MOVING CHARGES AND MAGNETISM

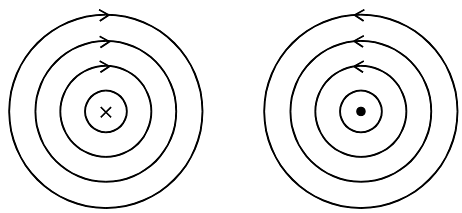
4.1 Introduction :

Q. Who did conclude that moving charges or currents produced a magnetic field in the surrounding space ?

Solution : Oersted concluded that moving charges or currents produced a magnetic field in the surrounding space.

Q. Draw the pattern of magnetic field due to a straight long current carring wire which is perpendicular to the plane of the paper and carrying the current (a) inward and (b) outward.

Solution :



(a) (b)

4.2 Magnetic Force :

Q. What is Lorentz Force ?

Solution : Consider a point charge q (moving with a velocity $\vec{v}$ and, located at $\vec{r}$ at a given time t) in presence of both the electric field $\vec{E}(\vec{r})$ and the magnetic field $\vec{B}(\vec{r})$. The force on an electric charge q due to both of them can be written as $\vec{F} = q[\vec{E}(\vec{r}) + \vec{v} \times \vec{B}(\vec{r})] \equiv \vec{F}_{electric} + \vec{F}_{magnetic}$. It is called the Lorentz force.

Q. Does a stationary charged particle experience a magnetic force ?

Solution : A stationary charged particle does not experience any magnetic force.

Q. What is the magnetic force if velocity and magnetic field are parallel or anti-parallel ?

Solution : As $\vec{F}_{magnetic} = q(\vec{v} \times \vec{B})$, if velocity and magnetic field are paralle or anti-parallel then $\vec{v} \times \vec{B}$ will become zero and hence the magnetic force will become zero.

Q. How can you give the direction of magnetic force ?

Solution : The force acts in a direction perpendicular to both the velocity and the magnetic field. Its direction is given by the screw rule or right hand rule for vector (or cross) product.

Q. Under what condition does a charged particle moving through a magnetic field experience maximum force and minimum force ?

Solution : A charged particle moving through a magnetic field experience maximum force when velocity of the charged particle is perpendicular to the magnetic field.

A charged particle moving through a magnetic field experience minimum force when velocity of the charged particle is parallel or anti-parallel to the magnetic field.

Q. In a certain arrangement, a proton does not get deflected while passing through a magnetic field region. Under what condition is it possible ?

Solution : It is possible when the proton is moving parallel or anti-parallel to the magnetic field.

Q. Which of the pair of vectors will be always perpendicular in magnetic force ?

Solution : From $\vec{F} = q(\vec{v} \times \vec{B})$, $\vec{F}$ **and** $\vec{B}$ & $\vec{F}$ **and** $\vec{v}$ are always perpendicular to each other.

Q. Define 1 tesla (T) or SI unit of magnetic field.

Solution : The magnitude of magnetic field B is 1 SI unit or 1 testa, when the force acting on a unit charge (1 C), moving perpendicular to $\vec{B}$ with a speed 1m/s, is one newton.

Q. What is the SI unit of magnetic field ?

Solution : tesla

Q. What is the smaller unit of magnetic field ?

Solution : gauss equals to 10^{-4} T

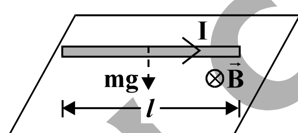
Q. What is the order of earth's magnetic field ?

Solution : 3.6×10^{-5} T

Q. Derive the expression for magnetic force (known as Lorentz force) on a current carrying conductor.

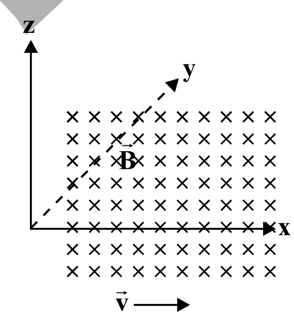
Solution : Consider a rod of a uniform cross-sectional area A and length l. Let the number density of these mobile charge carries in it be n. Then the total number of mobile charge carriers in it is nAl. For a steady current I in this conducting rod, we may assume that each mobile carrier has an average drift velocity $\vec{\mathbf{v}}_{\mathbf{d}}$. In the presence of an external magnetic field $\vec{\mathbf{B}}$, the force on these carries is : $\vec{\mathbf{F}} = (\mathbf{n}\mathbf{A}\boldsymbol{l})\mathbf{q}\vec{\mathbf{v}}_{\mathbf{d}} \times \vec{\mathbf{B}}$, where q is the value of the charge on a carrier. Now $\mathbf{nq}\vec{\mathbf{v}}_{\mathbf{d}}$ is the current density $\vec{\mathbf{j}}$ and $|(\mathbf{nq}\vec{\mathbf{v}}_{\mathbf{d}})|$ A is the current I. Thus, $\vec{\mathbf{F}} = [(\mathbf{nq}\vec{\mathbf{v}}_{\mathbf{d}})\mathbf{A}\boldsymbol{l}] \times \vec{\mathbf{B}} = [\vec{\mathbf{j}}\mathbf{A}\boldsymbol{l}] \times \vec{\mathbf{B}} = \mathbf{I}\vec{\boldsymbol{l}} \times \vec{\mathbf{B}}$, where $\vec{l}$ is a vector of magnitude l, the length of rod, and with a direction identical to the current I.

Q. A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field $\vec{\mathbf{B}}$. What is the magnitude of the magnetic field ? [NCERT Solved Example 4.1]



Solution : 0.65 T

Q. If the magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis, which way would the Lorentz force be for (a) an electron (negative charge), (b) a proton (positive charge). [NCERT Solved Example 4.2]



Solution : The velocity $\vec{v}$ of particle is along the x-axis, while $\vec{\mathbf{B}}$, the magnetic field is along the y-axis, so $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is along the z-axis (screw rule or right-hand thumb rule). So, (a) for electron it will be along -z axis. (b) for a positive charge (proton) the force is along +z axis.

4.3 Motion in Magnetic Field :

Q. Prove that no work is done by magnetic force ? OR, Prove that the power acted by the magnetic force equals to zero ? OR, Prove that the speed of the moving charged particle remains constant in magnetic field ? OR, Prove that the magnetic field can change only the direction of charged particle ?

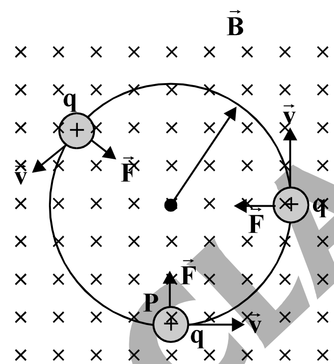
Solution : From $\vec{F} = q(\vec{v} \times \vec{B})$, $\vec{F}$ **and** $\vec{v}$ are always perpendicular to each other. The power acted by the magnetic force is given by $\vec{F} \cdot \vec{v}$ which equals to always zero. So no work is done and no change in the speed of the charged particle.

Q. What will be the path of the charged particle if it is projected parallel or anti-parallel to external magnetic field ?

Solution : As the force acting on the charged particle equals to zero if it is projected parallel or anti-parallel to external magnetic field, hence the path of the particle will be straight line.

Q. Discuss the motion of the charged particle if it is projected perpendicular to the direction of magnetic field ? OR, Prove that the charged particle will describe a circular path if its velocity is perpendicular to external magnetic field ? Also find the radius of the circular path and its frequency of revolution.

Solution :



The force experienced by charged particle equals to qvB (as v and B are perpendicular to each other) which acts as a certripetal force mv²/r (where m is the mass of particle and r is the radius of the circle) and produces the circular motion perpendicular to the magnetic field. Hence the particle will describe a circle if $\vec{v}$ and $\vec{B}$ are perpendicular to each other.

radius of circular path : $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$.

frequency of revolution : Time taken to one revolution is known as time period which is given by

$T = \frac{2\pi r}{v}$. From $r = \frac{mv}{qB}$, we get $T = \frac{2\pi m}{qB}$.

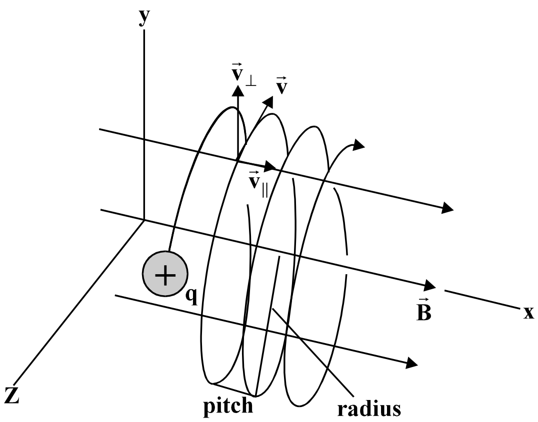
Frequency of revolution $= \frac{1}{T} = \frac{qB}{2\pi m}$.

