WAVES

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C1 Wave

A wave is a disturbance that travels or propagates and transports energy and momentum without the transport of matter. The ripples on a pond, the sound we hear, visible light, radio and TV signals are the examples of waves.

Mechanical waves such as water waves or sound waves require material medium for their propagation. These waves travels within or on the surface of material with elastic properties. These waves are governed by Newton's Laws.

Electromagnetic waves, such as light and TV signals, are **non-mechanical** and can propagate through vacuum.

Transverse and Longitudinal Waves

If a particle of the medium in which the wave is travelling moves perpendicular to the direction of wave propagation, the wave is called transverse. Example of transverse waves are EM waves, waves on the string.

If a particle of the medium move parallel to the direction of wave propagation, the wave is said to be longitudinal. Example of longitudinal wave are sound wave in any medium. Note that sound wave in solid may be transverse or may be longitudinal, depending upon the mode of excitation.

Non-mechanical waves are always transverse but the converse is not true.

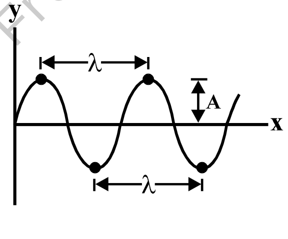
Mechanism of propagation of mechanical waves :

Mechanical waves are related to the elastic properties of the medium. In transverse waves, the constituents of the medium oscillate perpendicular to wave motion causing change in shape. That is, each element of the medium in subject to shearing stress. Solids and strings have shear modulus, that is they sustain shearing stress. Fluids have no shape of their own - they yield to shearing stress. This is why transverse waves are possible in solids and strings (under tension) but not in fluids. However, solids as well as fluids have bulk modulus, that is, they can sustain compressive strain. Since longitudinal waves involve compressive stress (pressure), they can be propagated through solids and fluids. Thus a steel bar possessing both bulk and sheer elastic moduli can propagate longitudinal as well as transverse waves. But air can propagate only longitudinal pressure waves (sound). When a medium such as steel bar propagaes both longitudinal and transverse waves, their speeds can be different since they arise from different elastic moduli.

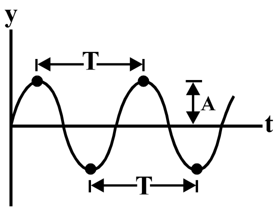
C2 Characteristics of a Wave

Three physical characteristics are important in describing a wave : wave length, frequency and wave speed.

Wave length (denoted by λ) : One wavelength is the minimum distance between any two identical points on a wave - for example, adjacent crests or adjacent troughs, as in figure, which is a graph of displacement versus position for a sinusoidal wave at a specific time.



Frequency : The frequency of sinusoidal waves is the same as the frequency of simple harmonic motion of a particle of the medium. The number of complete vibrations of a point that occur in one second or the number of wavelengths that pass a given point in one second is called frequency. The period T of the wave is the minimum time it takes a particle of the medium to undergo one complete oscillation, and is equal to the inverse of the frequency : $\mathbf{T = \frac{1}{f}}$. Figure shows position versus time for a particle of the medium as a sinusoidal wave is passing through its position.



The period is the time between instants when the particle has identical displacement and velocities.

Wave speed (denoted by v) : Waves travel through the medium with a specific wave speed, which depends on the properties of the medium being disturbed.

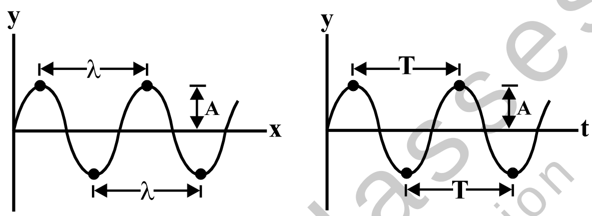
Another important parameter for the wave (in figure) is the amplitude of the wave denoted by A. This is the maximum displacement of a particle of the medium from the equilibrium position.

