## CURRENT ELECTRICITY

### 3.2 Electric Current :

## Q. Define current? What is the SI unit of current?

Solution : Let $\Delta \mathrm{Q}$ be the net charge flowing across a cross-section of a conductor during the time interval $\Delta t$ [i.e., between times $t$ and $(t+\Delta t)]$. Then, the current at time $t$ across the cross-section of the conductor is defined as the value of the ratio of $\Delta \mathrm{Q}$ to $\Delta \mathrm{t}$ in the limit of $\Delta \mathrm{t}$ tending to zero, $\mathbf{I}(\mathbf{t})=\lim _{\Delta t \rightarrow 0} \frac{\Delta \mathbf{Q}}{\Delta t}=\frac{\mathbf{d Q}}{\mathbf{d t}}$. In SI units, the unit of current is ampere.

## Q. Although current has the direction but it is a scalar quantity. Explain?

Solution : Algebraic operations are performed on this quantity that's why current is a scalar quantity although it has direction.

### 3.3 Electric Current in Conductors :

Q. Which charge particles will constitute current in conductor?

Solution : Electrons
Q. Why there is no current in the conductor in the absence of electric field ?

Solution : The electrons will be moving due to thermal motion during which they collide with the fixed ions. An electron colliding with an ion emerges with the same speed as before the collision. However, the direction of its velocity after the collision is completely random. At a given time, there is no preferential direction for the velocities of the electrons. Thus on the average, the number of electrons travelling in any direction will be equal to the number of electrons travelling in the opposite direction. So, there will be no net electric current.

### 3.4 Ohm's Law :

Q. What is Ohm's Law?

Solution : Imagine a conductor through which a current I is flowing and let V be the potential difference between the ends of the conductor. Then Ohm's law states that $\mathrm{V} \propto \mathrm{I}$ or, $\mathrm{V}=\mathrm{RI}$, where the constant of proportionality R is called the resistance of the conductor.

## Q. On what factors resistance of a coductor depends ?

Solution : From $\mathbf{R}=\rho \frac{l}{\mathbf{A}}$, where $\rho$ is the resistivity depends on the material of the conductor. $l$ is the length and A is the cross-sectional area of the conductor. Thus resistance depends also on the dimensions of the conductor.
Q. What is current density? Is it a vector or scalar? Write down its unit.

Solution : Current per unit area (taken normal to the current), I/A, is called current density and is denoted by j. It is a vector quantity. The SI unit of the current density is $\mathrm{A} / \mathrm{m}^{2}$.
Q. How current density is related to electric field applied in the conductor?

Solution : If E is the magnitude of uniform electric field in the conductor whose length is $l$, then the potential difference V across its ends is El . From Ohm's Law $\mathrm{V}=\mathrm{IR}$ and $\mathbf{R}=\rho \frac{\boldsymbol{l}}{\mathbf{A}}$,

$$
\mathbf{E} \boldsymbol{l}=\mathbf{I} \frac{\rho l}{\mathbf{A}} \Rightarrow \mathbf{E}=\frac{\mathbf{I}}{\mathbf{A}} \rho \Rightarrow \mathbf{E}=\mathbf{J} \rho \Rightarrow \mathbf{J}=\frac{\mathbf{1}}{\rho} \mathbf{E} \Rightarrow \mathbf{J}=\sigma \mathbf{E}
$$

### 3.5 Drift of Electrons and the Origin of Restivity :

Q. What is drift velocity of electron and how does it depend on relaxation or collision time ' $\tau$ ' ?

Solution : An electron will suffer collisions with the heavy fixed ions, but after collision, it will emerge with the same speed but in random directions. If we consider all the electrons, their average velocity will be zero since their directions are random. Thus, if there are N electrons and the velocity of the ith electron $(i=1,2,3, \ldots N)$ at a given time is $v_{i}$, then $\frac{\mathbf{1}}{\mathbf{N}} \sum_{i=1}^{\mathbf{N}} \overrightarrow{\mathbf{v}}_{\mathbf{i}}=\mathbf{0}$.

Consider now the situation when an electric field is present. Electrons will be accelerated due to this field by $\overrightarrow{\mathbf{a}}=\frac{-\mathbf{e} \overrightarrow{\mathbf{E}}}{\mathbf{m}}$, where -e is the charge and $m$ is the mass of an electron. Consider again the ith electron at a given time $t$. This electron would have had its last collision some time before $t$, and let $t_{i}$ be the time elapsed after its last collision. If $\mathrm{v}_{\mathrm{i}}$ was its velocity immediately after the last collision, then its velocity $\mathrm{V}_{\mathrm{i}}$ at time $\mathrm{t}_{\mathrm{i}}$
is $\overrightarrow{\mathbf{V}}_{\mathbf{i}}=\overrightarrow{\mathbf{v}}_{\mathbf{i}}-\frac{\mathbf{e} \overrightarrow{\mathbf{E}}}{\mathbf{m}} \mathbf{t}_{\mathbf{i}}$.
The average velocity of the electrons at time $t$ is the average of all the $V_{i}{ }_{i} s$. The average of $v_{i}$ 's is zero, since immediately after any collision, the direction of the velocity of an electron is completely random. The collisions of the electrons do not occur at regular intervals but at random times. Let us denote by $\tau$, the average time between successive collisions. The average value of $t_{i}$ then is $\tau$ (known as relaxation time).
Thus average value of $\overrightarrow{\mathbf{V}}_{\mathbf{i}}$ over the N-electrons at any given time $t$ gives us for the average velocity $v_{d}$

$$
\overrightarrow{\mathbf{v}}_{\mathrm{d}}=\left(\overrightarrow{\mathbf{V}}_{i}\right)_{\text {average }}=\left(\overrightarrow{\mathbf{v}}_{\mathbf{i}}\right)_{\text {average }}-\frac{\mathbf{e} \overrightarrow{\mathbf{E}}}{\mathrm{m}}\left(\mathbf{t}_{\mathbf{i}}\right)_{\text {average }}=0-\frac{\mathbf{e} \overrightarrow{\mathbf{E}}}{m} \tau=-\frac{\mathbf{e} \overrightarrow{\mathbf{E}}}{m} \tau
$$

In the above equation the velocity $\overrightarrow{\mathbf{v}}_{\mathbf{d}}$ is called the drift velocity.
Q. Establish the relation between current $I$ and drift velocity $v_{d}$ for electrons in conductor of cross-sectional area $A$. The number density of electrons is $n$.

## Solution :



Consider a planar area A, located inside the conductor such that the normal to the area is parallel to E. Then because of the drift, in an infinitesimal amount of time $\Delta t$, all electrons to the left of the area at distance upto $\left|y_{d}\right| \Delta t$ would have crossed the area. If $n$ is the number of the free electrons per unit volume in the metal, then there are $n \Delta t\left|V_{d}\right|$ A such electrons. Since each electron carries a charge -e, the total charge transported across this area A to the right in time $\Delta \mathrm{t}$ is $-\mathrm{ne} \mathrm{A}\left|\mathrm{v}_{\mathrm{d}}\right| \Delta \mathrm{t}$. E is directed towards the left and hence the total charge transported along E across the area is negative of this. The amount of charge crossing the area A in time $\Delta t$ is I $\Delta t$, where I is the magnitude of the current. Hence, I $\Delta t=+n$ e A $\left|v_{d}\right| \Delta t \Rightarrow I=n e A v_{d}$

## Q. How current density is related to drift speed?

Solution : From $I=n e A v_{d}$ and $I=|\overrightarrow{\mathbf{j}}|$ A, we will get $\mathbf{v}_{\mathbf{d}}=\frac{\mathbf{j}}{\mathbf{n e}}$.
Q. How the conductivity and resistivity of conductors depends on the relaxation time? Derive it.

Solution : From the relation $v_{d}=\frac{j}{n e}$ and $v_{d}=\frac{e E}{m} \tau$, we have $\frac{j}{n e}=\frac{e E}{m} \tau$.
$\because j=\sigma E$, we will have the relation $\sigma=\frac{\mathbf{n e}^{\mathbf{2}}}{\mathbf{m}} \tau$.
Q. Show the resistance of a conductor is given by $\frac{\mathrm{m} l}{\mathrm{Ane}^{2} \tau}$, where symbols have their usual meanings. Solution : From $\mathbf{R}=\frac{\boldsymbol{l}}{\sigma \mathbf{A}}$ and $\sigma=\frac{\mathbf{n e}^{\mathbf{2}}}{\mathbf{m}} \tau$, we will have the relation $\mathbf{R}=\frac{\mathbf{m} \boldsymbol{l}}{\mathbf{A n e}^{2} \tau}$.
Q. Define 'relaxation time' of electrons in a conductor. Explain how it varies with increase in temperature of a conductor. State the relation between resistivity and relaxation time.
Solution : The average time between successive collisions of conduction electrons in conductor is known as relaxation time.
Due to increase of temperature, the thermal speed of electrons will increase and hence the collision frequency will increase i.e., if the temperature will increase then relaxation time will decrease and vice versa.

