Properties of Matter

PROPERTIES OF MATTER

C1 All real "rigid" bodies are to some extent elastic, which means that we can change their dimensions slightly by pulling, pushing, twisting or compressing them.

Hooke's law states that in elastic deformations, stress (force per unit area) is proportional to strain (relative deformation) :

$$\frac{\textbf{Stress}}{\textbf{Strain}} = \textbf{Elastic modulus}$$

Three elastic moduli are used to describe the elastic bahaviour (deformations) of objects as they respond to forces that act on them.

1. Longitudinal stress and longitudinal strain : Longitudinal stress is defined as $\frac{F_\perp}{A}$, where $F_\perp$ is the force perpendicular to the plane of cross sectional A. There are two types of longitudinal stress :

(a) Tensile longitudinal stress, and

(b) Compresive longitudinal stress

Tensile stress is tensile force per unit area, $F_\perp / A$. Tensile strain is fractional change in length, $\Delta l/l_0$. Young's modulus Y is the ratio of tensile stress to tensile strain :

$$Y = \frac{F_\perp / A}{\Delta l / l_0} = \frac{F_\perp}{A}\frac{l_0}{\Delta l}$$

Compressives stress and strain are defined the same way as tensile stress and strain. For many materials, Young's modulus has the same value for both tension and compression.

2. Bulk stress or volume stress or hydraulic stress :

The bulk modulus B is the negative of the ratio of pressure change Δp (bulk stress) a fractional volume change $\Delta V/V_0$:

$$B = -\frac{\Delta p}{\Delta V / V_0}$$

Compressibility k is the reciprocal of bulk modulus : k = 1/B.

3. Shear stress is force per unit area $F_{||}/A$ for a force applied parallel to a surface. Shear strain is the angle ϕ. The shear modulus S is the ratio of shear stress to shear strain :

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{||}/A}{x/h} = \frac{F_{||}}{A}\frac{h}{x} = \frac{F_{||}/A}{\phi}$$

The proportional limit is the maximum stress for which stress and strain are proportional. Beyond the proportional limit, Hooke's law is not valid. The elastic limit is the stress beyond which irreversible deformation occurs. The breaking stress, or ultimate strength, is the stress at which the material breaks.

Energy stored in a stretched wire per unit volume equals to $\frac{1}{2}$ × stress × strain.

<u>**Practice Problems :**</u>

1. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied.

(a) length = 50 cm, diameter = 0.5 mm (b) length = 100 cm, diameter = 1 mm

(c) length = 200 cm, diameter = 2 mm (d) length = 300 cm, diameter = 3 mm

2. The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(a) 0.4 cm^3 (b) 4×10^{-5} cm^3 (c) 0.025 cm^3 (d) 0.004 cm^3

3. Young's modulus of steel is 2×10^{11} N/m^2. A steel wire has a length of 1 m and area of cross section 1 mm^2. The work required to increase its length by 1 mm is

(a) 0.1 J (b) 1 J (c) 10 J (d) 100 J

4. **A substance breaks down by a stress of 10^6 N/m^2. If the density of the material of the wire is 3×10^3kg/m^3, then the length of the wire of that substance which will break under its own weight when suspended vertically is**

   (a) 3.4 m (b) 34 m (c) 340 m (d) none of these

5. **A metal ring of initial radius r and cross-sectional area A is fitted onto a wooden disc of radius R > r. If Young's modulus of the metal is Y then the tenstion in the ring is**

   (a) $\frac{AYR}{r}$ (b) $\frac{AY(R-r)}{r}$ (c) $\frac{Y}{A}\left(\frac{R-r}{r}\right)$ (d) $\frac{Yr}{AR}$

6. **A massless rod AD consisting of three segments AB, BC and CD joined together is hanging vertically from a fixed support at A. The lengths of the segments are respectively 0.1 m, 0.2 m and 0.15 m. The cross-section of the rod is uniformly 10^{-4}m^2. A weight of 10 kg is hung from D. If $Y_{AB} = 2.5 \times 10^{10}$ N/m^2, $Y_{BC} = 4 \times 10^{10}$ N/m^2 and $Y_{CD} = 1 \times 10^{10}$ N/m^2 then the ratio of displacement of points B, C and D is**

   (a) 1 : 2 : 3 (b) 2 : 3 : 7 (c) 3 : 5 : 9 (d) none

7. **A steel wire (Young's modulus = 2×10^{11} N/m^2) of diameter 0.8 mm and length 1 m is clamped firmly at two points A and B which are 1 m apart and in the same plane. A body is hung from the middle point of the wire such that the middle point sags 1 cm lower from the original position. The mass of the body is**