Relation between wavelength, frequency and wave speed :

Since in one period T the wave advances by one wavelength λ, therefore, the wave velocity is $\mathbf{v = \frac{\lambda}{T} = \lambda f}$

Practice Problems :

1. What is the difference between the two figures ?



2. A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation ? (b) If the pulse rate is 1 after every 20 s. (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz ?

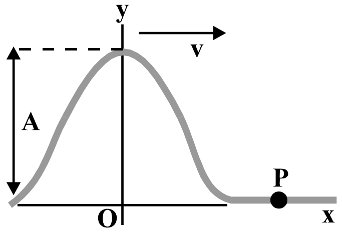
3. Earthquakes generate sound wave inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}, and that of P wave is 8.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight, line, at what distance does the earthquake occur ?

[Answers : (3) 1920 km]

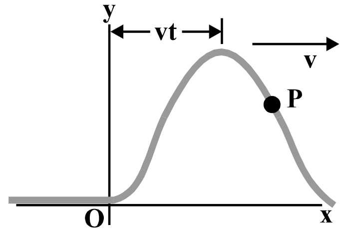
C3 Equation of a Travelling Wave

The equation of a wave traveling along the positive x-axis is given by y(x, t) = f(x – vt). The function y is called the wave function depends on the two variables position x and time t which is read "y as a function of x and t".

Meaning of y :



(a) Pulse at t = 0



(a) Pulse at time t

Figure represents the shape and position of the pulse at time t = 0. At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function that we will write as y(x, 0) = f(x). This function describes the vertical position y of the element of the string located at each value of x at time t = 0. Because the speed of the pulse is v, the pulse has traveled to the right at a distance vt at the time t. We adopt a simplification model in which the shape of the pulse does not change with time (In reality, the pulse changes its shape and gradually spreads out during the motion. This effect is called dispersion and is common to many mechanical waves; however we adopt a simplification model that ignores this effect). Thus, at time t, the shape of the pulse is the same as it was at time t = 0, as in figure. Consequently, an element of the string at x at this time has the same y position as an element located at x – vt has at time t = 0 :

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the displacement y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt)$$

Similarly, if the wave pulse travels to the left (negative x-axis), the displacement of the string is

$$y(x, t) = f(x + vt)$$

Consider a point P on the string, identified by a particular value of its x coordinates. As the pulse passes through P, the y coordinates of this point increases, reaches a maximum, and then decreases to zero. The wave function y(x, t) represents the y coordinates of any point P located at position x at any time t. Furthermore, if t is fixed (e.g., in the case of taking a snapshot of the pulse), then the wave function y as a function of x, sometimes called the waveform, defines a curve representing the actual geometric shape of the pulse at that time.

In general, the wave motion in one dimension is given by $y = f(x \pm vt)$

<u>Practice Problems :</u>

1. **You have learnt that a travelling wave in one dimension is represented by a function y = f(x, t) where x and t must appear in the combination x – vt or x + vt, i.e., y = f(x ± vt). Is the converse true ? Examine if the following functions for y can possibly represent a travelling wave :**

(a) $(x - vt)^2$ **(b)** $\log[(x + vt)/x_0]$

(c) $1/(x + vt)$ **(d)** $\sin(x - vt)$

(e) $\cos(x - vt)$ **(f)** $\tan(x - vt)$

(g) $\sqrt{x - vt}$

2. **A wave pulse moving along the x-axis is represented by the wave function**

$$y(x,t) = \frac{2.0}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is in seconds. Plot the waveform at t = 0, t = 1.0 s, and t = 2.0 s. Also find the direction of propagation of wave pulse, amplitude and velocity ?

[Answers (1) The converse is not true. An obvious requirement for an acceptable function for a travelling wave is that it should be finite everywhere and at all times. Only function (a), (d), (e) satisfies this condition, the remaining functions cannot possibly represent a travelling wave]

C4 Sine Harmonic Travelling Wave

The wave function that represents a sine harmonic travelling wave is given by

$$y(x, t) = A\sin[(kx \pm \omega t) + \phi]$$

The negative sign is used when the wave travels along the positive x-axis, and vice-versa.