Q. What is the motion of the charged particle if it is projected at any angle of θ (≠ zero, π, π/2) ?

Solution : The motion of the charged particle is helical motion also known as helix if it is projected at any angle of θ.

Q. Discuss the helical motion of the charged particle in the presence of magnetic field. What is the pitch and radius of helix ?

Solution : A charged particle is projected such that it has two components of velocity : one component is along $\vec{B}$, denoted by $v_{\parallel}$ and other component is perpendicular to $\vec{B}$ denoted by $v_{\perp}$. If velocity has a component along $\vec{B}$, this component remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. The motion in a plane perpendicular to $\vec{B}$ is as before a circular one, thereby producing a helical motion.



The time taken for one revolution is T = 2π/ω ≡ 1/ν. If there is a component of the velocity parallel to the magnetic field (denoted by $v_\parallel$), it will make the particle move along the field and the path of the particle would be helical one. The distance moved along the magnetic field in one rotation is called pitch p. Using equation, we have $p = v_\parallel T = 2\pi m\, v_\parallel/qB$

The radius of the circular component of motion is called the radius of the helix, given by $\mathbf{R = \frac{m\,v_\perp}{qB}}$.

Q. What is the radius of the path of an electron (mass 9 × 10^{-31} kg and charge 1.6 × 10^{-19} C) moving at a speed of 3 × 10^{7} m/s in a magnetic field of 6 × 10^{-4} T perpendicular to it ? What is its frequency ? Calculate its energy in keV. (1 eV = 1.6 × 10^{-19} J). [NCERT Solved Example 4.3]

Solution : r = 28 cm, f = 20MHz, K.E. = 2.5 keV

4.4 Motion in Combined Electric and Magnetic Fields :

Q. What is the use of velocity selector ? On which principle is it based ?

OR

Q. Describe velocity selector ?

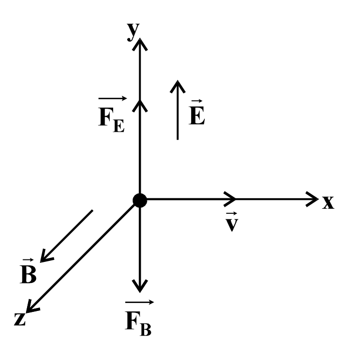
Solution : Velocity selector can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass).

Principle : We know that a charge q moving with velocity $\vec{v}$ in presence of both electric and magnetic fields experiences a force given by $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = \vec{F}_E + \vec{F}_B$.

We shall consider the simple case in which electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of the particle, as shown. We have,

$$\vec{E} = E\hat{j}, \vec{B} = B\hat{k}, \vec{B}, \vec{v} = v\hat{i}$$
$$\vec{F}_E = q\vec{E} = qE\hat{j}, \vec{F}_B = q\vec{v} \times \vec{B}, = q\left(v\hat{i} \times B\hat{k}\right) = -qB\,\hat{j}$$

Therefore, $\vec{F} = q(E - vB)\hat{j}$



Thus, electric and magnetic forces are in opposite directions as shown in the figure. Suppose, we adjust the value of $\vec{E}$ **and** $\vec{B}$ such that magnitude of the two forces are equal. Then, total force on the charge is zero and the charge will move in the fields undeflected. This happens when, $qE = qvB$ or $v = \frac{E}{B}$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds (irrespective of their charge and mass). The crossed E and B fields, therefore, serve as a velocity selector. Only particles with speed E/B pass undeflected through the region of crossed fields.

Q. Which method or principle was employed by J.J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron ?

Solution : The method or principle of velocity selector was employed by J.J. Thomson in 1897 to measure the charge to mass ratio (e/m) of an electron.

Q. What is mass spectrometer ? Which method or principle was employed in mass spectrometer ?

Solution : Mass spectrometer a device that separates charged particles, usually ions, according to their charge to mass ratio.

The principle of velocity selector is employed in Mass Spectrometer-charge to mass ratio.

Q. What is Cyclotron ?

Solution : The cyclotron is a machine to accelerate charged particles or ions to high energies.

Q. Who was invented Cyclotron ?

Solution : It was invented by E.O. Lawrence and M.S. Livingst in 1934.

Q. What is the purpose of invention of Cyclotron ?

Solution : The purpose of invention of Cyclotron to investigate nuclear structure.

Q. Explain with the help of a labelled diagram, the underlying principle, construction and working of a cyclotron.

Solution : The cyclotron uses both electric and magnetic fields in combination to increase the energy of charged particles. As the fields are perpendicular to each other they are called crossed fields. Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy. The particles move most of the time inside two semicircular disc-like metal containers, D_1 and D_2, which are called dees as they look like the letter D.

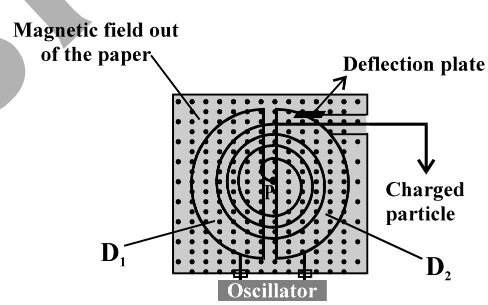


Figure shows a schematic view of the cyclotron. Inside the metal boxes the particle is shielded and is not acted on by the electric field. The magnetic field, however, acts on the particle and makes it go round in a circular path inside a dee. Every time the particle moves from one dee to another it is acted upon by the electric field. The sign of the electric field is changed alternately in tune with the circular motion of the particle. This ensures that the particle is always accelerated by the electric field. Each time the acceleration increases the energy of the particle. As energy increases, the radius of the circular path increases. So the path is a spiral one.

The whole assembly is evacuated to minimise collisions between the ions and the air molecules. A high frequency alternating voltage is applied to the dees. In the sketch shown in figure, positive ions or positively charged particles (e.g.. protons) are released at the centre P. They move in a semi-circular path in one of the dees and arrive in the gap between the dees in a time interval T/2, the period of revolution, is given by

$$T = \frac{1}{\nu_c} = \frac{2\pi m}{qB} \text{ or } \nu_c = \frac{qB}{2\pi m}$$

This frequency is called the cyclotron frequency for obvious reasons and is denoted by ν_c.

The frequency ν_a of the applied voltage is adjusted so that the polarity of the dees is reversed in the same time that it takes the ions to complete one half of the revolution. The requirement $\nu_a = \nu_c$ is called the resonance conditions. The phase of the supply is adjusted so that when the positive ions arrive at the edge of D_1, D_2 is at a lower potential and the ions are accelerated across the gap. Inside the dees the particle travel in a region free of the electric field. The increase in their kinetic energy is qV each time they cross from one dee to another (V refers to the voltage across the dees at that time). From equation, it is clear that the radius of their path goes on increasing each time their kinetic energy increases. The ions are repeatedly accelerated across the dees until they have the required energy to have a radius approximately that of the dees. They are then deflected by a magnetic field and leave the system via an exit slit. We have, $v = \frac{qBR}{m}$, where R is the radius of the trajectory at exit, and equals the radius of a dee. Hence, the kinetic energy of the ions is,

$$\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$$

Q. Write the uses of cyclotron ?

Solution : The cyclotron is used to bombard nucleous with energetic particles, so accelerated by it, and study the resulting nuclear reactions. It is also used to implant ions into solids and modify their properties or even synthesise new materials. It is used in hospitals to produce radioactive substances which can be used in diagnosis and treatment.

Q. What is cyclotron frequency ? Show that cyclotron frequency does not depend on the speed of the particles and radius of the circular path.

Solution : The frequency of revolution to complete circular path by a charged particle of charge q qnd mass m in the presence of magnetic field B is known as cyclotron frequency which is given by $\frac{qB}{2\pi m}$.

Time taken to one revolution is known as time period which is given by

$T = \frac{2\pi r}{v}$. From $r = \frac{mv}{qB}$, we get $T = \frac{2\pi m}{qB}$.

Frequency of revolution $= \frac{1}{T} = \frac{qB}{2\pi m}$. Hence cyclotron frequency does not depend upon the speed of charge particle or radius of its orbit.

Q. On which fact, the operation of the cyclotron is based ?

Solution : The operation of the cyclotron is based on the fact that the time for one revolution of charged particle is independent of its speed or radius of its orbit.

Q. What is resonance condition to operate cyclotron ?

Solution : The cyclotron frequency must be equal to the frequency of oscillator or applied voltage in oscillator. This is called the resonance condition.

Q. Prove that the kinetic energy of the ion or charged particles is proportional to square of the radius of circular path ?

Solution : As the speed of charged particle in circular path of radius R in the presence of magnetic field is given by $v = \frac{qBR}{m}$, where R is the radius of the trajectory at exit, and equals the radius of a dee.