The relation between resistivity and relaxation time is given by $\rho=\frac{\mathbf{m}}{\mathbf{n e}^{2} \tau}$.
Q. (a) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7} \mathrm{~m}^{2}$ carrying a current of 1.5 A . Assume that each copper atom contributes roughly one conduction electron. The density of copper is $9.0 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$, and its atomic mass is 63.5 u . (b) Compare the drift speed obtained above with, (i) thermal speeds of copper atoms at ordinary temperatures, (ii) speed of propagation of electric field along the conductor which causes the drift motion. [NCERT Solved Example 3.1]
Solution : (a) $1.1 \mathrm{~mm} / \mathrm{s}$ (b) (i) At a temperature T, the thermal speed of a copper atom of mass M is obtained from $\left[<(1 / 2) \mathrm{Mv}^{2}>=(3 / 2) \mathrm{k}_{\mathrm{B}} \mathrm{T}\right]$ and is thus typically of the order of $\sqrt{\mathbf{k}_{\mathbf{B}} \mathbf{T} / \mathbf{M}}$, where $\mathrm{k}_{\mathrm{B}}$ is the Boltzmann constant. For copper at 300 K , this is about $2 \times 10^{2} \mathrm{~m} / \mathrm{s}$. This figure indicates the random vibrational speeds of copper atoms in a conductor. Note that the drift speed of electrons is much smaller, about $10^{-5}$ times the typical thermal speed at ordinary temperatures. (ii) An electric field travelling along the conductor has a speed of an electromagnetic wave, namely equal to $3.0 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$. The drift speed is, in comparison, extremely small; smaller by a factor of $10^{-11}$
Q. (a) In Example 3.1, the electron drift speed is estimated to be only few $\mathrm{mm} \mathrm{s}^{-1}$ for currents in the range of a few amperes? How then is current established almost the instant a circuit is closed ?
(b) The electron drift arises due to the force experienced by electrons in the electric field inside the conductor. But force should cause acceleration. Why then do the electrons acquire a steady average drift speed?
(c) If the electron drift speed is so small, and the electron's charge is small, how can we still obtain large amounts of currents in a conductor?
(d) When electrons drift in metal from lower to higher potential, does it mean that all the 'free' electrons of the metal are moving in the same direction?
(e) Are the paths of electrons straight lines between successive collisions (with the positive ions of the metal) in the (i) absence of electric field, (ii) presence of electric field? [NCERT Solved Example 3.2]
Solution : (a) Electric field is established throughout the circuit, almost instantly (with the speed of light) causing at every point a local electron drift. Establishment of a current does not have to wait for electrons from one end of the conductor travelling to the other end. However, it does take a little while for the current to reach its steady value.
(b) Each 'free' electron does accelerate, increasing its drift speed until it collides with a positive ion of the metal. It loses its drift speed after collision but starts to accelerate and increases its drift speed again only to suffer a collision again and so on. On the average, therefore, electrons acquire only a drift speed.
(c) Simple, because the electron number density is enormous, $\sim 10^{29} \mathrm{~m}^{-3}$.
(d) By no means. The drift velocity is superposed over the large random velocities of electrons.
(e) In the absence of electric field, the paths are straight lines; in the presence of electric field, the paths are, in general, curved.
Q. What is mobility of electrons and how does it depend on relaxation time?

Solution : The mobility $\mu$ defined as the magnitude of the drift velocity per unit electric field : $\boldsymbol{\mu}=\frac{\left|\mathbf{v}_{\mathbf{d}}\right|}{\mathbf{E}}$ we have $\mathbf{v}_{\mathbf{d}}=\frac{\mathbf{e} \tau \mathbf{E}}{\mathbf{m}}$. Hence, $\boldsymbol{\mu}=\frac{\mathbf{v}_{\mathbf{d}}}{\mathbf{E}}=\frac{\mathbf{e} \tau}{\mathbf{m}}$, where $\tau$ is the average collision time for electrons.
Q.What is SI unit of mobility ?

Solution : The SI unit of mobility is $\mathrm{m}^{2} / \mathrm{Vs}$
Q. What is the order of mobility of electrons in conductor in unit ( $\left.\mathrm{cm}^{2} / \mathrm{Vs}\right)$ ?

Solution : The order of mobility is $10^{4}$ in practical units $\left(\mathrm{cm}^{2} / \mathrm{Vs}\right)$.
Q. Is mobility positive ?

Solution : Yes, mobility is positive.
3.6 Limitation of Ohm's Law :
Q. Draw the graph between voltage and current for Ohm's Law. What is the nature of graph? Which quantity is represented by the slope of this graph?

Solution :


The nature of graph is linear. The slope of the graph represents resistance of conductors.
Q. Name the material where Ohm's Law is not applicable ?

Solution : Semi-conductor material.
Q. Name the electronic device where Ohm's law is not applicable.

Solution : Diode
Q. Draw the graph for the variation of current vs. voltage for GaAs.

Solution :


### 3.7 Resistivity of Various Materials :

Q. What is the order of resistivity of metal ?

Solution : $10^{-8} \Omega \mathrm{~m}$ to $10^{-6} \Omega \mathrm{~m}$
Q. What are different types of commercially produced resistors?

Solution : Commercially produced resistors are of two types : wire bound resistors and carbon resistors.
Q. Write down the properties or characteristics of wire bound resistors (manganin, constantan, nichrome) ? OR Write special characteristics of manganin due to which it is used in making standard resistances. OR Why do we prefer manganin or constanton wire for making standard Resistance coils ?

Solution : Wire bound resistors are made by winding the wires of an alloy, viz, manganin, constantan, nichrome or similar ones. The choice of these materials is dictated mostly by the fact that their resistivities are relatively insensitive to temperature.
Q. What is the range of resistance of wire bound resistors?

Solution : These resistances are typically in the range of a fraction of an ohm to a few hundren ohms.
Q. Why do we prefer carbon resistors for higher range of resistance ?

Solution : They are compact and inexpensive, widely used in electronic circuits.
Q. The colours of a carbon resistor are orange, blue, yellow and gold. Find the value of resistance and its tolerence value?
Solution : $36 \times 10^{4} \Omega$ with a tolerance value of $5 \%$.
Q. A carbon resistor of $74 \mathrm{k} \Omega$ is to be marked with rings of different colours for its identification. Write the sequence of colours.
Solution : The sequence of the colour is violet, yellow and orange.
Q. The sequence of bands marked on a carbon resistor are yellow, red, orange and silver. What is its (i) resistance and (ii) tolerance?

Solution : $42 \mathrm{k} \Omega$ with a tolerance value of $10 \%$
3.8 Temperature Dependence of Resistivity :
Q. Define the term temperature coefficient of resistivity. Write its SI unit.

Solution : The temperature coefficient of resistivity is defined as $\alpha=\frac{1}{\rho}\left(\frac{d \rho}{d T}\right)$ where $\rho$ is known as resistivity. The unit of $\alpha$ is $\left({ }^{0} \mathrm{C}\right)^{-1}$.
Q. How the resistivity of the conductor changes with temperature?

Solution : The resistivity of the conductor increases with increase of temperature. The resistivity of the conductor is given by $\rho_{T}=\rho_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$ where $\rho_{0}$ is the resistivity at reference temperature $T_{0}$ and $\rho_{T}$ is the resistivity at temperature T . The factor $\alpha$ is called the temperature coefficient of resistivity.
Q. Plot a graph showing the variation of resistivity of copper, nichrome and semi-conductor with temperature.


## Q. Name some materials whose resistivity have weak dependence on temperature.

Solution : Nichrome, Manganin and constantan
Q. An electric toaster uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature $\left(27.0^{\circ} \mathrm{C}\right)$ is found to be $75.3 \Omega$. When the toaster is connected to a 230 V supply, the current settles, after a few seconds, to a steady value of 2.68 A . What is the steady temperature of the nichrome element ? The temperature coefficient of resistance of nichrome averaged over the temperature range involved, is $1.70 \times 10^{-4}{ }^{\mathbf{0}} \mathrm{C}^{\mathbf{- 1}}$. [NCERT Solved Example 3.3]
Solution : $847^{\circ} \mathrm{C}$
Q. How does the resistance of conductor depend on temperature?