Meaning of Standard Symbols :

y(x, t)	:	displacement as a function of position x and time t
A	:	amplitude of a wave
ω	:	angular frequency of the wave

k	:	angular wave number or propagation constant
$kx \pm \omega t + \phi$	:	phase of the wave
ϕ	:	initial phase or phase at x = 0 and t = 0

Relation between wave number (k) and wave length (λ) : $k = \frac{2\pi}{\lambda}$

Relation between angular frequency, frequency and time period : $\omega = \frac{2\pi}{T} = 2\pi f$

Wave velocity : $v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$.

Wave Velocity and Particle Velocity

Wave velocity is the velocity of the disturbance which propagates through a medium. It only depends on the properties of the medium and is independent of time and position.

Particle velocity is the rate at which particle's displacement vary as a function of time, i.e., $\frac{\partial y}{\partial t} = \pm \omega A \cos(kx \pm \omega t + \varphi)$.

Acceleration of the Particle :

The acceleration of the particle is obtained by differentiating above equation w.r.t. time $\frac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin(kx \pm \omega t + \varphi)$.

Practice Problems :

1. **A wave equation which gives the displacement along y-direction is given by**

   $y = 10^{-4} \sin(60t + 2x)$

   where x and y are in m and t in s. This represents a wave

   (a) travelling with velocity of 30 m/s in the negative x-direction

   (b) of wavelength (π) m

   (c) of frequency (30/π) Hz

   (d) all the above

2. **A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is traveling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10\ ms^{-1}$, then λ and f are given by**

   (a) $\lambda = 2\pi \times 10^{-2}$ m **(b) $\lambda = 10^{-3}$ m**

   (c) $f = 10^3/(2\pi)$ Hz **(d) both (a) and (c) are correct**

3. **A transverse wave is described by the equation $y = A \sin 2\pi(ft - x/\lambda)$. The maximum particle velocity is equal to n times the wave velocity then the relation between A and λ is**

   (a) $n\lambda = \pi A$ **(b) $n\lambda = 2\pi A$** **(c) $n\lambda = 3\pi A$** **(d) $n\lambda = 4\pi A$**

[Answers : (1) d (2) d (3) b]

C5 Wave speed on stretched string : The speed of a wave on a stretched string is set by properties of the string. The speed on a string with tension T and linear density μ is $v = \sqrt{\frac{T}{\mu}}$.

Practice Problems :

1. **Using the dimensional method or otherwise find the wave speed on a stretched string of mass per unit length μ and under the tension T ? Express this speed in terms of cross-sectional area A of the string and density of the string ρ.**

2. **Consider two strings of the same material but different cross-sectional radius in the ratio 1 : 2. Find the ratio of the tension in the string such that the wave speed in both string is same ?**

3. **A string of mass per unit length μ is suspended vertically from a roof and length of the string is L. A pulse is produced at the lowermost point.**
   - **(a) Find the speed of the pulse at the distance x from the lower end.**
   - **(b) Does it depend on the mass per unit length ? Comment ?**
   - **(c) What is the time taken by the pulse to reach uppermost point ?**

4. **Consider a wire of Young's Modulus of elasticity Y, density ρ and coefficient of linear expansion α. The wire is rigidly clamped between two points and the temperature is raised by ΔT. Find the wave speed of the transverse wave produced on the wire.**

C6 Energy Transmitted by a Wave

The average power transmitted by the wave is

$$P_{av} = \frac{dE_{av}}{dt} = \frac{1}{2}\mu\omega^2 A^2 v \quad \text{where } v = \frac{dx}{dt} \text{ is the wave velocity.}$$

The mass per unit length of a wire is given by μ = ρ a where ρ is the density and a is the cross-sectional area.

The intensity of the wave is given by $I = \frac{P_{av}}{a} = \frac{1}{2}\rho\omega^2 A^2 v$

Note that the power and intensity are proportional to the square of the frequency and square of the amplitude.