Hence, the kinetic energy of the ions is $\frac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}$. Hence proved.

Q. Write down the limitation of cyclotron.

Solution : It cannot be used to accelerate electrons and neutrons.

Q. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons ? If the radius of its 'dees' is 60 cm, what is the kinetic energy (in MeV) of the proton beam produced by the accelerator.

($e = 1.60 \times 10^{-19}$C, $m_p = 1.67 \times 10^{-27}$ kg, 1 MeV = 1.6×10^{-13} J). [NCERT Solved Example 4.4]

Solution : 0.66 T, 7 MeV

Q. Why is cyclotron not used to accelerate electrons ?

Solution : The mass of the electron is very small as compare to protons and other charged particles, due to which the speed of the electron will be near to the speed of light if they are accelerated by cyclotron. From relativistic effect, the mass of electron will change and hence the frequency of revolution in circular orbit will change. That's why the resonance condition will be violated which is necessary to operate a cyclotron. This is the reason cyclotron is not used to accelerate electrons.

4.5 Magnetic Field due to a Current Element, Biot-Savart Law :

Q. Write down Biot-Savart's Law ?

Solution : The magnetic field due to a current carring wire is given by the Biot-Savart's Law.

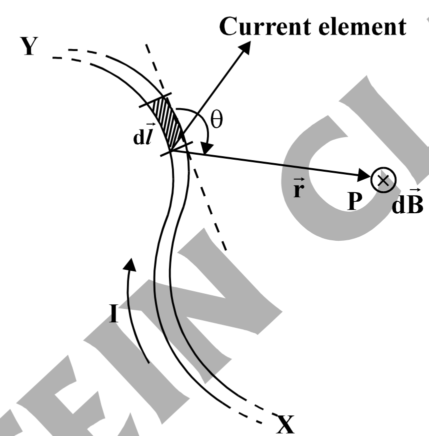


Figure shows a finite conductor XY carrying current I. Consider an infinitesimal element dl of the conductor. The magnetic field $d\vec{B}$ due to this element is to be determined at a point P which is at a distance r from it. Let θ be the angle between $d\vec{l}$ and the displacement vector $\vec{r}$. According to Biot-Savart's law, the magnitude of the magnetic field $d\vec{B}$ is proportional to the current I, the element length $|d\vec{l}|$, and inversely proportional to the square of the distance r. Its direction is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$. Thus, in vector notation,

$$d\vec{B} \propto \frac{Id\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \vec{r}}{r^3}$$

where $\mu_0/4\pi$ is a constant of proportionality. The above expression holds when the medium is vacuum.

The magnitude of this field is, $|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$.

The proportionality constant in SI units has the exact value, $\frac{\mu_0}{4\pi} = 10^{-7}\,\text{Tm/A}$. We call μ_0 the permeability of free space (or vacuum).

Q. What are the similarities and differences of Biot-Savart's Law with the Coulomb's Law ?

Solution : The Biot-Savart law for the magnetic field has certain similarities as well as differences with the Coulomb's law for the electrostatic field. Some of these are :

(i) Both are long range, since both depend inversely on the square of distance from the source to the point of interest. The principle of superposition applies to both fields.

(ii) The electrostatic field is produced by a scalar source, namely, the electric charge. The magnetic field is produced by a vector source I $\mathbf{d}\vec{l}$.

(iii) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector $\vec{\mathbf{r}}$ and the current element I $\mathbf{d}\vec{l}$.

(iv) There is an angle dependence in the Biot-Savart law which is not present in the electrostatic case.

Q. What is the value of magnetic field at any point along the direction of $\mathbf{d}\vec{l}$ of a current carrying wire ?

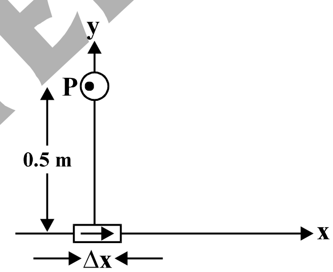
Solution : The magnetic field at any point in the direction of $\mathbf{d}\vec{l}$ (the dashed line) is zero. Along this line, θ = 0, sin θ = 0 and hence, $|\mathbf{d}\vec{\mathbf{B}}| = \mathbf{0}$.

Q. What is the relation of speed of light with permittivity and permeability of free space ?

Solution : There is relation between ε_0, the permittivity of free space; μ_0, the permeability of free space; and c, the speed of light in vacuum : $\varepsilon_0\mu_0 = (4\pi\varepsilon_0)\left(\frac{\mu_0}{4\pi}\right) = \left(\frac{1}{9\times10^9}\right)(10^{-7}) = \frac{1}{(3\times10^8)^2} = \frac{1}{c^2}$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

Q. An element $\Delta\vec{l} = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10 A. What is the magnetic field on the y-axis at a distance of 0.5 m. Δx = 1 cm. [NCERT Solved Example 4.5]



Solution : 4×10^{-8} T

4.6 Magnetic Field on the Axis of a Circular Current Loop :

Q. Derive the expression of magnetic field in free space (vacuum) on the axis at the distance x from the centre of a circular current loop (placed in the yz plane) of radius R carrying the current I. Hence obtain the expression of magnetic field at the centre of the loop.

Solution :

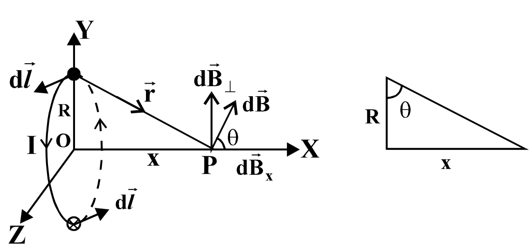


Figure depicts a circular loop carrying a steady current I. The loop is placed in the y-z plane with its centre at the origin O and has a radius R. The x-axis is the axis of the loop. We wish to calculate the magnetic field at the point P on this axis. Let x be the distance of P from the centre O of the loop.

Consider a conducting element $\mathbf{d\vec{l}}$ of the loop. This is shown in figure. The magnitude dB of the magnetic field due to $\mathbf{d\vec{l}}$ is given by the Biot-Savart law, $\mathbf{dB = \frac{\mu_0}{4\pi}\frac{I|d\vec{l}\times\vec{r}|}{r^3}}$.

Now $r^2 = x^2 + R^2$. Further, any element of the loop will be perpendicular to the displacement vector from the element to the axial point. Hence $\mathbf{|d\vec{l}\times\vec{r}| = rd\mathit{l}}$. Thus, $\mathbf{dB = \frac{\mu_0}{4\pi}\frac{Id\mathit{l}}{(x^2+R^2)}}$. The direction of $\mathbf{d\vec{B}}$ is shown in figure. It is perpendicular to the plane formed by $\mathbf{d\vec{l}}$ **and** $\mathbf{\vec{r}}$. It has an x-component $\mathbf{d\vec{B}_x}$ and a component perpendicular to x-axis, $\mathbf{d\vec{B}_\perp}$. When the components perpendicular to the x-axis are summed over, they cancel out and we obtain a null result. For example, the $\mathbf{d\vec{B}_\perp}$ component due to $\mathbf{d\vec{l}}$ is cancelled by the contribution due to the diametrically opposite $\mathbf{d\vec{l}}$ element. Thus, only the x-component survives. The net contribution along x-direction can be obtained by integrating $dB_x = dB\cos\theta$ over the loop.

$$\mathbf{\cos\theta = \frac{R}{(x^2+R^2)^{1/2}}}$$

Hence, $$\mathbf{dB_x = \frac{\mu_0 Id\mathit{l}}{4\pi}\frac{R}{(x^2+R^2)^{3/2}}}.$$

The integration of element dl over the loop yields $2\pi R$, the circumference of the loop. Thus, the magnetic field at P due to entire circular loop is $\mathbf{\vec{B} = B_x\hat{i} = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}\hat{i}}$.

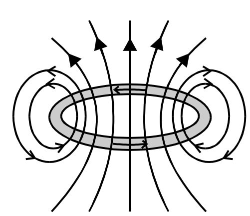
As a special case of the above result, we may obtain the field at the centre of the loop. Here x = 0, and we obtain, $\mathbf{\vec{B}_0 = \frac{\mu_0 I}{2R}\hat{i}}$.

Q. What will be the magnetic field on the axis at the distance 'x' from the centre of a circular current loop of N number of turns. The loop has the radius R and carrying the current I. Also write down the expression for the magnetic field at the centre of the loop.

Solution : $\mathbf{\vec{B} = B_x\hat{i} = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}\hat{i}}$, $\mathbf{\vec{B}_0 = \frac{\mu_0 NI}{2R}\hat{i}}$

Q. Draw the magnetic field lines due to circular wire.

Solution : The magnetic field lines due to a circular wire form closed loops and are shown in figure.