   (a) 82 gm (b) 41 gm (c) 22.5 gm (d) 11 gm

8. **The bulk modulus of water if its volume changes from 100 litre to 99.5 litre under a pressure of 100 atmosphere is**

   (a) 1.026×10^9 N/m^2 (b) 2.026×10^9 N/m^2
   (c) 3.026×10^9 N/m^2 (d) 4.026×10^9 N/m^2

9. **A rubber cord of length L is suspended vertically. Density of rubber is D and Young's modulus is Y. If the cord extends by a length *l* under its own weight, then *l* is**

   (a) L^2Dg/Y (b) $L^2Dg/2Y$ (c) $L^2Dg/4Y$ (d) $\frac{2L^2Dg}{Y}$

[Answers : (1) a (2) a (3) a (4) b (5) b (6) d (7) a (8) b (9) b]

C2 Density : Density is mass per unit volume. If a mass m of material has volume V, its density ρ is $\rho = \frac{m}{V}$.

Specific gravity is the ratio of the density of a material to the density of water.

Practice Problems :

1. **If equal masses of two liquids of densities d_1 and d_2 are mixed together, the density of the mixture is**

   (a) $(d_1 + d_2)$ (b) $2d_1d_2/(d_1 + d_2)$ (c) $d_1d_2/(d_1 + d_2)$ (d) $(d_1 + d_2)/2$

2. **If equal volume of two liquids of density is d_1 and d_2 are mix together then the density of the mixture is**

   (a) $(d_1 + d_2)$ (b) $2d_1d_2/(d_1 + d_2)$ (c) $d_1d_2/(d_1 + d_2)$ (d) $(d_1 + d_2)/2$

3. **Due to the change of pressure the density of the liquid will change. If the change in pressure is ΔP and the bulk modulus of liquid is B then the fractional change in density of the liquid equals to**

   (a) $\frac{\Delta P}{B}$ (b) $\frac{2\Delta P}{B}$ (c) $\frac{\Delta P}{2B}$ (d) $\frac{3}{2}\frac{\Delta P}{B}$

[Answers : (1) b (2) d (3) a]

C3 **Pressure :** Pressure is normal force per unit area.

Pressure (a scalar quantity) on a surface is defined as

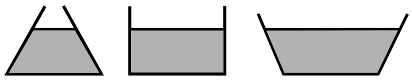
$$p = \lim_{\Delta s \to 0} \frac{\Delta F_{\perp}}{\Delta S} = \frac{dF_{\perp}}{dS}$$

The units for pressure are Nm^{-2} or pascal (Pa), or mm of mercury (or any other substance).

C4 **Hydrostatic pressure distribution :** Pressure in a fluid at rest increases with vertical height 'h' according to the relation $\frac{dp}{dh} = \rho g$.

If the density of the liquid is constant at each point then the pressure at a point A at a depth h below the free surface is given by $p_A = \rho gh + p_0$, where p_0 is the pressure at the free surface (atmospheric pressure). Absolute pressure is the total pressure in a fluid; gauge pressure is the difference between absolute pressure and atmospheric pressure.

Hydrostatic Paradox :



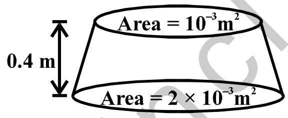
Three vessels of equal base area but containing different amounts of liquid upto the same height will have same force at their bottom.

<u>Practice Problems :</u>

1. **The pressure in a water tap at the base of a building is 3×10^6 dynes/cm^2 and on its top it is 1.6×10^6 dynes/cm^2. The height of the building is approximately**

(a) 7 m **(b)** 14 m **(c)** 70 m **(d)** 140 m

2. **A uniformly tapering vessel is filled with a liquid of density 900 kg/m^3. The thrust on the base of the vessel due to the liquid is (g = 10 m/s^2)**



(a) 3.6 N **(b)** 7.2 N **(c)** 10.8 N **(d)** 14.4 N

3. **Consider a liquid of density ρ is placed in a container upto the height h. If the force exerted by the liquid on the side wall is directly proportional to h^n, then the value of n is**

(a) 0 **(b)** ½ **(c)** 1 **(d)** 2

[Answers : (1) b (2) b (3) d]

C5 **Pascal Law :** Pascal's law states that pressure applied to the surface of an enclosed fluid is transmitted undiminished to every portion of the fluid.