Solution : The resistance of conductor increases with increase of temperature. The resistance of the conductor is given by $R_{T}=R_{0}\left[1+\alpha\left(T-T_{0}\right)\right]$ where $R_{0}$ is the resistance at reference temperature $T_{0}$ and $R_{T}$ is the resistance at temperature T . The factor $\alpha$ is called the temperature coefficient of resistance.
Q. The resistance of the platinum wire of a platinum resistance thermometer at the ice point is $5 \Omega$ and at steam point is $5.23 \Omega$. When the thermometer is inserted in a hot bath, the resistance of the platinum wire is $5.795 \Omega$. Calculate the temperature of the bath. [NCERT Solved Example 3.4]
Solution : $345.65{ }^{\circ} \mathrm{C}$
Q. V - I graph for a given metallic wire at two temperatures are shown, which of these is for a higher temperature?


Solution: $\mathrm{T}_{2}>\mathrm{T}_{1}$
3.9 Electrical Energy, Power :
Q. Define Electric Power. Write down the SI unit of power.

Solution : The energy delivered or energy dissipiated per unit time in electrical circuit is known as electric power which is defined as the product of current passing through the circuit and voltage across the circuit i.e. $\mathrm{P}=\mathrm{V}$ I. SI unit of electric power is 1 watt $(1 \mathrm{~W})$, where 1 watt $=1$ volt $\times 1$ ampere.
Q. What is the practical unit of electric energy ?

Solution : The practical unit of electric energy is kWh and 1 kWh equal to $3.6 \times 10^{6} \mathrm{~J}$.
Q. Which one will have greater resistance : (i) 1 kW electric heater at 220 V (ii) 100 W bulb at 220 V (iii) 1 kW bulb at 220 V (iv) $\mathbf{6 0 W}$ bulb at 220 V ?

Solution : 60 W bulb at 220 V .
Q. What happens to the power dissipation if the value of electric current passing through a conductor of constant resistance is doubled ?
Solution : From $\mathrm{P}=I^{2} R, \mathrm{P}$ is directly proportional to $\mathrm{I}^{2}$ if R is constant. Hence if current will become twice then the power dissipation will become four times.
Q. Two bulbs whose resistances are in the ratio of $1: 2$ are connected in parallel to a source of constant voltage. What will be the ratio of power dissipation in these?
Solution : From $\mathrm{P}=\mathrm{V}^{2} / \mathrm{R}, \mathrm{P}$ is inversely proportional to R when V is constant. Hence the ratio of power dissipation is $2: 1$.
Q. A heater joined in series with a 60 W bulb is connected to the mains. If 60 W bulb is replaced by a 100 W bulb, will the rate of heat produced by the heater be more, less or remain the same ?
Solution : The resistance of a 100 W bulb is less than that of 60 W bulb. Hence, on joining 100 W bulb (instead of 60 W bulb) with heater, the resistance of the circuit decreases and consequently, circuit current increases. Hence, heat produced by the heater rises.
Q. Two heater wires of the same dimensions are first connected in series and then in parallel to a source of supply. What will be the ratio of heat produced in the two cases?

Solution : From $H=\frac{V^{2} t}{R}$, we have $\frac{\mathbf{H}_{\text {series }}}{\mathbf{H}_{\text {parallel }}}=\frac{\mathbf{R}_{\mathbf{p}}}{\mathbf{R}_{\mathrm{s}}}=\frac{\mathbf{R} / \mathbf{2}}{2 R}=\frac{\mathbf{1}}{\mathbf{4}}$.
3.10 Combinations of Resistors - Series and Parallel :
Q. Consider two resistors of resistance $R_{1}$ and $R_{2}$ are connected in series combination. Derive the expression for equivalent resistance.

Solution


Consider two resistors $R_{1}$ and $R_{2}$ in series as shown in above figure. The charge which leaves $R_{1}$ must be entering $\mathrm{R}_{2}$. Since current measures the rate of flow of charge, this means that the same current I flows through $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$. By Ohm's law :

Potential difference across $\mathrm{R}_{1}=\mathrm{V}_{1}=\mathrm{I} \mathrm{R}_{1}$, and Potential difference across $\mathrm{R}_{2}=\mathrm{V}_{1}=\mathrm{I} \mathrm{R}_{2}$.
The potential difference V across the combination is $\mathrm{V}_{1}+\mathrm{V}_{2}$. Hence, by Ohm's law is $\mathbf{R}_{\mathrm{eq}} \equiv \frac{\mathbf{V}}{\mathbf{I}}=\left(\mathbf{R}_{\mathbf{1}}+\mathbf{R}_{\mathbf{2}}\right)$
Q. Consider two resistors of resistance $R_{1}$ and $R_{2}$ are connected in parallel combination. Derive the expression for equivalent resistance.

Solution


Consider the parallel combination of two resistors as shown in above figure. The charge that flows in at A from the left rows out partly through $\mathrm{R}_{1}$ and partly through $\mathrm{R}_{2}$. The currents $\mathrm{I}, \mathrm{I}_{1}, \mathrm{I}_{2}$ shown in the figure are the rates of flow of charge at the points indicated. Hence, $I=I_{1}+I_{2}$.
The potential difference between $A$ and $B$ is given by the Ohm's law applied to $R_{1}, V=I_{1} R_{1}$
Also, Ohm's law applied to $\mathrm{R}_{2}$ gives $\mathrm{V}=\mathrm{I}_{2} \mathrm{R}_{2}$
$\therefore \quad I=I_{1}+I_{2}=\frac{V}{R_{1}}+\frac{V}{R_{2}}=V\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)$
If the combination was replaced by an equivalent resistance $R_{\text {eq }}$, we would have, by Ohm's law
$I=\frac{V}{R_{\text {eq }}}$

Hence, $\frac{\mathbf{1}}{\mathbf{R}_{\mathrm{eq}}}=\frac{\mathbf{1}}{\mathbf{R}_{\mathbf{1}}}+\frac{\mathbf{1}}{\mathbf{R}_{\mathbf{2}}}$

### 3.11 Cells, EMF, Internal Resistance :

Q. Define EMF (electro motive force) of a battery.

Solution : This is the potential difference between the positive and negative terminal of the cell when there is no current through it.
Q. What is the potential difference between the terminal of the battery of emf $E$ and internal resistance $r$ when current I will be drawn from the battery?
Solution : E - Ir
$Q$. A cell of emf $E$ and internal resistance $r$ is connected across a load of resistance $R$. Draw the graph for the variation of current through $R$ if $R$ will change. Under what condition there will be maximum current drawn from the cell and what is value of this? Why in most cells the maximum allowed current is much lower than the maximum value?

Solution :


The maximum current that can be drawn from a cell is for $R=0$ and it is $I_{\max }=E / r$.
In most cells the maximum allowed current is much lower than this to prevent permanent damage to the cell.
Q. A network of resistors is connected to a 16 V battery with internal resistance of $1 \Omega$ as shown in figure

(a) Compute the equivalent resistance of the network. (b) Obtain the current in each resistor.
(c) Obtain the voltage drops across each resistor. [NCERT Solved Example 3.5]

Solution : (a) $7 \Omega$ (b) $4 / 3 \mathrm{~A}$ (c) $4 \mathrm{~V}, 2 \mathrm{~V}, 8 \mathrm{~V}$

### 3.12 Cells in Series And in Parallel :

Q. Consider two cells of emf's $\varepsilon_{1}$ and $\varepsilon_{2}$ and internal resistance $r_{1}$ and $r_{2}$ are connected in series combination with same polarity. Find the equivalent cell i.e., equivalent emf and internal resistance ? Also find the equivalent emf and internal resistance if the cells are connected with opposite polarity?

Solution


Consider first two cells in series, where one terminal of the two cells is joined together leaving the other terminal in either cell free. $\varepsilon_{1}, \varepsilon_{2}$ are the emf's of the two cells and $r_{1}, r_{2}$ their internal resistances, respectively.
Let $V(A), V(B), V(C)$ be the potentials at point $A, B$ and $C$ shown in figure. Then $V(A)-V(B)$ is the potential difference between the positive and negative terminals of the first cell.
$\mathrm{V}_{\mathrm{AB}} \equiv \mathrm{V}(\mathrm{A})-\mathrm{V}(\mathrm{B})=\varepsilon_{1}-\mathrm{Ir}_{1}$
Similarly, $\mathrm{V}_{\mathrm{BC}} \equiv \mathrm{V}(\mathrm{B})-\mathrm{V}(\mathrm{C})=\varepsilon_{2}-\mathrm{Ir}_{2}$
Hence, the potential difference between the terminals A and C of the combination is
$V_{A C} \equiv V(A)-V(C)=[V(A)-V(B)]+[V(B)-V(C)]=\left(e_{1}+e_{2}\right)-I\left(r_{1}+r_{2}\right)$
If we wish to replace the combination by a single cell between $A$ and $C$ of emf $\varepsilon_{\text {eq }}$ and internal resistance $r_{\text {eq }}$, we would have
$\mathrm{V}_{\mathrm{AC}}=\varepsilon_{\mathrm{eq}}-\mathrm{I} \mathrm{r}_{\mathrm{eq}}$
Comparing the least two equations, we get
$\varepsilon_{\mathrm{eq}}=\varepsilon_{1}+\varepsilon_{2}$ and $\mathrm{r}_{\mathrm{eq}}=\mathrm{r}_{1}+\mathrm{r}_{2}$
In figure, we had connected the negative electrode of the first to the positive electrode of the second. If instead we connect the two negatives. Equation would change to $\mathrm{V}_{\mathrm{BC}}=-\varepsilon_{2}-\mathrm{Ir}_{2}$ and we will get
$\varepsilon_{\text {eq }}=\varepsilon_{1}-\varepsilon_{2}\left(\varepsilon_{1}>\varepsilon_{2}\right)$
Q. What is the rule for equivalent emf's and equivalent internal resistance of series combination of cells?