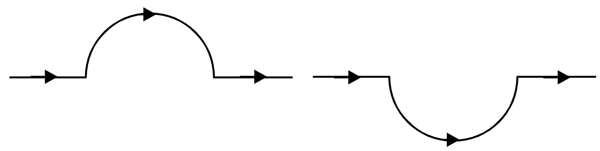


Q. Which rule does give the direction of the magnetic field due to circular current carrying loop ? Write down the statement of this rule.

Solution : The direction of the magnetic field is given by right-hand thumb rule stated below :

Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current.

Q. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in figure(a). Consider the magnetic field $\vec{B}$ at the centre of the arc. (a) What is the magnetic field due to the straight segments ? (b) In what way the contribution to $\vec{B}$ from the semicircle differs from that of a circular loop and in what way does it resemble ? (c) Would your answer be different if the wire were bent into a semi-circular arc of the same radius but in the opposite way as shown in figure (b) ? [NCERT Solved Example 4.6]



(a) **(b)**

Solution : (a) $\vec{dl}$ and $\vec{r}$ for each element of the straight segments are parallel. Therefore, $\vec{dl} \times \vec{r} = 0$. Straight segments do not contribute to $|\vec{B}|$.

(b) For all segments of the semicircular, $\vec{dl} \times \vec{r}$ are all parallel to each other (into the plane of the paper). All such contributions add up in magnitude. Hence direction of $\vec{B}$ for a semicircular arc is given by the right hand rule and magnitude is half that of a circular loop. Thus $\vec{B}$ is 1.9×10^{-4} T normal to the plane of the paper going into it. (c) Same magnitude of $\vec{B}$ but opposite in direction to that in (b).

Q. Consider a tightly wound 100 turn coil of radius 10 cm, carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil ? [NCERT Solved Example 4.7]

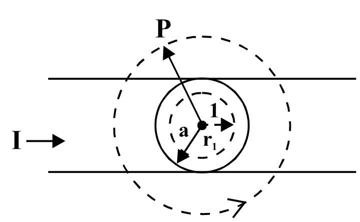
Solution : 6.28×10^{-4} T

4.7 Ampere's Circuital Law :

Q. Write down the Ampere's circuital (loop) law.

Solution : Ampere's law states that the integral of $\oint \vec{B} \cdot \vec{dl}$ is equal to μ_0 times the total current passing through the surface, i.e., $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$, where I is the total current through the surface. The integral is taken over the closed loop coinciding with the boundary of the surface.

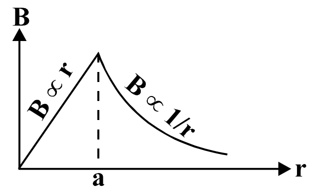
Q. Figure shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region r < a and r > a. [NCERT Solved Example 4.8]



Solution : $\mathbf{B = \frac{\mu_0 I}{2\pi r}(r < a)}$, $\mathbf{B = \frac{\mu_0 I}{2\pi a^2} r (r \geq a)}$

Q. Plot the magnitude of magnetic field with distance r from the centre of a long straight wire of circular cross-section.

Solution :



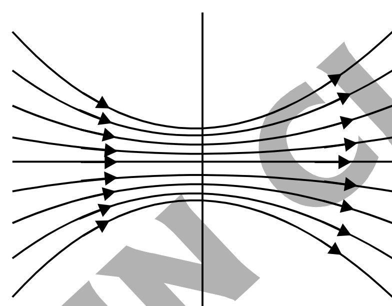
4.8 The Solenoid and the Toroid :

Q. What is long solenoid ?

Solution : A long solenoid means that the solenoid's length is large compared to its radius. It consists of a long wire wound in the form of a helix where the neighbouring turns are closely spaced. So each turn can be regarded as a circular loop. The net magnetic field is the vector sum of the fields due to all the turns. Enamelled wires are used for winding so that turns are insulated from each other.

Q. Draw the magnetic field lines for a finite solenoid.

Solution :

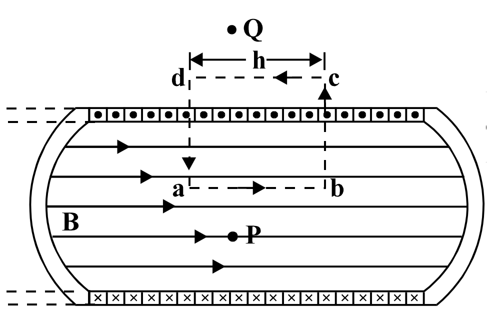


Q. What is the value of magnetic field outside the solenoid ?

Solution : The field at the exterior point is weak and for ideal solenoid it will be taken as zero.

Q. Derive the expression for the magnetic field inside the very long solenoid.

Solution : As the solenoid is made longer it appears like a long cylindrical metal sheet. Figure represents this idealised picture.



The field outside the solenoid approaches zero. We shall assume that the field outside is zero.The field inside becomes everywhere parallel to the axis.

Consider a rectangular Amperian loop abcd. Along cd the field is zero. Along transverse sections bc and ad, the field component is zero. Thus, these two sections make no contribution. Let the field along ab be B. Thus, the relevant length of the Amperian loop is, L = h.

Let n be the number of turns per unit length, then the total number of turns is nh. The enclosed current is, I_e = I (nh), where I is the current in the solenoid. From Ampere's circuital law

$BL = \mu_0 I_e$, $\quad$ $B\,h = \mu_0 I\,(n\,h)$

$B = \mu_0\, n\, I$

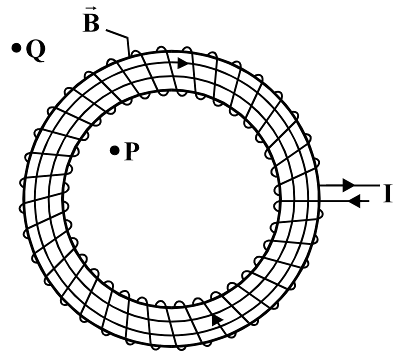
The direction of the field is given by the right-hand rule.

Q. What is the use of solenoid ?

Solution : The solenoid is used to obtain a uniform magnetic field.

Q. What is the toroid ?

Solution :



The toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. It can be viewed as a solenoid which has been bent into a circular shape to close on itself. It is shown in figure carrying a current I.

Q. What is the value of magnetic field in the open space inside and exterior to the toroid ?

Solution : The magnetic field in the open space inside and exterior to the toroid is zero.

Q. What is the nature of magnetic field inside the toroid ?

Solution : The field $\vec{B}$ inside the toroid is constant in magnitude for the ideal toroid of closely wound turns.

Q. Derive the expression for the magnetic field due to a toroid (a) in the open space (b) inside the toroid.

Solution :

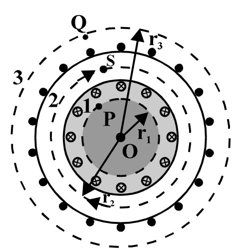


Figure shows a sectional view of the toroid. The direction of the magnetic field inside is clockwise as per the right-hand thumb rule for circular loops. Three circular Amperian loops 1, 2 and 3 are shown by dashed lines. By symmetry, the magnetic field should be tangential to each of them and constant in magnitude for a given loop. The circular areas bounded by loops 2 and 3 both cut the toroid : so that each turn of current carrying wire is cut once by the loop 2 twice by the loop 3.

Magnetic field in the open space at P :

Let the magnetic field along loop 1 be B_1 in magnitude. Then in Ampere's circuital law, $L = 2\pi\, r_1$. However, the loop encloses no current, so $I_e = 0$. Thus,

$B_1\,(2\,\pi\, r_1) = \mu_0(0)$, $B_1 = 0$

Thus, the magnetic field at any point P in the open space inside the toroid is zero.

Magnetic field in open space at Q :

We shall now show that magnetic field at Q is likewise zero. Let the magnetic field along loop 3 be B_3. Once again from Ampere's law $L = 2\pi r_3$. However, from the sectional cut, we see that the current coming out of the plane of the paper is cancelled exactly by the current going into it. Thus, $I_e = 0$, and $B_3 = 0$.

Magnetic field inside the toroid :

Let the magnetic field inside the solenoid be B. Once again we employ Ampere's law. We find, $L = 2\pi r$.

The current enclosed I_e is (for N turns of toroidal coil) N I.

$$B(2\pi r) = \mu_0 NI$$

$$\mathbf{B = \frac{\mu_0 NI}{2\pi r}}$$

Q. Compare the magnetic field due to the solenoid and toroid.

Solution : Let r be the average radius of the toroid and n be the number of turns per unit length. Then

$N = 2\pi r\,n$ = (average) perimeter of the toroid × number of turns per unit length

and thus, $B = \mu_0 n I$,

i.e., the result for the solenoid and the toroid are the same.

Q. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid ? [NCERT Solved Example 4.9]

Solution : 6.28×10^{-3} T

4.9 Force between two Parallel Currents, the Ampere :

Q. Consider long two parallel conductors carrying the currents I_a and I_b in the same direction, separeted by a distance d. Derive the expression for the force between them. Also prove that the force between them is attractive.