<u>Practice Problems :</u>

1. **A piston of cross-sectional area 100 cm^2 is used in a hydraulic press to exert a force of 10^7 dynes on the water. The cross-sectional area of the other piston which supports a truck of mass 2000 kg is**

(a) $9.8 \times 10^2 cm^2$ **(b)** $9.8 \times 10^3 cm^2$ **(c)** $1.96 \times 10^3 cm^2$ **(d)** $1.96 \times 10^4 cm^2$

2. **A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be**

(a) 1.12 **(b)** 1.1 **(c)** 1.05 **(d)** 1.0

3. **A U-tube is partly filled with a liquid A. Another liquid B, which does not mix with A, is poured into one side until it stands a height h above the level of A on the other side, which has meanwhile risen a height *l*. The density of B relative to that of A is**

(a) $\frac{l}{h + l}$ **(b)** $\frac{l}{h + 2l}$ **(c)** $\frac{2l}{h + 2l}$ **(d)** $\frac{l}{2h + l}$

[Answers : (1) d (2) b (3) c]

C6 Archimede's Principle

When a body is immersed partly or wholly in a fluid, there acts an upward force on it called the buoyancy and its magnitude is equal to the weight of the fluid displaced. The point of the application of buoyancy is at the centre of mass of the displaced fluid and is called the centre of buoyancy. Buoyancy exists because of pressure gradient. Thus in case of a free fall situation buoyancy is zero.

Principle of floatation

Weight of the object = Buoyancy

$$\rho_s V g = \rho_l V_s g$$

V : total volume of the object

V_s : submerged volume of the object

ρ_s : density of object

ρ_l : density of liquid

Practice Problems :

1. A piece of wood of relative density 0.36 floats in oil of relative density 0.90. The fraction of volume of wood above the surface of oil is

(a) 0.3 **(b)** 0.4 **(c)** 0.6 **(d)** 0.8

2. A large block of ice 10 m thick with a vertical hole drilled through it is floating in a lake. The minimum length of the rope required to scoop out a bucket full of water through the hole is (density of ice = 0.9 g/cm³)

(a) 0.5 m **(b)** 1.0 m **(c)** 1.2 m **(d)** 1.8 m

3. A streamlined body of relative density d_1 falls from a height h on the surface of a liquid of relative density d_2, where $d_2 > d_1$. The time for which the body will fall inside the liquid is

(a) $\frac{d_1}{d_2}\sqrt{\frac{2h}{g}}$ **(b)** $\frac{d_2}{d_1}\sqrt{\frac{2h}{g}}$

(c) $\frac{d_1}{d_2 - d_1}\sqrt{\frac{2h}{g}}$ **(d)** $\frac{d_2 - d_1}{d_2}\sqrt{\frac{2h}{g}}$

4. A small ball of density ρ is immersed in a liquid of density σ(σ > ρ) to a depth h and then released. The height above the surface of water up to which the ball will jump is

(a) $\frac{\sigma h}{\rho}$ **(b)** $\left(\frac{\sigma}{\rho} - 1\right)h$ **(c)** $\left(1 - \frac{\rho}{\sigma}\right)h$ **(d)** $\frac{\rho h}{\sigma}$

5. A small ball of density ρ is dropped from a height h into a liquid of density σ (σ > ρ). Neglecting damping forces, the maximum depth to which the body sinks is

(a) $\frac{h\sigma}{\sigma - \rho}$ **(b)** $\frac{h\rho}{\sigma - \rho}$ **(c)** $\frac{h(\sigma - \rho)}{\rho}$ **(d)** $\frac{h(\sigma - \rho)}{\sigma}$

6. A block (density ρ) is suspended from a spring and produces an extension 'x'. If the whole system is dipped in a liquid (density σ) then new extention is

(a) $x\rho/\sigma$ **(b)** $x\,\sigma/\rho$ **(c)** $x(1 - \sigma/\rho)$ **(d)** $x(1 - \rho/\sigma)$

[Answers : (1) c (2) b (3) c (4) b (5) b (6) c]

C7 Fluid Dynamics :

An ideal fluid is incompressible and has no viscosity. A flow line is the path of the fluid particle; a streamline is a curve tangent at each point to the velocity vector at that point. A flow tube is a tube bounded at its sides by flow lines. In laminar flow, layers of fluid slide smoothly past each other. In turbulent flow there is great disorder and a constantly changing flow pattern.

Principle of Continuity : Conservation of mass in an incompressible fluid is expressed by the equation of continuity; for two cross sections A_1 and A_2 in a flow tube, the flow speed v_1 and v_2 are related by $A_1v_1 = A_2v_2$.