<u>**Practice Problems :**</u>

1. **Derive the expression for the average power and average intensity carried by the wave on a string of mass per unit length μ. The angular frequency, amplitude and speed of the wave are ω, A and v respectively. Also prove that propagation of average kinetic energy per unit time and average potential energy per unit time carried by the wave are equal.**

2. **A string with linear mass density μ = 5.00 × 10^{-2}kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm ?**

3. **Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density to 4.00 × 10^{-2} kg/m. If the source can deliver a maximum power of 300 W and the string is under a tension of 100 N, what is the highest vibrational frequency at which the source can operate ?**

4. **A is singing a note and at the same time B is singing a note with exactly one-eighth the frequency of the note of A. The energies of the two sounds are equal. The amplitude of the note of B is**
   - **(a) Same as that of A**
   - **(b) Twice that of A**
   - **(c) Four times that of A**
   - **(d) Eight times that of A**

[Answers : (2) 512 W (3) 55.1 Hz (4) d]

C7 Velocity of Longitudinal Waves

The velocity of longitudinal wave is given by $\sqrt{\frac{E}{\rho}}$ where E is the modulus of elasticity of the medium and ρ is the density of the medium in which wave will propagate. If the longitudinal wave will propagate in solid then E = Y (Young's modulus of elasticity) and if the longitudinal wave will propagate in fluid then E = B (Bulk modulus of elasticity).

C8 Velocity of Sound Wave

The speed of longitudinal waves in a fluid is given by $\mathbf{v} = \sqrt{\frac{\mathbf{B}}{\rho}}$

where B is the bulk modulus defined as $\mathbf{B} = -\frac{\Delta \mathbf{p}}{\Delta \mathbf{V}/\mathbf{V}}$, where $\frac{\Delta \mathbf{V}}{\mathbf{V}}$ is the fractional change in volume produced by the change in pressure Δp.

The propagation of sound in gas is an adiabatic process. For gas the bulk modulus in adiabatic condition is given by B = γp, where γ is an adibatic exponent.

Thus, velocity of sound in air is given by $\mathbf{v} = \sqrt{\frac{\gamma \mathbf{p}}{\rho}}$

Using ideal gas equation $\frac{\mathbf{p}}{\rho} = \frac{\mathbf{RT}}{\mathbf{M}}$, $\mathbf{v} = \sqrt{\frac{\gamma \mathbf{RT}}{\mathbf{M}}}$

Substituting the values of γ and M for air we obtain the approximate value of speed of sound in air at absolute temperature T as $\mathbf{v} \approx 20\sqrt{\mathbf{T}}$

<u>Practice Problems :</u>

1. The ratio (V_s/V_{rms}) of the velocity of sound in a gas (V_s) and the root mean square velocity (V_{rms}) of its molecular at the same temperature is : (γ = ratio of the two specific heats of the gas)

(a) $\sqrt{\gamma/3}$ (b) $\sqrt{3/\gamma}$ (c) $\sqrt{3\gamma}$ (d) $\frac{1}{\sqrt{3\gamma}}$

2. The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(a) $\sqrt{(2/7)}$ (b) $\sqrt{(1/7)}$ (c) $(\sqrt{3})/5$ (d) $(\sqrt{6})/5$

3. The temperature of air is increased from 300 K to 301 K. The fractional change in velocity of sound is

(a) **1/300** (b) **1/600** (c) **1/900** (d) **1/1200**

4. The speed of sound in a gas at NTP is 300 m/sec. If the pressure is increased four times, without change in temperature, the velocity of sound will be

(a) **150 m/sec** (b) **300 m/sec** (c) **600 m/sec** (d) **1200 m/sec**

5. Estimate the speed of sound in air at standard temperature and pressure. The mass of 1 mole of air is 29.0 × 10^{-3} kg.

6. What is the percentage change in velocity of sound if (i) percentage change in temperature is 1% (ii) percentage change in temperature is 21% (iii) percentage change in pressure is 100% keeping the temperature constant.

7. Find the velocity of sound in a mixture of gases consists of one mole of He gas and two moles of O_2 gas at the temprature of 0^0C.