Solution :

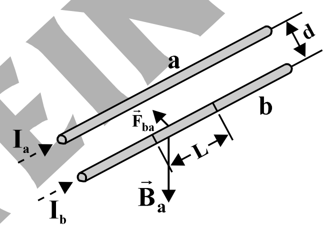


Figure shows two long parallel conductors a and b separated by a distance d and carrying (parallel) currents I_a and I_b, respectively. The conductor 'a' produces, the same magnetic field $\vec{\mathbf{B}}_\mathbf{a}$ at all points along the conductor 'b'. The right-hand rule tells us that the direction of this field is downwards (when the conductors are placed horizontally). Its magnitude is given by $\mathbf{B_a = \frac{\mu_0 I_a}{2\pi d}}$

The conductor 'b' carrying a current I_b will experience a sideways force due to the field $\vec{\mathbf{B}}_\mathbf{a}$. The direction of this force is towards the conductor 'a' (using right hand rule). We label this force as $\vec{\mathbf{F}}_\mathbf{ba}$, the force on a segment L of 'b' due to 'a'. The magnitude of this force is given by : $\mathbf{F_{ba} = I_b\, L\, B_a = \frac{\mu_0 I_a I_b}{2\pi d} L}$

It is of course possible to compute the force on 'a' due to 'b'. From considerations similar to above we can find the force $\vec{\mathbf{F}}_\mathbf{ab}$, on a segment of length L of 'a' due to the current in 'b'. It is equal in magnitude of F_{ba}, and directed towards 'b'. Thus, $\vec{\mathbf{F}}_\mathbf{ba} = -\vec{\mathbf{F}}_\mathbf{ab}$

We have seen from above that currents flowing in the same direction attract each other.

Q. Which expression is used to define the ampere (A) and hence define ampere (A) ?

Solution : Let f_{ba} represent the magnitude of the force $\vec{F}_{ba}$ per unit length. Then this expression

$$f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}$$ is used to define the ampere (A).

The ampere is the value of that steady current which, when maintained in each of the two very long, straight, parallel conductors of negligible cross-section, and placed one metre apart in vacuum, would produce on each of these conductors a force equal to 2×10^{-7} newtons per metre of length.

Q. Define 1 coulomb ?

Solution : When a steady current of 1A is set up in a conductor, the quantity of charge that flows through its cross-section in 1s is one coulomb (1C).

Q. The horizontal component of the earth's magnetic field at a certain place is 3.0×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is (a) east to west; (b) south to north ? [NCERT Solved Example 4.10].

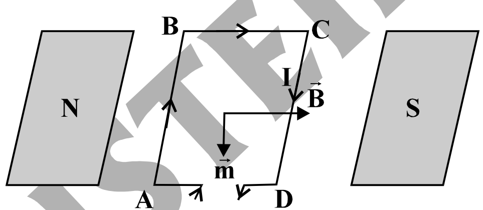
Solution : (a) 3×10^{-5} Nm^{-1} (b) zero

4.10 Torque on Current Loop, Magnetic Dipole :

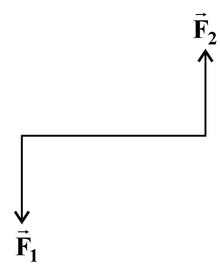
Q. A current carrying loop is placed in a uniform magnetic field. What is the force does it experience ? Explain. OR Prove that the net force on a current carrying loop in uniform magnetic field is zero.

Solution : The net force on the loop is zero.

Consider a rectangular loop carrying a steady current I and placed in a uniform magnetic field such that the uniform magnetic field $\vec{B}$ is in the plane of the loop. This is illustrated in Figure.



(a)



(b)

The force on a current carrying wire in external magnetic field is given by $I(\vec{L} \times \vec{B})$.

The field exerts no force on the two arms AD and BC of the loop, as these arms are parallel to the field. It is perpendicular to the arm AB of the loop and exerts a force $\vec{F}_1$ on it which is directed into the plane of the loop. Its magnitude is, $F_1 = I\, b\, B$

Similarly it exerts a force $\vec{F}_2$ on the arm CD and $\vec{F}_2$ is directed out of the plane of the paper.

$F_2 = I\, b\, B = F_1$

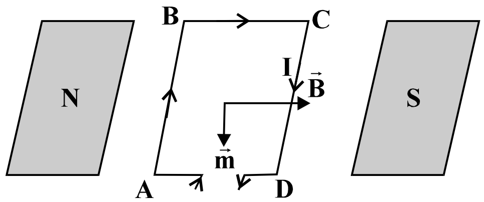
Thus, the net force on the loop is zero.

Q. Consider a rectangular loop carring a stady current I and placed in a uniform magnetic field $\vec{B}$ such that the field in the plane of the loop. Derive the expression for the torque acting on the loop.

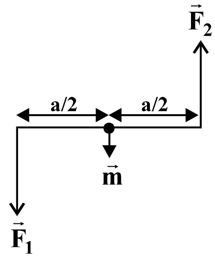
OR

Derive the expression for the maximum torque acting on the loop.

Solution : Consider a rectangular loop carrying a steady current I and placed in a uniform magnetic field such that the uniform magnetic field $\vec{B}$ is in the plane of the loop. This is illustrated in Figure.



(a)



(b)

The field exerts no force on the two arms AD and BC of the loop. It is perpendicular to the arm AB of the loop and exerts a force $\vec{F}_1$ on it which is directed into the plane of the loop. Its magnitude is, $F_1 = I\ b\ B$

Similarly it exerts a force $\vec{F}_2$ on the arm CD and $\vec{F}_2$ is directed out of the plane of the paper. Both $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction i.e., $F_2 = I\ b\ B = F_1$

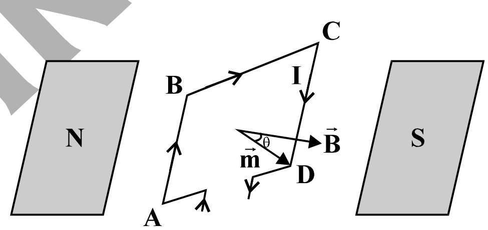
There is a torque on the loop due to the pair of forces $\vec{F}_1$ **and** $\vec{F}_2$. Figure shows a view of the loop from the AD end. It shows that the torque on the loop tends to rotate it anti-clockwise. This torque is (in magnitude)

$$\tau = F_1 \frac{a}{2} + F_2 \frac{a}{2}$$

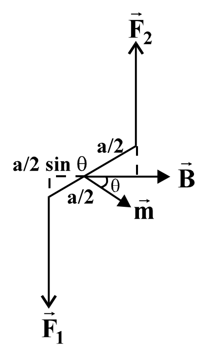
$$= IbB\frac{a}{2} + IbB\frac{a}{2} = I(ab)B$$ = I A B, where A = ab is the area of the rectangle.

Q. Consider a rectangular loop carring a stady current I and placed in a uniform magnetic field $\vec{B}$. Derive the general expression for the torque acting on the loop.

Solution : Consider the case when the plane of the loop, is not along the magnetic field, but makes an angle with it. We take the angle between the field and the normal to the coil to be angle θ. Figure illustrates this general case.



(a)



(b)

The forces on the arms BC and DA are equal, opposite, and act along the axis of the coil, which connects the centres of mass of BC and DA. Being collinear along the axis they cancel each other, resulting in no net force or torque. The forces on arms AB and CD are $\vec{F}_1$ **and** $\vec{F}_2$. They too are equal and opposite, with magnitude, $F_1 = F_2 = I\ b\ B$

But they are not collinear and results in a couple. Figure (b) is a view of the arrangement from the AD end and it illustrates these two forces constituting a couple. The magnitude of the torque on the loop is,

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta = \text{I ab B} \sin\theta = \text{I A B} \sin\theta$$

The torques can be expressed as vector product of the magnetic moment of the coil and the magnetic field.

We define the magnetic moment of the current loop as, $\vec{m} = I\vec{A}$.

Then as the angle between $\vec{m}$ **and** $\vec{B}$ is θ, hence the torque acting on the loop can be expressed by $\vec{\tau} = \vec{m} \times \vec{B}$.

Q. Define magnetic moment of the current loop. Also write down its units and dimensions.

Solution : The magnetic moment of the current loop as, $\vec{m} = I\vec{A}$, where the direction of the area vector $\vec{A}$ is given by the right-hand thumb rule. The dimensions of the magnetic moment are $[AL^2]$ and its unit is Am^2.

Q. In the expression, $\vec{\tau} = \vec{m} \times \vec{B}$, which of the vectors will be always perpendicular to each other ?

Solution : Torque ($\vec{\tau}$) and magnetic dipole moment ($\vec{m}$) will be always perpendicular to each other and torque and magnetic field $\vec{B}$ will be always perpendicular to each other.