The product Av is the volume flow rate, dV/dt, the rate at which volume crosses a section of the tube : $\frac{dV}{dt} = Av$.

Bernoulli's equation relates the pressure p, flow speed v, and elevation y for steady flow in an ideal fluid which is based on conservation of energy principle. For any two points, denoted by subscripts 1 and 2.

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Practice Problems :

1. **Two large tanks a and b, open at the top, contains different liquids. A small hole is made in the side of each tank at the same depth h below the liquid surface, but the hole in a has twice the area of the hole in b. The ratio of the densities of the liquids in a and b so that the mass flux is the same for each hole should be**

   (a) 2 **(b)** 0.5 **(c)** 4 **(d)** 0.25

2. **In the above problem the ratio of flow rates (volume flux) from the holes in a and b is**

   (a) 2 **(b)** 0.5 **(c)** 4 **(d)** 0.25

3. **Air is streaming past a horizontal aeroplane wing such that its speed is 120 m/s over the upper surface and 90 m/s at the lower surface. If the density of air is 1.3 kg/m³. If the wing is 10 m long and has an average width 2 m, the gross lift of the wing is**

   (a) 5.2×10^4N **(b)** 6.2×10^4N **(c)** 7.2×10^4N **(d)** 8.2×10^4N

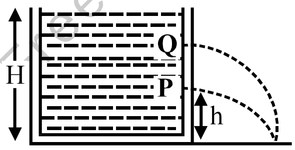
4. **A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm², the water velocity is 1 m/s and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm² is**

   (a) 500 Pa **(b)** 750 Pa **(c)** 900 Pa **(d)** 1100 Pa

5. **The rate of flow of glycerine of density 1.25×10^3 kg/m³ through the conical section of a pipe, if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length is 10 N/m² is**

   (a) 6.28×10^{-3} m³/s **(b)** 6.28×10^{-4} m³/s
   (c) 3.9×10^{-4} m³/s **(d)** 3.9×10^{-3} m³/s

6. **Water flows out of two small holes P and Q in a wall of a tank and the two streams strike the ground at the same point. If the hole P is at a height h above the ground and the level of water stands at a height H above the ground, then the height of Q is**



   (a) $\frac{H+h}{2}$ **(b)** H – h **(c)** H – h/2 **(d)** $\frac{H-h}{2}$

[Answers : (1) b (2) a (3) d (4) a (5) b (6) b]

C8 **Viscosity :** The viscosity of a fluid characterizes its resistance to shear strain. In a Newtonian fluidthe viscous force is propotional to strain rate. The viscous force between two layers of a fluid of area A having a velocity gradient dv/dx is given by $F = -\eta A \frac{dv}{dx}$ where η is called the coefficient of viscosity. In SI unit of η is poiseuille (1 PI = 1 Ns m⁻²) and the dimension of η is $ML^{-1}T^{-1}$.

Practice Problems :

1. **The velocity of water (viscosity = 10^{-3} poiseuille) in a river is 18 km/hr at the surface. If the river is 5 m deep, then the shearing stress between the horizontal layers of water is**

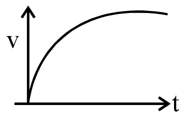
(a) 0.5×10^{-3} N/m^2 **(b)** 0.8×10^{-3} N/m^2

(c) 10^{-3} N/m^2 **(d)** 1.2×10^{-3} N/m^2

[Answers : (1) c]

C9 Stoke's Law and Terminal Speed : A sphere of radius r moving with speed v through a fluid having viscosity η experiences a viscous resisting force F given by Stoke's law : $F = 6\pi\eta rv$.

The following graph shows the variation of velocity v with time t for a small spherical body falling vertically in a long column of viscous liquid



The terminal speed acheived by a sphere is given by $v_t = \frac{2}{9}\frac{r^2 g}{\eta}(\sigma - \rho)$ where σ is the density of the sphere and ρ is the density of the fluid in which sphere is moving.