[Answers : (1) a (2) c (3) b (4) b (5) 280 m s^{-1} (6) (i) 0.5% (ii) 10% (iii) 0%]

C9 Relationship Between Pressure Waves and Displacement Waves

The displacement accompanying a sound wave in air are longitudinal displacements of small elements of the fluid from their equilibrium positions. Such displacement results if the source of the waves oscillates in air, for example diaphragm of a loud-speaker. As the vibrating source moves forward, it compresses the medium past it, increasing the density locally. This part of the medium compresses the layer next to it by collision. The compression travels in the medium at a speed which depends on the elastic and inertia properties of the medium. As the source moves back, it drags the medium and produces a rarefaction in the layer. The layer next to it is then dragged back and thus the rarefaction pulse passes forward. In this way, compression and rarefaction pulses are produced which travel in the medium.

A longitudinal wave in a fluid (liquid or gas) can be described either in terms of the longitudinal displacement suffered by the particles of the medium or in terms of the excess pressure generated due to the compression or rarefaction.

For a harmonic wave, the longitudinal displacement (y) is given by y = A sin (kx – wt) .

We know $\mathbf{p = -B\frac{\Delta V}{V}}$

Since change in volume is produced by the displacement of the particles, therefore,

$$\frac{\Delta V}{V} = \frac{\partial y}{\partial x}$$

Thus $\mathbf{p = -B\frac{\partial y}{\partial x}}$ = –BAk cos (kx – wt) or $p = -p_0 \cos(kx - wt)$

where p_0 = BAk is the pressure amplitude.

Note that displacement wave and pressure wave amplitudes are π/2 out of phase.

__Practice Problems :__

1. A sinusoidal sound wave is described by the displacement

$s(x, t) = (2.00\ \mu m) \cos[(15.7\ m^{-1})x - (858\ s^{-1})t]$

(a) Find the amplitude, wavelength, and speed of this wave. (b) Determine the instantaneous displacement of an element of air at the position x = 0.050 0 m at t = 3.00 ms. (c) Determine the maximum speed of an element's oscillatory motion.

2. Calculate the pressure amplitude of a 2.00-kHz sound wave in air if the displacement amplitude is equal to 2.00×10^{-8} m.

3. An experimenter wishes to generate in air a sound wave that has a displacement amplitude of 5.50×10^{-6} m. The pressure amplitude is to be limited to 0.840 N/m². What is the minimum wavelength the sound wave can have ?

4. A sound wave traveling in air has a pressure amplitude of 4.00 N/m² and a frequency of 5.00 kHz. Take ΔP = 0 at the point x = 0 when t = 0. (a) What is ΔP at x = 0 when t = 2.00×10^{-4} s ? (b) What is ΔP at x = 0.020 0 m when t = 0 ?

5. Write an expression that describes the pressure variation as a function of position and time for a sinusoidal sound wave in air, if λ = 0.100 m and ΔP_{max} = 0.200 N/m².

[Answers : (1) (a) 2.00 µm, 40.0 cm, 54.6 m/s (b) –0.433 µm (c) 1.72 mm/s (3) 5.81 m (5) ΔP = (0.200 N/m²) sin(62.8x/m – 2.16 × 10⁴t/s)]

C10 Intensity of sound wave and Sound level

The intensity I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface : $\mathbf{I = \frac{P}{A}}$, where P is the time rate of energy transfer (power) of the sound wave and A is the area of the surface intercepting the sound. The intensity I carried by the sound wave is given by $\frac{p_0^2}{2\rho v}$ where p_0 is the pressure amplitude and ρ is the density of medium in which the wave will propagate with speed v.

Sound Level in Descibels : The sound level β in decibels (dB) is defined as $\beta = (10dB)\log\frac{I}{I_0}$, where I_0 (= 10^{-12} W/m²) is a reference intensity to which all intensities are compared. For every factor of 10 increases in intensity, 10 dB is added to the sound level.