Q. What is the value of torque acting on the loop if magnetic moment of the loop is either parallel or antiparallel to the magnetic field ?

Solution : zero

Q. When will be the current carrying loop in equilibrium in the external magnetic field ?

Solution : The current carrying loop will be in equilibrium in the external magnetic field if magnetic moment of the loop is either parallel or antiparallel to the magnetic field.

Q. When will be the current carrying loop in stable equilibrium in the external magnetic field ?

Solution : The current carrying loop will be in stable equilibrium in the external magnetic field if magnetic moment of the loop is parallel to the magnetic field.

Q. When will be the current carrying loop in unstable equilibrium in the external magnetic field ?

Solution : The current carrying loop will be in unstable equilibrium in the external magnetic field if magnetic moment of the loop is anti-parallel to the magnetic field.

Q. What is the magnetic dipole moment of a loop with N closely wound turns ?

Solution : $\vec{m} = NI\vec{A}$

Q. A 100 turn closely wound circular coil of radius 10 cm carries a current of 3.2 A. (a) What is the field at the centre of the coil ? (b) What is the magnetic moment of this coil ?

The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 2T in the horizontal direction exists such that initially the axis of the coil is in the direction of the field. The coil rotates through an angle of 90^0 under the influence of the magnetic field. (c) What are the magnitudes of the torques on the coil in the initial and final position ? (d) What is the angular speed acquired by the coil when it has rotated by 90^0 ? The moment of inertia of the coil is 0.1 kg m^2. [NCERT Solved Example 4.11]

Solution : (a) 2×10^{-3} T (b) 10 A m^2 (c) initial torque $\tau_i = 0$, final torque $\tau_f = 20$ N m. (d) 20 s^{-1}

Q. (a) A current-carrying circular loop lies on a smooth horizontal plane.Can a uniform magnetic field be set up in such a manner that the loop turns around itself (i.e., turns about the vertical axis). (b) A current-carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium ? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum. (c) A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to a circular shape ? [NCERT Solved Example 4.12]

Solution : (a) No, because that would require $\vec{\tau}$ to be in the vertical direction. But $\vec{\tau} = \mathbf{I}\vec{\mathbf{A}} \times \vec{\mathbf{B}}$, and since $\vec{\mathbf{A}}$ of the horizontal loop is in the vertical direction, τ would be in the plane of the loop for any $\vec{\mathbf{B}}$.
(b) Orientation of stable equilibrium is one where the area vector $\vec{\mathbf{A}}$ of the loop is in the direction of external magnetic field. In this orientation, the magnetic field produced by the loop is in the same direction as external field, both normal to the plane of the loop, thus giving rise to maximum flux of the total field.
(c) It assumes circular shape with its plane normal to the field to maximize flux, since for a given perimeter, a circle encloses greater area than any other shape.

Q. Prove that the magnetic field due to current in a circular current loop at very large distance from the centre has very similar behaviour to the electric field of an electric dipole.

Solution : The magnetic field on the axis of a circular loop, of a radius R, carrying a steady current I. The magnitude of this field is $\mathbf{B} = \dfrac{\mu_0 \mathbf{I}\mathbf{R}^2}{2(\mathbf{x}^2 + \mathbf{R}^2)^{3/2}}$.

Here, x is the distance along the axis from the centre of the loop. For x >> R, we may drop the R^2 term in the denominator. Thus, $\mathbf{B} = \dfrac{\mu_0 \mathbf{I}\mathbf{R}^2}{2\mathbf{x}^3}$. Note that the area of the loop A = πR^2. Thus, $\mathbf{B} = \dfrac{\mu_0 \mathbf{IA}}{2\pi \mathbf{x}^3}$.

As earlier, we define the magnetic moment $\vec{\mathbf{m}}$ to have a magnitude IA. $\vec{\mathbf{m}} = \mathbf{I}\vec{\mathbf{A}}$.

Hence, $\vec{\mathbf{B}} \cong \dfrac{\mu_0 \vec{\mathbf{m}}}{2\pi \mathbf{x}^3} = \dfrac{\mu_0}{4\pi}\dfrac{2\vec{\mathbf{m}}}{\mathbf{x}^3}$

The expression is very similar to an expression obtained earlier for the electric field of a dipole. The similarity may be seen if we substitute.

$$\mu_0 \to 1/\varepsilon_0,\ \vec{\mathbf{m}} \to \vec{\mathbf{p}}_e \text{ (electrostatic dipole)}, \vec{\mathbf{B}} \to \vec{\mathbf{E}} \text{ (electrostatic field)}$$

We then obtain, $\vec{\mathbf{E}} = \dfrac{2\vec{\mathbf{p}}_e}{4\pi\varepsilon_0 \mathbf{x}^3}$, which is precisely the field for an electric dipole at a point on its axis.

Q. What is the difference between the electric dipole and magnetic dipole ?

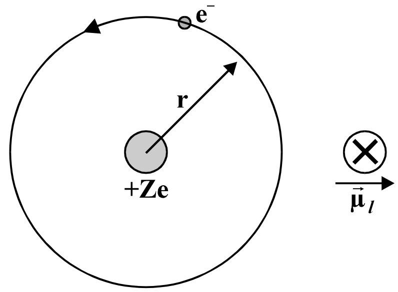
Solution : An electric dipole is built up of two elementary units - the charges (or electric monopoles) whereas a magnetic dipole (or a current loop) is the most elementary element in magnetism.

Q. Derive the expression for the magnetic dipole moment of a revolving electron in a circular orbit of radius r with the speed v.

OR

Consider an electron of charge (–e) is revolving with speed v in the hydrogen atom of radius r. Find the magnetic dipole moment of this electron.

Solution :



The electron of charge (–e) (e = + 1.6 × 10^{-19} C) performs uniform circular motion around a stationary heavy nucleus of charge +Ze. This constitutes a current I, where, $\mathbf{I} = \dfrac{\mathbf{e}}{\mathbf{T}}$ and T is the time period of revolution. Let r be the orbital radius of the electron, and v the orbital speed. Then, $\mathbf{T = \frac{2\pi r}{v}}$, substitution in $\mathbf{I = \frac{e}{T}}$.

We have I = ev/2πr. There will be a magnetic moment, usually denoted by μ_l, associated with this circulating current and its magnitude is, $\mu_l = I\pi r^2 = evr/2$.

Q. Relate the magnetic dipole moment of the revoving electron in Bohr atom with angular momentum of this electron.

Solution : $\mu_l = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} l$, Here, l is magnitude of the angular momentum of the electron about the central nucleus known as ("orbital" angular momentum). Vectorially, $\boldsymbol{\mu}_l = -\frac{e}{2m_e}\vec{l}$.

Q. What is gyromagnetic ratio ?

Solution : The ratio $\frac{\mu_1}{l} = \frac{e}{2m_e}$ is called the gyromagnetic ratio and is a constant. Its value is 8.8×10^{10}C/kg for an electron.

Q. What is Bohr magneton ? Find the value of this quantity ?

Solution : The orbital magnetic dipole moment of the circulating electron in the first orbit of hydrogen or Bohr atom is known as Bohr magneton. The angular momentum of electron in first orbit of hydrogen atom equals to $\frac{h}{2\pi}$, where h is known as Plank's constant.

$$\because \qquad \mu_l = \frac{e}{2m} l = \frac{e}{2m}\left(\frac{h}{2\pi}\right) = \frac{eh}{4\pi m}$$

The numerical value of μ_l is given by $\mu_l = \frac{eh}{4\pi m} = \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}} = 9.27 \times 10^{-24}\ \text{Am}^2$.

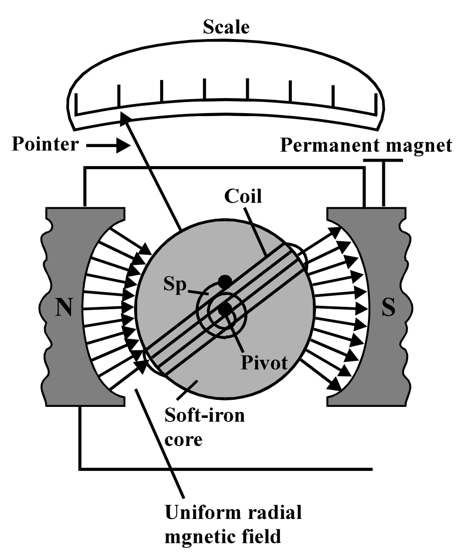
4.11 The Moving Coil Galvanometer :

Q. What is the moving coil galvanometer (MCG) ?

Solution : It is a device which is used to detect the current the current in the electrical circuit.

Q. With the help of a neat and labelled diagram, explain the underlying principle and working of a moving coil galvanometer.

Solution : The moving coil galvanometer is shown in the figure.