Practice Problems :

1. **The velocity of a small ball of mass m and density d_1 when dropped in a container filled with glycerine becomes constant after some time. The viscous force acting on the ball if density of glycerine is d_2 is**

(a) $mg\left(1 - \frac{d_2}{d_1}\right)$ **(b)** mg **(c)** $mg\left(1 - \frac{d_1}{d_2}\right)$ **(d)** $mg\left(\frac{d_2}{d_1}\right)$

2. **The viscosity of glycerine (having density 1.3 gm/cc) if a steel ball of 2 mm radius (density = 8 gm/cc) acquires a terminal velocity of 4 cm/sec in falling freely in the tank of glycerine is**

(a) 13.6 poise **(b)** 14.6 poise **(c)** 15.6 poise **(d)** 16.6 poise

3. **An air bubble of radius 1 mm is allowed to rise through a long cylindrical column of a viscous liquid of radius 5 cm and travels at a steady rate of 2.1 cm per sec. If the density of the liquid is 1.47 gm per cc, then its viscosity is**

(a) 1.324 poise **(b)** 1.424 poise **(c)** 1.524 poise **(d)** 1.624 poise

4. **'n' equal drops of water are falling through air with a steady velocity v. If the drops coalesced, then the new velocity is**

(a) $(n^{1/3})$ v **(b)** nv **(c)** $(n^{1/2})$ v **(d)** $(n^{2/3})$ v

5. **A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water (viscosity of water is 9.8×10^{-6} N-s/m^2). If after entering the water the velocity of the ball does not change, the value of h is**

(a) 20.4 m **(b)** 22.4 m **(c)** 24.4 m **(d)** 26.4 m

[Answers : (1) a (2) b (3) c (4) d (5) a]

C10 Poiseuille's Equation :

When such a fluid flows in a cylindrical pipe of inner radius R, and length L is the length if pipe, the total volume rate is given by Poiseuille's equation :

$$\frac{dV}{dt} = \frac{\pi}{8}\left(\frac{R^4}{\eta}\right)\left(\frac{p_1 - p_2}{L}\right)$$

where p_1 and p_2 are the pressures at the two ends and η is the viscosity.

Practice Problems :

1. **Under a pressure head the rate of orderly volume flow of a liquid through a capillary tube is Q. If the length of the capillary tube is doubled and the diameter of the bore is halved, the rate of flow would become**

(a) Q/32 **(b)** Q/8 **(c)** Q/4 **(d)** 8 Q

2. **Two liquids of coefficients of viscosity η_1 and η_2 are made to flow through a tube in succession under the same pressure difference. If V_1 and V_2 are, respectively, the volumes of the two liquids flowing per second, then V_1/V_2 is**

(a) $\frac{\eta_2}{\eta_1}$ **(b)** $\frac{\eta_1}{\eta_2}$ **(c)** $\frac{\eta_2^{\,2}}{\eta_1^{\,2}}$ **(d)** $\frac{\eta_1^{\,2}}{\eta_2^{\,2}}$

3. **The graph for the variation of capillary rise and radius of the tube for the given liquid is**

(a) linear **(b)** constant **(c)** hyperbolic **(d)** exponential

[Answers : (1) a (2) a (3) c]

C11 Reynolds Number : The turbulence flow of a fluid is determined by a dimensionless parameter called the Reynolds number given by $\mathbf{R_e = \frac{\rho vd}{\eta}}$ where ρ is the density of liquid, v its velocity, η its viscosity and d is the diameter of tube in which liquid will flow. For most cases $R_e < 1000$ signifies laminar flow; $1000 < R_e < 2000$ is unsteady flow and $R_e > 2000$ implies turbulent flow.

C12 Surface Tension : The surface of a liquid behaves like a membrane under tension; the force per unit length across a line on the surface is called the surface tension, denoted by T.

C13 Excess Pressure : Excess pressure inside a liquid drop of radius r is given by $\frac{2T}{r}$. Excess pressure inside a liquid bubble or air bubble of radius r is given by $\frac{4T}{r}$.

C14 Capillary Rise or Fall : The rise or fall of a liquid in a capillary tube is given by $\mathbf{h = \frac{2T\cos\theta}{\rho gr}}$, where θ is the angle of contact, ρ is the density of liquid in the tube and r is the radius of the tube. For a clean glass plate in contact with pure water, θ = 0.

Practice Problems :

1. **A liquid rises to a height h in a capillary tube on the earth. The height to which the same liquid would rise in the same tube on the moon is about**

(a) 6 h **(b)** $\sqrt{6}$ h **(c)** h/6 **(d)** h/$\sqrt{6}$

2. **n identical spherical drops of a liquid of surface tension T, each of radius r, coalesce to form a single drop. The surface energy**

(a) decreases by $4\pi r^2(n - n^{1/3})T$ **(b)** increases by $4\pi r^2(n - n^{1/3})T$

(c) decreases by $4\pi r^2(n - n^{2/3})T$ **(d)** increases by $4\pi r^2(n - n^{2/3})T$

[Answers : (1) a (2) c]