Solution : The rule for series combination clearly can be extended to any number of cells.
(i) The equivalent emf of a series combination of $n$ cells is just the sum of their individual emf's, and
(ii) The equivalent internal resistance of a series combination of $n$ cells is just the sum of their internal resistances.
Q. Consider two cells of emf's $\varepsilon_{1}$ and $\varepsilon_{2}$ and internal resistance $r_{1}$ and $r_{2}$ are connected in parallel combination with same polarity. Find the equivalent cell i.e., equivalent emf and internal resistance? Also find the equivalent emf and internal resistance if the cells are connected with opposite polarity?


Consider a parallel combination of the cells. $I_{1}$ and $I_{2}$ are the currents leaving the positive electrodes of the cells. At the point $B_{1}, I_{1}$ and $I_{2}$ flows in whereas the current I flows out. Since as much charge flows in as out, we have $\mathrm{I}=\mathrm{I}_{1}+\mathrm{I}_{2}$
Let $V\left(B_{1}\right)$ and $V\left(B_{2}\right)$ be the potentials at $B_{1}$ and $B_{2}$, respectively. Then, considering the first cell, the potential difference across its terminals is $V\left(B_{1}\right)-V\left(B_{2}\right)$. Hence, from equation,
$\mathrm{V} \equiv \mathrm{V}\left(\mathrm{B}_{1}\right)-\mathrm{V}\left(\mathrm{B}_{2}\right)=\varepsilon_{1}-\mathrm{I}_{1} \mathrm{r}_{1}$
Points $B_{1}$ and $B_{2}$ are connected exactly similarly to the second cell. Hence considering the second cell, we also have $\mathrm{V} \equiv \mathrm{V}\left(\mathrm{B}_{1}\right)-\mathrm{V}\left(\mathrm{B}_{2}\right)=\varepsilon_{2}-\mathrm{I}_{2} \mathrm{r}_{2}$
Combining the last three equations
$\mathrm{I}=\mathrm{I}_{1}+\mathrm{I}_{2}=\frac{\varepsilon_{1}-\mathrm{V}}{\mathrm{r}_{1}}+\frac{\varepsilon_{2}-\mathrm{V}}{\mathrm{r}_{2}}=\left(\frac{\varepsilon_{1}}{\mathrm{r}_{1}}+\frac{\varepsilon_{2}}{\mathrm{r}_{2}}\right)-\mathrm{V}\left(\frac{1}{\mathrm{r}_{1}}+\frac{1}{\mathrm{r}_{2}}\right)$

Hence, $V$ is given by, $V=\frac{\varepsilon_{1} \mathbf{r}_{2}+\varepsilon_{2} r_{1}}{\mathbf{r}_{1}+\mathbf{r}_{2}}-I \frac{\mathbf{r}_{1} \mathbf{r}_{2}}{\mathbf{r}_{1}+\mathbf{r}_{2}}$
If we want to replace the combination by a single cell, between $\mathrm{B}_{1}$ and $\mathrm{B}_{2}$, of emf $\varepsilon_{\text {eq }}$ and internal resistance $\mathrm{r}_{\mathrm{eq}}$, we would have $\mathrm{V}=\varepsilon_{\mathrm{eq}}-\mathrm{Ir}_{\mathrm{eq}}$
The last two equations should be the same and hence
$\varepsilon_{\mathrm{eq}}=\frac{\varepsilon_{1} \mathrm{r}_{2}+\varepsilon_{2} \mathrm{r}_{1}}{\mathrm{r}_{1}+\mathrm{r}_{2}}, \mathrm{r}_{\mathrm{eq}}=\frac{\mathrm{r}_{1} \mathrm{r}_{2}}{\mathrm{r}_{1}+\mathrm{r}_{2}}$
We can put these equations in a simpler way,
$\frac{1}{\mathrm{r}_{\mathrm{eq}}}=\frac{1}{\mathrm{r}_{1}}+\frac{1}{\mathrm{r}_{2}}, \frac{\varepsilon_{\mathrm{eq}}}{\mathrm{r}_{\mathrm{eq}}}=\frac{\varepsilon_{1}}{\mathrm{r}_{1}}+\frac{\varepsilon_{2}}{\mathrm{r}_{2}}$
In figure, we had joined the positive terminals together and similarly the two negative ones, so that the currents $I_{1}, I_{2}$ flow out of positive terminals. If the negative terminal of the second is connected to positive terminal of the first, equation and would still be valid with $\varepsilon_{2} \rightarrow-\varepsilon_{2}$

### 3.13 Kirchhoff's Rules :

Q. State Kirchhoff's laws for an electric network. On which basic principle are they based?

Solution : Junction Rule : At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction. This rule is based on conservation of charge principle.
Loop Rule : The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. This rule is based on conservation of energy.
Q. In the following circuit write the equation to calculate $I_{1}, I_{2}, I_{3}$ uning KVL ?


Solution : $\mathrm{I}_{3}=\mathrm{I}_{1}+\mathrm{I}_{2},-30 \mathrm{I}_{1}-4 \mathrm{I}_{3}+45=0,-30 \mathrm{I}_{1}+21 \mathrm{I}_{2}-80=0$
Q. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical netrowk consisting of 12 resistors each of resistance $1 \Omega$. Determine the equivalent resistance of the network and the current along each edge of the cube. [NCERT Solved Example 3.6]


Solution : The equivalent resistance is $\frac{\mathbf{5}}{\mathbf{6}} \mathbf{R}$ and $\mathrm{I}=4 \mathrm{~A}$.
Q. Determine the current in each branch of the network as shown in figure : [NCERT Solved Example 3.7]


Solution : The current in the various branches of the network are

$$
\mathrm{AB}: \frac{5}{8} \mathrm{~A}, \mathrm{CA}: 2 \frac{1}{2} \mathrm{~A}, \mathrm{DEB}: 1 \frac{7}{8} \mathrm{~A}, \mathrm{AD}: 1 \frac{7}{8} \mathrm{~A}, \mathrm{CD}: 0 \mathrm{~A}, \mathrm{BC}: 2 \frac{1}{2} \mathrm{~A}
$$

3.14 Wheatstone Bridge :
Q. What is Wheatstone bridge ? Find the balance condition in Wheatstone bridge using the Krichoff's law or otherwise.

Solution:


The bridge has four resistors $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}$ and $\mathrm{R}_{4}$. Across one pair of diagonally opposite points (A and C in the figure) a source is connected. This (i.e., AC) is called the battery arm. Between the other two vertices, B and D , a galvanometer G (which is a device to detect currents) is connected. This line shows as BD in the figure, is called the galvanometer arm.

For simplicity, we assume that the cell has no internal resistance. In general there will be currents flowing across all the resistors as well as a current $\mathrm{I}_{\mathrm{g}}$ through G . Of special interest, is the case of a balanced bridge where the resistors are such that $\mathrm{I}_{\mathrm{g}}=0$. We can easily get the balance condition, such that there is no current through G. In this case, the Kirchhoff's junction rule applied to junctions D and B immediately gives us the relations $I_{1}=I_{3}$ and $I_{2}=I_{4}$. Next, we apply Kirchoff's loop rule to closed loops ADBA and CBDC. The first loop gives

$$
-\mathrm{I}_{1} \mathrm{R}_{1}+0+\mathrm{I}_{2} \mathrm{R}_{2}=0 \quad\left(\mathrm{I}_{\mathrm{g}}=0\right)
$$

From equation, we obtain $\frac{\mathbf{I}_{\mathbf{1}}}{\mathbf{I}_{\mathbf{2}}}=\frac{\mathbf{R}_{\mathbf{2}}}{\mathbf{R}_{\mathbf{1}}}$
and the second loop gives, upon using $I_{3}=I_{1}, I_{4}=I_{2}$

$$
\mathrm{I}_{2} \mathrm{R}_{4}+0-\mathrm{I}_{1} \mathrm{R}_{3}=0
$$

From Equation, we obtain $\frac{\mathbf{I}_{\mathbf{1}}}{\mathbf{I}_{\mathbf{2}}}=\frac{\mathbf{R}_{\mathbf{4}}}{\mathbf{R}_{\mathbf{3}}}$
Hence, we obtain the condition $\frac{\mathbf{R}_{\mathbf{2}}}{\mathbf{R}_{\mathbf{1}}}=\frac{\mathbf{R}_{\mathbf{4}}}{\mathbf{R}_{\mathbf{3}}}$


This last equation relating the four resistors is called the balance condition for the galvanometer to give zero or null deflection.
Q. Which device is based on Wheatstone bridge ?