The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it. This torque is given by τ = NI AB, where the symbols have their usual meaning. Since the field is radial by design, we have taken sin θ = 1 in the above expression for the torque. A spring S_p provides a counter torque $k\phi$ that balances the magnetic torque NIAB; resulting in a steady angular deflection ϕ. In equilibrium $k\phi$ = NI AB where k is the torsional constant of the spring; i.e., the restoring torque per unit twist. The deflection ϕ is indicated on the scale by a pointer $\phi = \left(\frac{NAB}{k}\right)I$.

Q. What is the nature of magnetic field in moving coil galvanometer ?

Solution : The nature of the magnetic field is uniform and radial.

Q. Why is the nature of magnetic field radial in moving coil galvanometer ?

OR

What is the function of radial magnetic field in moving coil ganvanometer ?

Solution : The nature of magnetic field is radial in moving coil galvanometer such that the magnetic dipole moment of coil is always perpendicular to the magnetic field.

Q. How can the uniform radial magnetic field achieve in moving coil galvanometer ?

Solution : By using a cylindrical soft iron core, one can achieve the uniform radial magnetic field in moving coil galvanometer.

Q. Why is the soft iron core cylindrical in moving coil galvanometer ?

OR

What is the function of soft iron core in moving coil galvanometer ?

Solution : A cylindrical soft iron core makes the field radial and increases the strength of magnetic field. That's why, the soft iron core is cylindrical in moving coil galvanometer.

Q. What are the various uses of galvanometer ?

Solution : (i) The galvanometer can be used to check if a current is flowing in the circuit. (ii) The galvanometer can be converted into an ammeter and voltmeter.

Q. Why the galvanometer cannot be used as an ammeter ?

OR

Why is there is need to convert the galvanometer into ammeter to measure the current ?

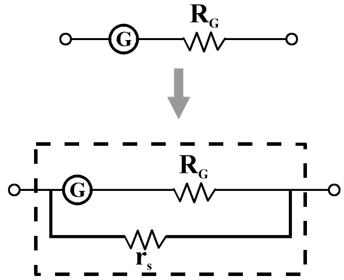
Solution : The galvanometer cannot as such be used as an ammeter to measure the value of the current in a given circuit. This is for two reasons (i) Galvanometer is a very sensitive device, it gives a full-scale deflection for a current of the order of μA. (ii) For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of the current in the circuit.

Q. How can you use a galvanometer as an ammeter ? Also find the resistance of this arrangement.

OR

Convert the galvanometer into ammeter.

Solution : To convert the galvanometer into ammeter, one attaches a small resistance r_s, called shunt resistance, in parallel with the galvanometer coil as shown in figure; so that most of the current passes through the shunt.



The resistance of this arrangement is, $R_G r_s/(R_G + r_s) \cong r_s$ if $R_G >> r_s$.

Q. Define the current sensitivity of a galvanometer. Which way we can increase the sensitivity of the galvanometer ?

Solution : The current sensitivity of the galvanometer is defined as the deflection per unit current, which is given by $\frac{\phi}{I} = \frac{NAB}{k}$.

A convenient way to increase the sensitivity is to increase the number of turns N.

Q. Why the resistance of ammeter should be small ?

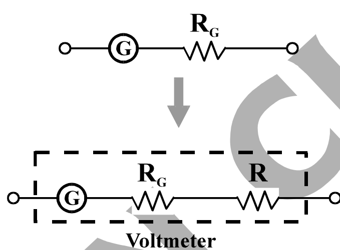
OR

Why a small resistance should be connected in parallel with galvanometer, to convert this into ammeter ?

Solution : If the resistance of the ammeter will be small then the effect of introducing the ammeter in the circuit is also small and negligible. Hence the ammeter will measure the accurate current. To achieve this, a small resistance should be connected in parallel with galvanometer, to convert galvanometer into ammeter.

Q. How can you convert the galvanometer in a volt meter ?

Solution :



The galvanometer can also be used as a voltmeter to measure the voltage across a given section of the circuit. To convert the galvanometer into voltmeter, a large resistance R is connected in series with the galvanometer. This arrangement is schematically depicted in figure. Note that the resistance of the voltmeter is now, $R_G + R = R$. As $R >> R_G$, hence the approximately resistance of the voltmeter is R.

Q. Define voltage sensitivity of galvanometer.

Solution : The voltage sensitivity is the deflection per unit voltage, which is given by $\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R}$.

Q. Why is the resistance of voltmeter very large ?

OR

Why should be a large resistance connected in series with galvanometer, to convert this into voltmeter ?

Solution : It must draw a very small current, otherwise the voltage measurement will disturb the original set up by an amount which is very large. That why resistance of voltmeter should be very large. To achieve this a large resistance should be connected in series with galvanometer, to convert the galvanometer into voltmeter.

Q. Comment on the following statement : By increasing the current sensitivity may not necessarily increased the voltage sensitivity.

OR

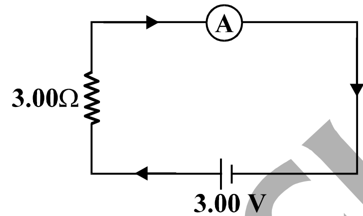
Comment on the following statement : The modification needed for conversion of a galvanometer to an ammeter will be different from what is needed for converting it into a voltmeter.

Solution : As the current sensitivity is proportional to number of turns hence, if number of terms (N) will become twice then current sensitivity will also become twice. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. Thus the voltage sensitivity remains unchanged because voltage sensitivity is equal to $\frac{NAB}{kR}$. So in general, the modification needed for conversion of a galvanometer to an ammeter will be different from what is needed for converting it into a voltmeter.

Q. Which one of the two, an ammeter or a milliammeter, has a higher resistance and why ?

Solution : A miliammeter has higher resistance than an ammeter. To measure the larger value of current, we require lower value of shunt resistance which will be connected in parallel with the galvanometer while converting it into ammeter. As we know the resistance of this arrangement nearly equals to the resistance of the shunt and hence a miliammeter has higher resistance than an ammeter.

Q. In the circuit the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance R_G = 60.00 Ω; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance r_s = 0.02 Ω; (c) is an ideal ammeter with zero resistance ?



Solution : (a) 0.048 A (b) 0.99 A (c) 1.00 A

NCERT EXERCISE

4.1 A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field $\vec{B}$ at the centre of the coil ?

4.2 A long straight wire carries a current of 35 A. What is the magnitude of the field $\vec{B}$ at a point 20 cm from the wire ?

4.3 A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of $\vec{B}$ at a point 2.5 m east of the wire.

4.4 A horizontal overhead power the carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line ?

4.5 What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30^0 with the direction of a uniform magnetic field of 0.15 T ?

4.6 A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire ?

4.7 Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

4.8 A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of 4.1 $\vec{B}$ inside the solenoid near its centre.

4.9 A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30^0 with the direction of a uniform horizontal magnetic field of magnitude 0.80 T.What is the magnitude of torque experienced by the coil ?

4.10 Two moving coil metres, M_1 and M_2 have the following particulars :

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

4.11 In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg).

4.12 In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron ? Explain.

4.13 (a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60^0 with the terminal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area ? (All other particulars are also unaltered.)

ADDITIONAL EXERCISES

4.14 Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

4.15 A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not feromagnetic.

4.16 For a circular coil of radius R and N turns carrying current I, the magnitude of the magnetic field at a point on its axis at a distance x from its centre is given by, $B = \dfrac{\mu_0 I R^2 N}{2(x^2 + R^2)^{3/2}}$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius R, and number of turns N, carrying equal currents in the same direction, and separated by a distance R. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to R, and is given by, $B = 0.72\dfrac{\mu_0 N I}{R}$, approximately.

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as Helmholtz coils]

4.17 A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

4.18 Answer the following questions :

(a) A magnetic field that varies in magnitude from point to point but has a constant direction (east or west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle ?

(b) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal to initial speed if it suffered no collision with the environment ?

(c) An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflection from its straight line path.

4.19 An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30^0 with the initial velocity.

4.20 A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^{-5} V m^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique ?

4.21 A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before ? (Ignore the mass of the wires) g = 9.8 m s^{-2}.

4.22 The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart ? Is the force attractive or repulsive ?

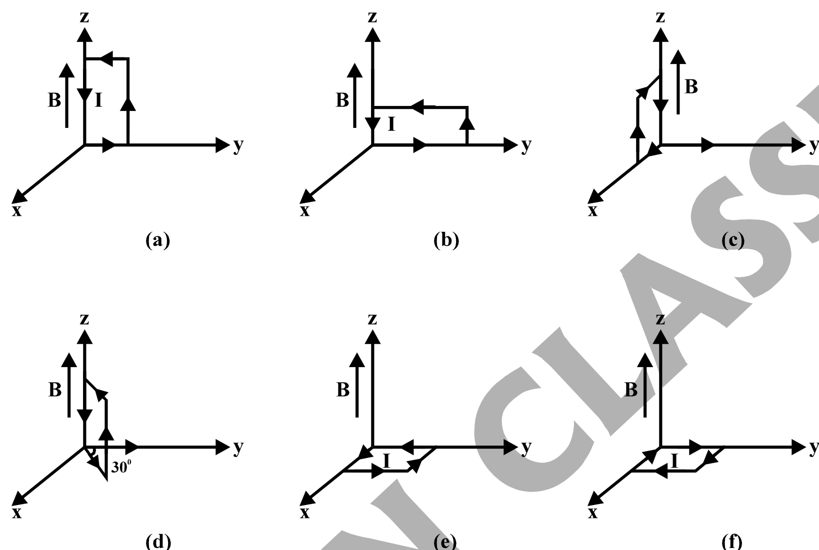
4.23 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis long east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) the wire intersects the axis,

(b) the wire is turned from N-S to northeast-northwest direction

(c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm ?

