\,\hat{a}(t) = -4.0\hat{j}$ (b) 8.54 m $s^{-1}$, $70^0$ with x-axis

4.21 (a) 2s, 24 m (b) 21.26 m $s^{-1}$

4.22 $\sqrt{2}$, $45^0$ with the x-axis; $\sqrt{2}$ – $45^0$ with the x-axis, $(5/\sqrt{2}, -1/\sqrt{2})$

4.23 (b) and (e)

4.24 Only (e) is true

4.25 182 m $s^{-1}$

4.27 No. Rotations in general cannot be associated with vectors

4.28 A vector can be associated with a plane area

4.29 No

4.30 At an angle of $\sin^{-1}(1/3) = 19.5^0$ with the vertical; 16 km.

4.31 0.86 m $s^{-2}$, $54.5^0$ with the direction of velocity

# ADDITIONAL QUESTIONS AND PROBLEMS

**Q. Can a particle accelerate if its speed is constant ? Can it accelerate if its velocity is constant ? Explain.**

A. When a particle describes a uniform circular motion, its speed is constant but it has centripetal acceleration acting along the radius directed towards the centre of the circular path.
When the particle is moving with a constant velocity, there is no change in velocity with time and hence its acceleration is zero.

**Q. The magnitude and direction of the acceleration of a body both are constant. Will the path of the body be necessarily be a straight line ?**

A. No ; The acceleration of a body remaining constant, the magnitude and direction of the velocity of the body may change. For example, under a constant acceleration (i.e., acceleration due to gravity g), the path of body in angular projection is a parabolic path.

**Q. A ball is dropped horizontally and at the same time another ball is dropped vertically from the top of a tower. (a) Will both the balls reach the ground at the same ? (b) Will both the balls strike the ground with same velocity ?**

A. (a) Yes, because initial vertical downward velocity of both the balls is the same and both will cover the same vertical distance in a given time under the same vertical downward acceleration g.
(b) No. It is because the vertical velocity of both the balls while striking the ground will be same but horizontal velocities of two balls are different. The horizontal velocity of one ball will be the same as the velocity of throw while the velocity of other ball is zero. Hence their resultant velocities will be different.

**Q. Is the maximum height attained by projectile is largest when its horizontal range is maximum ?**

A. No ; horizontal range is maximum when $\theta = 45^0$ and maximum height attained by projectile is largest when $\theta = 90^0$.

**Q. A person sitting in a running train throws a ball vertically upwards. What is the nature of the path described by the ball to a person (a) sitting inside the train (b) standing on the ground outside the train.**

A. (a) Vertical st. line path because w.r.t. person sitting in the train the ball has only one velocity acting vertically (b) Parabolic path because w.r.t. person outside the train the ball has the vertical as well as horizontal component velocities.

**Q. A body slides down a smooth inclined plane when released from the top, while another body falls freely from the same point. Which one will strike the ground earlier ?**

A. A body falling freely will reach the ground earlier because its acceleration is g (i.e., acceleration due to gravity) which is greater than the acceleration of other body = g sin θ ; where θ is the inclination of the plane with the horizontal.

**Q. At what angle to the horizontal should an object be projected so that the maximum height reached is equal to the horizontal range.**

A. Here, $u^2 \sin^2 \theta/2g = u^2 \sin 2\theta/g$ ; On solving we get $\tan \theta = 4$ or $\theta = \tan^{-1}(4)$.

**Q. A body is projected with a speed u at an angle to the horizontal to have maximum range. What is its velocity at the highest point ?**

A. For range to be maximum $\theta = 45^0$. At the highest point, the vertical component velocity is zero but the horizontal component velocity $= u \cos \theta = u \cos 45^0 = u/\sqrt{2}$.

**Q. Is it true that the instantaneous velocity of a projectile is tangential to its parabolic path.**

A. Yes.

**Q. A body is thrown horizontally with a velocity v from a tower H metre high. After how much time and at what distance from the base of the tower will the body strike the ground ?**

A. Time, $\mathbf{t = \sqrt{2H/g}}$ ; Horizontal distance, $x = vt = \mathbf{v\sqrt{2H/g}}$.

**Q. If a body is released from the aeroplane in flight, is it an example of projectile.**

A. Yes, because the body released from the aeroplane has two dimensional motion, hence it is an example of projectile.

**Q. What are the assumptions made in the study of a projectile motion ?**

A. To study the motion of a projectile, we assume that (i) there is no frictional resistance of air. (ii) The effect due to rotation of earth and curvature of the earth is negligible. (iii) The acceleration due to gravity is constant in magnitude and direction at all points of the motion of projectile.

**Q. Prove that there are two angles of projection for the same horizontal range.**

**Q. Prove that the horizontal range is same when angle of projection is (i) greater than $45^0$ by certain value and (ii) less than $45^0$ by the same value.**

**Q. At what point of the projectile-path, the speed is (i) minimum and (ii) maximum ?**

A. A projectile when given angular projection has two rectangular component velocities, acting horizontally and vertically. The horizontal component velocity remains constant throughout the projectile path but vertical component velocity of projectile decreases as it goes up and becomes zero at the highest point. That is why, the projectile on its path, has minimum velocity at the highest point and maximum velocity at the projection point or at a point where it strikes the horizontal ground during its flight.

**Q. A ball is projected with velocity u at an angle $\alpha$ with horizontal plane. What is its speed when it makes an angle $\beta$ with the horizontal plane ?**

A. Let v be the velocity of ball at an instant, when it makes an angle $\beta$ with the horizontal. The horizontal component velocity of the ball = v cos $\beta$.