Solution : Meter Bridge.
Q. What is the use of Wheatstone bridge ?

Solution : The Wheatstone bridge and its balance condition provide a practical method for determination of an unknown resistance.
Q. The four arms of a Wheatstone bridge have the following resistances :
$\mathrm{AB}=100 \Omega, \mathrm{BC}=10 \Omega, \mathrm{CD}=5 \Omega$, and $\mathrm{DA}=60 \Omega$


A galvanometer of $15 \Omega$ resistance is connected across $B D$. Calculate the current through the galvanometer when a potential difference of 10 V is maintained across AC. [NCERT Solved Example 3.8]
Solution : 4.87 mA

### 3.15 Meter Bridge :

Q. Draw a circuit diagram for a metre bridge to determine the unknown resistance of a resistor. Obtain the balance condition for a metre bridge.
Solution : The meter bridge is shown.


It consists of a wire of length 1 m and of uniform cross sectional area stretched taut and clamped between two thick metallic strips. The metallic strip has two gaps across which resistors can be connected. The end points where the wire is clamped are connected to a cell through a key. One end of a galvanometer is connected to the metallic strip midway between the two gaps. The other end of the galyanometer is connected to a 'jockey'.
R is an unknown resistance whose value we want to determine. It is connected across one of the gaps. Across the other gap, we connect a standard known resistance S . The jockey is connected to some point D on the wire, a distance $l \mathrm{~cm}$ from the end A . The jockey can be moved along the wire. The portion AD of the wire has a resistance $R_{c m} l$, where $R_{c m}$ is the resistance of the wire per unit centimetre. The portion $D C$ of the wire similarly has a resistance $\mathrm{R}_{\mathrm{cm}}(100-l)$.
The four arms AB, BC, DA and CD [with resistances $\mathrm{R}, \mathrm{S}, \mathrm{R}_{\mathrm{cm}} l$ and $\mathrm{R}_{\mathrm{cm}}(100-l)$ obviously form a Wheatstone bridge. If the jockey is moved along the wire, then there will be one position where the galvanometer will show no current. Let the distance of the jockey from the end $A$ at the balance point be $l=l_{1}$.

$$
\frac{\mathrm{R}}{\mathrm{~S}}=\frac{\mathrm{R}_{\mathrm{cm}} l_{1}}{\mathrm{R}_{\mathrm{cm}}\left(100-l_{1}\right)}=\frac{l_{1}}{100-l_{1}}
$$

Thus, once we have found out $l_{1}$, the unknown resistance R is known in terms of the standard known resistance S by

$$
\mathrm{R}=\mathrm{S} \frac{l_{1}}{100-l_{1}}
$$

By choosing various values of $S$, we would get various values of $l_{1}$, and calculate R each time.
Q. How can you minimise the percentage error in measurement of resistance using meter bridge ?

Solution : The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, i.e., when $l_{1}$ is close to 50 cm .
Q. In a meter bridge, the null point is found at a distance of 33.7 cm from $\mathbf{A}$. If now a resistance of $12 \Omega$ is connected in parallel with $S$, the null point occurs at 51.9 cm . Determine the values of $R$ and S. [NCERT Solved Example 3.9]

Solution : $\mathrm{R}=6.86 \Omega, \mathrm{~S}=13.5 \Omega$.
3.16 Potentiometer :
Q. Describe potentiometer. Define the term potential gradient. With the help of circuit diagram, explain how a potentiometer can be used to compare the emf of two battery and to find internal resistance of a battery.
Solution : It is basically a long piece of uniform wire, sometimes a few meters in length across which a standard cell is connected. In the figure, the wires run from A to C . The small vertical portions are the thick metal strips connecting the various sections of the wire.
A current I flows through the wire which can be varied by a variable resistance (rheostat, R ) in the circuit. Since the wire is uniform, the potential difference between A and any point at a distance $l$ from A is $\varepsilon(l)=\phi l$ where $\phi$ is the potential drop per unit length. It is also known as potential gradient.


Figure shows an application of the potentiometer to compare the emf of two cells of emf $\varepsilon_{1}$ and $\varepsilon_{2}$. The points marked 1, 2, 3 form a two way key. Consider first a position of the key where 1 and 3 are connected so that the galvanometer is connected to $\varepsilon_{1}$. The jockey is moved along the wire till at a point $\mathrm{N}_{1}$, at a distance $l_{1}$ from A, there is no deflection in the galvanometer. We can apply Kirchhoff's loop rule to the closed loop $\mathrm{AN}_{1}$ G31A and get,

$$
\phi l_{1}+0-\varepsilon_{1}=0
$$

Similarly, if another $\operatorname{emf} \varepsilon_{2}$ is balanced against $l_{2}\left(\mathrm{AN}_{2}\right)$

$$
\phi l_{2}+0-\varepsilon_{2}=0
$$

From the last two equations $\frac{\varepsilon_{1}}{\varepsilon_{2}}=\frac{l_{1}}{l_{2}}$
Internal resistance of cell using potentiometer :


We can also use a potentiometer to measure internal resistances of a cell. For this the cell (emf $\varepsilon$ ) connected across a resistance box through a key $\mathrm{K}_{2}$, as shown in the figure. With key $\mathrm{K}_{2}$ open, balance is obtained at length $l_{1}\left(\mathrm{AN}_{1}\right)$. Then,

$$
\varepsilon=\phi l_{1}
$$

When key $\mathrm{K}_{2}$ is closed, the cell sends a current (I) through the resistance box (R). If V is the terminal potential difference of the cell and balance is obtained at length $l_{2}\left(\mathrm{AN}_{2}\right)$,

$$
\mathrm{V}=\phi l_{2}
$$

So, we have e $/ \mathrm{V}=l_{1} / l_{2}$
But, $\varepsilon=\mathrm{I}(\mathrm{r}+\mathrm{R})$ and $\mathrm{V}=\mathrm{IR}$. This gives
$\varepsilon / V=(r+R) / R$
From both equation we have $(\mathrm{R}+\mathrm{r}) / \mathrm{R}=l_{1} / l_{2}$
$\mathrm{r}=\mathrm{R}\left(\frac{l_{1}}{l_{2}}-1\right)$
Using Equation we can find the internal resistance of a given cell.
Q. What is the advantage of using potentiometer to measure the emf of the battery?

Solution : The potentiometer has the advantage that it draws no current from the voltage source being measured. As such it is unaffected by the internal resistance of the source.
Q. Write two possible causes of potentiometer giving only one sided deflection. OR Write any two important precautions while measuring the emf of the battery using potentiometer.
Solution : (i) The emf of battery may be less than the emf of cells.
(ii) Positive terminals of cells and battery may not be connected at the same points.
Q. A resistance of $\mathbf{R} \Omega$ draws current from a potentiometer. The potentiometer has a total resistance $R_{0} \Omega$. A voltage $V$ is applied to the potentiometer. Derive an expression for the voltage across $R$ when the sliding contact is in the middle of the potentiometer. [NCERT Solved Example 3.10]