4.24 A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in the figure ? What is the force on each case ? Which case corresponds to stable equilibrium ?



4.25 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) total torque on the coil.

(b) total force on the coil.

(c) average force on each electron in the coil due to the magnetic field ?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}).

4.26 A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid arein the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire ? g = 9.8 m s^{-2}.

4.27 A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V ?

4.28 A galvanometer coil has a resistance of 15 Ω and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A ?

ANSWERS

4.1 $\pi \times 10^{-4}$ T $\cong$ 3.1×10^{-4} T

4.2 3.5×10^{-5} T

4.3 4×10^{-6} T, vertical up

4.4 1.2×10^{-5} T, towards south

4.5 0.6 N m^{-1}

4.6 8.1×10^{-2} N; direction of force given by Fleming's left-hand rule

4.7 2×10^{-5} N; attractive force normal to A towards B

4.8 $8\pi \times 10^{-3}$ T $\cong$ 2.5×10^{-2} T

4.9 0.96 N m

4.10 (a) 1.4, (b) 1

4.11 4.2 cm

4.12 18 MHz

4.13 (a) 3.1 Nm, (b) No, the answer is unchanged because the formula $\tau = N\,I\,A \times B$ is true for a planar loop of any shape.

4.14 $5\pi \times 10^{-4}$ T $\cong$ 1.6×10^{-3} T towards west.

4.15 Length about 50 cm, radius about 4 cm, number of turns about 400, current about 10 A. These particulars are not unique. Some adjustments with limits is possible.

4.16 (b) In a small region of length 2d about the mid-point between the coils.

$$B = \frac{\mu_0 I R^2 N}{2} \times \left[\left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-3/2} + \left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{-3/2}\right]$$

$$\cong \frac{\mu_0 I R^2 N}{2} \times \left(\frac{5R^2}{4}\right)^{-3/2} \times \left[\left(1+\frac{4d}{5R}\right)^{-3/2} + \left(1-\frac{4d}{5R}\right)^{-3/2}\right]$$

$$\cong \frac{\mu_0 I R^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{3/2} \times \left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

where in the second and third steps above, terms containing d^2/R^2 and higher powers of d/R are neglected since $\frac{d}{R} << 1$. The terms linear in d/R cancel giving a uniform field B in a small region :

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 I N}{R} \cong 0.72 \frac{\mu_0 I N}{R}$$

4.17 Hint : B for a toroid is given by the same formula as for a solenoid : $B = \mu_0 nI$, where n in this case is given by $n = \frac{N}{2\pi r}$. The field is non-zero only inside the core surrounded by the windings. (a) Zero, (b) 3.0×10^{-2} T, (c) zero. Note, the field varies slightly across the cross-section of the toroid as r varies from the inner to outer radius. Answer (b) corresponds to the mean radius r = 25.5 cm.

4.18 (a) Initial $\vec{v}$ is either parallel or anti-parallel to $\vec{B}$.

(b) Yes, because magnetic force can change the direction of $\vec{v}$, not its magnitude.

(c) $\vec{B}$ should be in a vertically downward direction.

4.19 **(a) Circular trajectory of radius 1.0 mm normal to $\vec{B}$.**

(b) Helical trajectory of radius 0.5 mm with velocity component 2.3×10^7 m s^{-1} along $\vec{B}$.

4.20 **Deuterium ions or deuterons; the answer is not unique because only the ratio of charge to mass is determined. Other possible answers are He^{++}, Li^{+++}, etc.**

4.21 **(a) A horizontal magnetic field of magnitude 0.26 T normal to the conductor in such a direction that Fleming's left-hand rule gives a magnetic force upward.**

(b) 1.176 N.

4.22 **1.2 N m^{-1}; repulsive. Note, obtaining total force on the wire as $1.2 \times 0.7 = 0.84$ N, is only approximately correct because the formula $F = \frac{\mu_0}{2\pi r} I_1 I_2$ for force unit length is strictly valid for infinitely long conductors.**

4.23 **(a) 2.1 N vertically downwards.**

(b) 2.1 N vertically downwards (true for any angle between current direction and $\vec{B}$ since $l \sin\theta$ remains fixed, equal to 20 cm)

(c) 1.68 N vertically downwards.

4.24 **Use $\tau = I\vec{A} \times \vec{B}$ and $\vec{F} = I\vec{l} \times \vec{B}$**

(a) 1.8×10^{-2} N m along –y direction

(b) same as in (a)

(c) 1.8×10^{-2} N m along –x direction

(d) 1.8×10^{-2} N m at an angle of 240^0 with the +x direction

(e) zero

(f) zero

Force is zero in each case. Case (e) corresponds to stable, and case (f) corresponds to unstable equilibrium.

4.25 **(a) Zero, (b) Zero, (c) force on each electron is evB = IB/(nA) = 5×10^{-25} N.**

Note : Answer (c) denotes only the magnetic force.

4.26 **108 A**

4.27 **Resistance in series = 5988 Ω**

4.28 **Shunt resistance = 10 mΩ**

ADDITIONAL QUESTIONS AND PROBLEMS

Q. **Derive the expression for magnetic field due to circular loop of radius R at the centre of loop. The loop is carrying the current I.**

Q. **Using Ampere's loop law, derive the expression of magnetic field due to infinite current carrying wire at the distance r from the wire. The wire is carrying the current I.**

Q. **Prove that oppositely directed currents repel each other and find the force with which they will repel each other.**

Q. **Prove that a circular current loop is treated as a magnetic dipole.**

Q. **A straight wire of length $\frac{\pi}{2}$m is bent into a circular shape. If the wire was to carry a current of 5 A, calculate the magnetic field due to it, before bending at a point distance 0.01 times the radius of the circle formed from it. Also calculate the magnetic field at the centre of the circular loop formed for the same value of current.**

A. 4×10^{-4}T, 1.256×10^{-5}T

Q. **(a)** **Which one of the following will expeirence maximum force, when projected with the same velocity 'v' perpendicular to the magnetic field :**

(i) α-particle (ii) β-particle (iii) proton (iv) deutron

(b) **Which one of the following will describe the smallest circle when projected with the same velocity v perpendicular to the magnetic field B : (i) α-particle (ii) β-particle (iii) proton (iv) deutron ? Also compare the time period to complete one revolution and frequency of revolution. Does their speed change in the magnetic field, give reasons ?**

Q. **A charge q moving in a straight line is accelerated by a potential difference V. It enters a uniform magnetic field B perpendicular to its path. Deduce in terms of V an expression for the radius of the circular path in which it travels.**

Q. **A long straight conductor PQ, carrying a current of 60 A, is fixed horizontally. Another long conductor XY is kept parallel to PQ at a distance of 4 mm in air. Conductor XY is free to move and carries a current I. Calculate the magnitude and direction of current I for which the magnetic repulsion just balances the weight of conductor XY. Mass per unit length for conductor XY is 10^{-2} kg m^{-1}.**

A. 32.67 A

Q. **The coil of a galvanometer has a resistance of 100 Ω. It shows full scale deflection for a current of 5×10^{-4}A. How will you convert it into a voltmeter to read a maximum potential difference of 5 V ?**

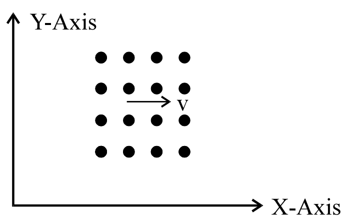
A. 9900 Ω

Q. **Calculate the value of resistance needed to convert a galvanometer of resistance 120 Ω, which gives a full scale deflection for a current of 5 mA, into a voltmeter of 0 - 50 V range.**

Q. **A galvanometer having a coil resistance of 100 Ω gives full scale deflection when a current of 1 mA is passed through it. Calculate the value of resistance required to convert it into an ammeter of the range of 1 A.**

A. 0.1 Ω

Q. **In the figure**



What should be magnitude and direction of electric field so that charge goes undeviated ? The magnitude of magnetic field is B.

A. The magnitude of electric field is vB and the direction will be along positive y-axis.