Solution : $\frac{2 V R}{\mathbf{R}_{\mathbf{0}}+\mathbf{4 R}}$

## NCERT EXERCISE

3.1 The storage battery of a car has an emf of 12 V . If the internal resistance of the battery is $0.4 \Omega$, what is the maximum current that can be drawn from the battery?
3.2 A battery of emf 10 V and internal resistance $3 \Omega$ is connected to a resistor. If the current in the circuit is 0.5 A , what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?
3.3 (a) Three resistors $1 \Omega, 2 \Omega$ and $3 \Omega$ are combined in series. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.
3.4 (a) Three resistors $2 \Omega, 4 \Omega$ and $5 \Omega$ are combined in parallel. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.
3.5 At room temperature $\left(27.0^{\circ} \mathrm{C}\right)$ the resistance of a heating element is $100 \Omega$. What is the temperature of the element if the resistance is found to be $117 \Omega$, given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4} \mathbf{C}^{\mathbf{1}}$.
3.6 A negligibly small current is passes through a wire of length 15 m and uniform cross-sectional $6.0 \times 10^{-7} \mathrm{~m}^{2}$, and its resistance is measured to be $5.0 \Omega$. What is the resistivity of the material at the temperature of the experiment?
3.7 A silver wire has a resistance of $2.1 \Omega$ at $27.5^{\circ} \mathrm{C}$, and a resistance of $2.7 \Omega$ at $100{ }^{\circ} \mathrm{C}$. Determine the temperature coefficient of resistivity of silver.
3.8 A heating element using nichrome connected to a 230 V supply draws an initial current of 3.4 A which settles after a few seconds to a steady value of 2.8 A . What is the steady temperature of the heating element if the room temperature is $27.0^{\circ} \mathrm{C}$ ? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}{ }^{0} \mathrm{C}^{-1}$.
3.9 Determine the current in each branch of the network shown in figure :

3.10 (a) In a metre bridge, the balance point is found to be at 39.5 cm from the end $A$, when the resistor $Y$ is of $12.5 \Omega$. Determine the resistance of $X$. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?
(b) Determine the balance point of the bridge above if $X$ and $Y$ are interchanged.
(c) What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?
3.11 A storage battery of emf 8.0 V and internal resistance $0.5 \Omega$ is being charged by a 120 V de supply using a series resistor of $15.5 \Omega$. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?
3.12 In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm , what is the emf of the second cell?
3.13 The number density of free electrons in a copper conductor estimated is $8.5 \times 10^{28} \mathbf{m}^{-3}$. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is $2.0 \times 10^{-6} \mathbf{m}^{2}$ and it is carrying a current of 3.0 A .

## ADDITIONAL EXERCISES

3.14 The earth's surface has a negative surface charge density of $10^{-9} \mathbf{C ~ m}^{-2}$. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe). (Radius of earth $=6.37 \times 10^{6} \mathrm{~m}$ ).
3.15 (a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance $0.015 \Omega$ are joined in series to provide a supply to a resistance of $8.5 \Omega$. What are the current drawn from the supply and its terminal voltage?
(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of $380 \Omega$. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?
3.16 Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables ( $\rho_{\mathrm{Al}}=2.63 \times 10^{-8} \Omega \mathrm{~m}, \rho_{\mathrm{Cu}}=1.72 \times 10^{-8} \Omega \mathrm{~m}$, Relative density of $\mathrm{Al}=2.7$, of $\mathrm{Cu}=8.9$ ).
3.17 What conclusion can you draw from the following observations on a resistor made of alloy manganin ?

| Current (A) | Voltage (V) | Current (A) | Voltage (V) |
| :--- | :--- | :--- | :--- |
| 0.2 | 3.94 | 3.0 | 59.2 |
| 0.4 | 7.87 | 4.0 | 78.8 |
| 0.6 | 11.8 | 5.0 | 98.6 |
| 0.8 | 15.7 | 6.0 | 118.5 |
| 1.0 | 19.7 | 7.0 | 138.2 |
| 2.0 | 39.4 | 8.0 | 158.0 |

3.18 Answer the following questions :
(a) A steady current flows in a metallic conductor of non-uniform cross-section. Which of these quantities is constant along the conductor : current, current density, electric field, drift speed?
(b) Is Ohm's law universally applicable for all conducting elements? If not, give examples of elements which do not obey Ohm's law.
(c) A low voltage supply from which one needs high currents must have very low internal resistance. Why?
(d) A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?
3.19 Choose the correct alternative :
(a) Alloys of metals usually have (greater/less) resistivity than that of their constituents metals.
(b) Alloys usually have much (lower/higher) temperature coefficient of resistance than pure metals.
(c) The resistivity of the alloy manganin is nearly independent of / increases rapidly with increase of temperature.
(d) The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of $\left(10^{22} / 10^{3}\right)$.
3.20 (a) Given $n$ resistors each of resistance $R$, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?
(b) Given the resistances of $1 \Omega, 2 \Omega, 3 \Omega$, how will be combine them to get an equivalent resistance of (i) (11/3) $\Omega$ (ii) (11/5) $\Omega$, (iii) $6 \Omega$, (iv) $(6 / 11) \Omega$ ?
(c) Determine the equivalent resistance of networks shown in figure.

3.21 Determine the current drawn from a 12 V supply with internal resistance $0.5 \Omega$ by the infinite network shown in figure. Each resistor has $1 \Omega$ resistance.

3.22 Figure shows a potentiometer with a cell of 2.0 V and internal resistance $0.40 \Omega$ maintaining a potential drop across the resistor wire $A B$. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents upto a few mA ) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of $600 \mathrm{k} \Omega$ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf $\varepsilon$ and the balance point found similarly, turns out to be at $\mathbf{8 2 . 3} \mathbf{~ c m}$ length of the wire.

(a) What is the value of $\varepsilon$ ?
(b) What purpose does the high resistance of $600 \mathrm{k} \Omega$ have ?
(c) Is the balance point affected by this high resistance?
(d) Is the balance point affected by the internal resistance of the driver cell ?
(e) Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V ?
(f) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?
3.23 Figure shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R=10.0 \Omega$ is found to be 58.3 cm , while that with the unknown resistance $X$ is 68.5 cm .


Determine the value of $\mathbf{X}$. What might you do if you failed to find a balance point with the given cell of emf $\varepsilon$ ?
3.24 Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell is open circuit is 76.3 cm . When a resistor of $9.5 \Omega$ is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire.


Determine the internal resistance of the cell.
$3.1 \quad 30 \mathrm{~A}$
$3.217 \Omega, 8.5 \mathrm{~V}$
3.3 (a) $6 \Omega$ (b) $2 \mathrm{~V}, 4 \mathrm{~V}, 6 \mathrm{~V}$
3.4 (a) $(20 / 19) \Omega$ (b) $10 \mathrm{~A}, 5 \mathrm{~A}, 4 \mathrm{~A} ; 19 \mathrm{~A}$
$3.5 \quad 1027{ }^{\circ} \mathrm{C}$
$3.6 \quad 2.0 \times 10^{-7} \Omega \mathrm{~m}$
$3.7 \quad 0.0039{ }^{0} \mathrm{C}^{-1}$
$3.8 \quad 867^{\circ} \mathrm{C}$
3.9 Current in branch $A B=(4 / 17) A$,
in $B C=(6 / 17) A$, in $C D=(-4 / 17) A$,
in $\mathrm{AD}=(6 / 17) \mathrm{A}$, in $\mathrm{BD}=(-2 / 17) \mathrm{A}$, total current $=(10 / 17) \mathrm{A}$.
3.10 (a) $8.2 \Omega$; to minimise resistance of the connection which are not accounted for in the bridge formula. (b) 60.5 cm from $A(c)$ The galvanometer will show no current
3.11 11.5 V ; the series limits the current drawn from the external source. In its absence, the current will be dangerously high.
$3.12 \quad 2.25 \mathrm{~V}$
$3.13 \quad 7.5 \mathrm{~h}$
3.14283 s
3.15 (a) $1.4 \mathrm{~A}, 11.9 \mathrm{~V}$
(b) 0.005 A ; impossible because a starter motor requires large current ( $\sim \mathbf{1 0 0} \mathbf{A}$ ) for a few seconds
3.16 The mass (or weight) ratio of copper to aluminium wire is $(1.72 / 2.63) \times(8.9 / 2.7) \cong 2.2$. Since aluminium is lighter, it is preferred for long suspensions of cables.
3.17 Ohm's law is valid to a high accuracy; the resistivity of the alloy manganin is nearly independent of temperature.
3.18 (a) Only current (because it is given to be steady !). The rest depends on the area of cross-section inversely.
(b) No, examples of non-ohmic elements : Vacuum diode, semiconductor diode.
(c) Because the maximum current drawn from a source $=\varepsilon / r$.
(d) Because, if the circuit is shorted (accidentally), the current drawn will exceed safety limits, if internal resistance is not large.
3.19 (a) greater, (b) lower, (c) nearly independent of, (d) $10^{22}$
3.20 (a) (i) series (ii) parallel, $n^{2}$
(b) (i) Join $1 \Omega, 2 \Omega$ in parallel and the combination in series with $3 \Omega$, (ii) parallel combination of $2 \Omega$ and $3 \Omega$ in series with $1 \Omega$ (iii) all in series, (iv) all in parallel.
(c) (i) $16 / 3 \Omega$ (ii) 5 R
$3.21 \quad 3.7 \mathrm{~A}$
3.22 (a) $\varepsilon=1.25 \mathrm{~V}$
(b) To reduce current through the galvanometer when the movable contact is far from the balance point.
(c) No.
(d) No.
(e) No. If $\varepsilon$ is greater than the emf of the driver cell of the potentiometer, there will be no balance point on the wire $A B$.
(f) The circuit, as it is, would be unsuitable, because the balance point (for $\varepsilon$ of the order of a few mV ) will be very close to the end $A$ and the percentage error in measurement will be very large. The circuit is modified by putting a suitable resistor $R$ in series with the wire $A B$ so that potential drop across $A B$ is only slightly greater than the emf to be measured. Then, the balance point will be at larger length of the wire and the percentage error will be much smaller.
$3.23 X=11.75 \Omega$ or $11.8 \Omega$. If there is no balance point, it means potential drops across $R$ and $X$ are greater than the potential drop across the potentiometer wire AB. The obvious thing to do is to reduce current in the outside circuit (and hence potential drops across $R$ and $X$ ) suitably by putting a series resistor.
$3.241 .7 \Omega$

## ADDITIONAL QUESTIONS AND PROBLEMS

Q. Give reasons why the electrical conductance of electrolytes is less than that of metals.
A. Metals have an exceedingly high number of conduction electrons whose mobility is quite high. In electrolysis number density of ions is comparatively less and their mobility too is small.
Q. State the condition in which terminal voltage across a secondary cell is equal to its emf.
A. When the cell is in an open circuit i.e., when no current is being drawn from the cell.
Q. A wire of resistivity $\rho$ is stretched to twice its length. What will be its new resisvity ?
A. Resistivity will remain unchanged at $\rho$, because resistivity of a material is independent of its dimensions.
Q. Define conductance. What is its unit ?
A. Reciprocal of resistance is called conductance. Thus, Conductance $\sigma=\frac{\mathbf{1}}{\mathbf{R}}=\frac{\mathbf{I}}{\mathbf{V}}$

SI unit of conductance is $\Omega^{-1}$ or mho or siemen (S).
Q. Define electrical conductivity of a conductor and give its SI unit.
A. Reciprocal or resistivity of a conductor is called its conductivity. Alternatively conductance of a unit cube of a conductor is called its electrical conductivity. Its SI unit is $\mathrm{S} \mathrm{m}^{-1}$.
Q. Two wires A and B are of same metal, have the same area of cross-section and have their lengths in the ratio 2: 1. What will be the ratio of currents flowing through them respectively, when the same potential difference is applied across length of each of them ?
A. As $V$ is constant, hence is $\frac{\mathbf{I}_{\mathrm{A}}}{\mathrm{I}_{\mathrm{B}}}=\frac{\mathbf{R}_{\mathrm{B}}}{\mathbf{R}_{\mathrm{A}}}=\frac{\boldsymbol{l}_{\mathrm{B}}}{l_{\mathrm{A}}}=\frac{\mathbf{1}}{\mathbf{2}}$.
Q. Two wires $A$ and $B$, of the same metal and of the same length, have their areas of cross-sections in the ratio 2 : 1 . If the same potential difference is applied across each wire is turn, what will be the ratio of the currents flowing in $A$ and $B$ ?
A. As V is constant, hence $\frac{\mathbf{I}_{A}}{\mathbf{I}_{\mathbf{B}}}=\frac{\mathbf{R}_{\mathrm{B}}}{\mathbf{R}_{\mathrm{A}}}=\frac{\mathbf{A}_{\mathbf{A}}}{\mathbf{A}_{\mathrm{B}}}=\frac{\mathbf{2}}{\mathbf{1}}$
Q. If potential difference $Y$ applied across a conductor is increased to 2 V , how will the drift velocity of the electrons change?
A. $\because$ Drift velocity $\mathbf{v}_{\mathbf{d}}=\frac{\mathbf{e E}}{\mathbf{m}} \cdot \tau=\frac{\mathbf{e}}{\mathbf{m}} \cdot \frac{\mathbf{V}}{l} \cdot \tau$.
Q. Sketch a graph showing variation of resistivity of carbon with temperature.
A.

Q. The variation of potential difference with length in case of two potentiometer wires $A$ and $B$ is given. Which of the two is more sensitive?

OR
The variation of potential difference $\mathbf{V}$ with length $l$ in case of two potentiometers $A$ and $B$ is as shown. Which one of these two will you prefer for comparing emfs of two primary cells?

A. B is more sensitive because potential gradient for it is smaller. Hence, we prefer potentiometer B to compare emfs of two primary cells.
Q. The voltage-current graph for two resistors of the same material and same radii with lengths $L_{1}$ and $L_{2}$ are shown in figure. If $L_{1}>L_{2}$, state with reason, which of these graphs represent $V-I$ change for $L_{1}$.

A. We know that slope of V-I graph gives the value of resistance of a conductor is given by the relation

$$
R=\frac{\rho L}{A}
$$

In present case, material and radii (i.e., the cross-section area) being the same, hence

$$
\mathrm{R} \propto \mathrm{~L} \quad \text { or } \quad \frac{\mathbf{R}_{\mathbf{1}}}{\mathbf{R}_{\mathbf{2}}}=\frac{\mathbf{L}_{\mathbf{1}}}{\mathbf{L}_{\mathbf{2}}}
$$

It is given that $L_{1}>L_{2}$, hence $R_{1}>R_{2}$. In figure, slope of $V-I$ graph for $B$ is more. It means that graph $B$ represents $V-I$ change for $L_{1}$
Q. Distinguish between emf and terminal potential difference of a cell. Give only $\mathbf{3}$ points of differences.
A. Emf of a cell : (1) Emf of a cell is the amount of work done in moving a unit charge once around a complete circuit outside as well as inside the cell. (2) It is a constant quantity and depends on the nature and composition of cell. (3) It is equal to terminal potential difference when the cell circuit is open.
Terminal potential difference of a cell : (1) Terminal potential difference of a cell is the amount of work done in order to move a unit charge once from +ve terminal of cell to -ve terminal of cell through the external resistance. (2) It is a variable quantity and depends on the value of external resistance. (3) It is the terminal potential difference in actual closed circuit condition when current is being drawn from the cell.
Q. Why do we prefer a potentiometer with a longer bridge wire. OR How can you increase the sensitivity of a potentiometer.
A. We prefer a potentiometer with a longer bridge wire to ensure a lower value of the potential gradient. It makes the instrument more sensitive.
Q. A potential difference $V$ is applied across a conductor of length $L$ and diameter $D$. How are the electric field $E$ and the resistance $R$ of conductor affected when in turn
(i) $\quad V$ is halved,
$L$ is halved, and
(iii) D is doubled?

Justify your answer in each case.
A. We know that electric field $\mathbf{E}=\frac{\mathbf{V}}{\mathbf{L}}$
and resistance of conductor $R=\frac{m}{n e^{2} \tau} \cdot \frac{L}{A}=\frac{m}{n e^{2} \tau} \cdot \frac{4 L}{\pi D^{2}}$.
(i) If V is halved then obviously electric field is halved but resistance R remains unaffected.
(ii) If L is halved then electric field E is doubled and resistance R is halved.
(iii) If diameter D is doubled then electric field E remains unaffected but resistance is reduced to one-fourth of its original value.
Q. A conductor of length ' $l$ ' is connected to a d.c. source of potential ' $V$ '. If the length of the conductor is triplet by stretching it, keeping $V$ constant, explain how do the following factors vary in the conductor :
(i)
Drift speed of electrons,
Resistance, and
(iii) Resistivity
A. (i) As drift speed $\mathbf{v}_{\mathbf{d}}=\frac{\mathbf{e E}}{\mathbf{m}} \tau=\frac{\mathbf{e V}}{\mathbf{m} \boldsymbol{l}} \boldsymbol{\tau}$, hence on increasing the length to three times of its original value, the drift speed becomes one-third of its initial value.
(ii) When wire is stretched to three times its original length $\left(l^{\prime}=\mathbf{3 l}\right)$, its area of cross-section is reduced to $\frac{\mathbf{1}}{\mathbf{3}}$ rd of its previous value, i.e., $\mathbf{A}^{\prime}=\frac{\mathbf{A}}{\mathbf{3}}$.

Resistance $\mathbf{R}=\frac{\mathbf{m}}{\mathbf{n e}^{\mathbf{2} \tau}} \cdot \frac{\mathbf{1}}{\mathbf{A}}$ i.e., $\mathbf{R} \propto \frac{\boldsymbol{l}}{\mathbf{A}}$. Hence, resistance will have a value nine times its original value ( $R^{\prime}=9 R$ ).
(iii) Resistivity $\rho=\frac{\mathbf{m}}{\mathbf{n e}^{2} \tau}$. Hence, resistivity remains unaffected and does not change.
Q. $\quad V$ - I graph for parallel and series combination of two metallic resistors are as shown in the figure.

Which graph shows parallel combination? Justify your answer.
Q. Two cells of emfs 1.5 V and 2.0 V and internal resistances $2 \Omega$ and $1 \Omega$, respectively have their negative terminals joined by a wire of $6 \Omega$ and positive terminals by a wire of $4 \Omega$ resistance. A third resistance wire of $8 \Omega$ connects middle points of these wires. Draw the circuit diagram. Using Kirchhoff's laws, find the potential difference at the ends of this third wire.
A. $\quad 92 / 73 \mathrm{Y}$
Q. A cylindrical metallic wire is stretched to increase its length by $5 \%$. Calculate the percentage change in its resistance.
A. $10 \%$
Q. A student has two wires of iron and copper of equal length and diameter. He first joins the two wires in series and passes electric current through the combination whicn increases gradually. After that he joins the two wires in parallel and repeats the process of passing the current. Which wire will glow first in each case and why?
A. (i) In series arrangement same current is passed in both wires and, hence, heat produced is directly proportional to the resistance $(H \propto R)$. As resistivity and hence resistance of iron wire is more, iron wire is heated more and begins to glow first.
(ii) In parallel arrangement, voltage is constant and hence $\mathbf{H} \propto \frac{\mathbf{1}}{\mathbf{R}}$. As a result, copper wire, having lesser resistance, is heated rapidly and begins to glow first.
Q. Determine the voltage drop across the resistor $R_{1}$ in the circuit given below with $E=65 \mathrm{~V}, \mathrm{R}_{1}=50 \Omega$, $R_{2}=100 \Omega, R_{3}=100 \Omega$ and $R_{4}=300 \Omega$.

A. $\quad 25 \mathrm{~V}$
Q. Calculate the current shown by the ammeter in the circuit diagram given below

A. 5 A
Q. Using Kirchhoff's laws, calculate the value of the electric current $I_{1}, I_{2}$ and $I_{3}$ in the given electrical network

A. $\quad I_{1}=\frac{30}{31} A, I_{2}=\frac{48}{31} A$ and $I_{3}=\frac{18}{31} A$.
Q. State Kirchoff's rules of current distribution in an electrical network.

Using these rules determine the value of the current $I_{1}$ in the electric circuit in figure.

A. $I_{1}=-1.2 \mathrm{~A}$
Q. Find the current drawn from a cell of emf 1 V and internal resistance $\frac{2}{3} \Omega$ connected to the network shown in figure.

A. $\quad 1.0 \mathrm{~A}$
Q. A 20 V battery of internal resistance $1 \Omega$ is connected to three coils of $12 \Omega, 6 \Omega$ and $4 \Omega$ in parallel, a resistor of $5 \Omega$ and a reversed battery (emf 8 V and internal resistance $2 \Omega$ ) as shown in figure. Calculate
(i) the current in the circuit,
(ii) the current in resistor of $12 \Omega$ coil, and
(iii) potential difference across each battery.

A. (i) 1.2 A (ii) 0.2 A (iii) 10.4 V
Q. A battery of emf 3 V and internal resistance $r$ is connected in series with a resistor of $55 \Omega$ through an ammeter of resistance $1 \Omega$. The ammeter reads 50 mA . Draw the circuit diagram and calculate the value of r .
A. $4 \Omega$
Q. In the circuit diagram, the cells $E_{1}$ and $E_{2}$ have emfs of 4 V and 8 V and internal resistance $0.5 \Omega$ and $1.0 \Omega$, respectively. Calculate the current in each resistance.

A. $\quad 0.33 \mathrm{~A}, 0.17 \mathrm{~A}$
Q. A potential difference of 2 V is applied between the points $A$ and $B$ as shown in the network drawn in the figure. Calculate
(i) equivalent resistance of the network across the points $A$ and $B$, and
(ii) the magnitude of currents flowing in the arms AFCEB and AFDEB.

A. $2 \Omega, 0.5 \mathrm{~A}$
Q. Find the value of the unknown resistance $X$ in the circuit of figure, if no current flows through the section $A O$. Also calculate the current drawn by the circuit from the battery of emf 6 V and negligible internal resistance.

A. $\quad 1.0 \mathrm{~A}$
Q. Six resistors, each of value $4 \Omega$, are joined together in a circuit as shown. Calculate equivalent resistance across the point $A$ and $B$. If a cell of emf 2 V is connected across $A B$, compute the current through the arms $A B$ and DF of the circuit.

A. 1 A , there is no current flowing through the arm DF , as the bridge is balanced one.
Q. In the given network shown in figure, find the values of the currents $I_{1}, I_{2}$ and $I_{3}$.

A. $\quad I_{1}=\frac{5}{13} A, I_{2}=-\frac{1}{13} A$ and $I_{3}=\frac{6}{13} A$
Q. When two known resistances $R$ and $S$ are connected in the left and right gaps of a metre bridge, the balance point is found at a distance $l_{1}$ from the zero end of the metre bridge wire. An unknown resistance $X$ is now connected in parallel to the resistance $S$ and the balance point is now found at a distance $l_{2}$ from the zero end of the metre bridge wire. Obtain a fourmula for $X$ in terms of $l_{1}, l_{2}$ and S.
A. $\mathrm{X}=\frac{\mathrm{S}}{\frac{l_{2}}{l_{1}} \cdot\left(\frac{100-l_{1}}{100-l_{2}}\right)-1}$
Q. In a potentiometer, a standard cell of emf 5 V and of negligible resistance maintains a steady current through the potentiometer wire of length 5 m . Two primary cells of emfs $E_{1}$ and $E_{2}$ are joined in series with (a) same polarity, and (b) opposite polarity. The combination is connected through a galvanometer and a jockey to the poentiometer. The balancing lengths in the two cases are found to be 350 cm and 50 cm , respectively.
(i) Draw the necessary circuit diagram.
(ii) Find the value of the emfs of the two cells.
A. (ii) 1.5 V
Q. In a hydrogen atom the electron moves in an orbit of radius $5.0 \times 10^{-11} \mathrm{~m}$ with a speed of $2.2 \times 10^{6} \mathbf{~ m ~ s}^{-1}$. Find the equivalent current.
A. $\quad 1.12 \times 10^{-3} \mathrm{~A}$
Q. Two cells of emf 3.0 V and 4.0 V and of internal resistances $1 \Omega$ and $2 \Omega$, respectively are connected in parallel so as to send current in the same direction through an external resistance of $5 \Omega$. Calculate the current in each branch of the network.
A. $\frac{1}{17} \mathrm{~A}, \frac{9}{17} \mathrm{~A}, \frac{10}{17} \mathrm{~A}$
Q. A 10 V battery of internal resistance $0.5 \Omega$ is connected to three coils of $6 \Omega, 3 \Omega$ and $2 \Omega$ in parallel, a resistor of 2.5 and a reversed battery $(\mathrm{emf}=4 \mathrm{~V}$ and internal resistance $=1 \Omega)$ as shown. Calculate (i) the current in the circuit, (ii) current in resistor of $6 \Omega$ coil, and (iii) potential difference across each battery.

A. (i) 1.2 A (ii) 0.2 A (iii) $9.4 \mathrm{~V}, 5.2 \mathrm{~V}$
Q. A battery $E_{1}$ of $4 V$ and a variable resistance $R h$ are connected in series with the wire $A B$ of the potentiometer. The length of the wire of the potentiometer is 1 m . When a cell $E_{2}$ of emf 1.5 V is connected between points $A$ and $C$, no current flows through $E_{2}$. Length of $A C=60 \mathrm{~cm}$.
(i) Find the potential difference between the ends $A$ and $B$ of the potentiometer.
(ii) Would the method work, if the battery $E_{1}$ is replaced by a cell of emf 1 V ?

A. (i) 2.5 V (ii) No
Q. $\quad A B$ is 2 m long uniform wire of $20 \Omega$ resistance. The other data is as shown in the circuit diagram given in figure.

Calculate (i) potential gradient along AB , and (ii) length AO of the wire, when the galvanometer shows no deflection.

A. (i) $0.008 \mathrm{~V} \mathrm{~cm}^{-1}$ (ii) 75 cm
Q. Calculate the equivalent resistance between $X$ and $Y$ in the circuit shown in figure, if $r_{1}=r_{2}=r_{3}=4 \Omega$.

A. $3 \Omega$
Q. Calculate the equivalent resistance between $X$ and $Y$ in the circuit shown in figure.

Given that $r_{1}=r_{3}=3 \Omega$ and $r_{2}=r_{4}=2 \Omega$.

A. $2.5 \Omega$