__Practice Problems :__

1. **Two sound waves move in the same direction in the same medium. The pressure amplitudes of the waves are equal but the wavelength of the first wave is double the second. Let the average power transmitted across a cross-section by the first wave be P_1 and that by the second wave be P_2. Find the value of $\frac{P_1}{P_2}$.**

2. **A sound wave of frequency 300 Hz has an intensity of 1.00 μ W/m^2. What is the amplitude of the air oscillations caused by this wave ?**

3. **Two sounds differ in sound level by 1.00 dB. What is the ratio of the greater intensity to the smaller intensity ?**

4. **A certain sound is increased in sound level by 30 dB. By what multiple is (a) its intensity increased and (b) its pressure amplitude increased ?**

5. **(a) If two sound waves, one in air and one in (fresh) water, are equal in intensity, what is the ratio of the pressure amplitude of the wave in water to that of the wave in air ? Assume the water and the air are at 20^{0}C. (b) If the pressure amplitudes are equal instead, what is the ratio of the intensities of the waves ? The density of air is 1.21 kg/m^3 and for water 0.998 × 10^3 kg/m^3 at 20^{0}C.**

6. **Find the ratios (greater to smaller) of (a) the intensities, (b) the pressure amplitudes, and (c) the particle displacement amplitudes for two sounds whose sound levels differ by 37 dB.**

7. **Derive the formula of intensity carried by the sound wave.**

[Answers : (1) 1 (2) 36.8 nm (4) (a) 1000 (b) 32 (5) (a) 59.7 (b) 2.81 × 10^{-4}]

C11 Variation of intensity with distance

(a) For point source, the wavefronts are spherical and the intensity at the distance r from the source of power P is $\frac{P}{4\pi r^2}$.

(b) For line source, the wavefronts are cylinderical and the intensity at the distance r from the source of power P is $\frac{P}{2\pi r L}$ where L is the length of the source.

(c) For plane wave front, the intensity does not change with distance.

<u>Practice Problems :</u>

1. **(a) Show that the intensity I of a wave is the product of the wave's energy per unit volume u and its speed v. (b) Radio waves travel at a speed of 3.00 × 10^8 m/s. Find u for a radio wave 480 km from a 50,000 W source, assuming the wavefronts are spherical.**

2. **A source emits sound wave isotropically. The intensity of the waves 2.50 m from the source is 1.91 × 10^{-4} W/m^2. Assuming that the energy of the waves is conserved, find the power of the source.**

3. **A sound wave travels out uniformly in all directions from a point source. (a) Justify the following expression for the displacement s of the transmitting medium at any distance r from the source : $s = \frac{b}{r}\sin k(r - vt)$, where b is a constant. Consider the speed, direction of propagation, periodicity, and intensity of the wave. (b) What is the dimension of the constant b ?**

4. **A point source emits 30.0 W of sound isotropically. A small microphone intercepts the sound in an area of 0.750 cm^2, 200 m from the source. Calculate (a) the sound intensity there and (b) the power intercepted by the microphone.**

5. **An electric spark jumps along a straight line of length L = 10m, emitting a pulse of sound that travels radially outward from the spark. The power of the emission is P_s = 1.6 × 10^4 W. (a) What is the intensity I of the sound when it reaches a distance r = 12 m from the spark ? (b) At what time rate P_d is sound energy intercepted by an acoustic detector of area A_d = 2.0 cm^2, aimed at the spark and located a distance r = 12 m from the spark ?**

6. **The sound level 46 m in front of the speaker systems was β_2 = 120 dB. What is the ratio of the intensity I_2 of the band at that spot to the intensity I_1 of a jackhammer operating at sound level β_1 = 92 dB ?**

[Answers : (1) (b) 5.76 × 10^{-17} J/m^3 (2) 15.0 mW (3) (b) L^2 (5) (a) 21 W/m^2 (b) 4.2 mW (6) 630]

C12 Superposition of Waves

When two or more waves traverse in the same medium, the displacement of any particle of the medium is the sum of the displacement that the individual waves would give it.

The wave do not alter one another and each propagates through the medium as if the other were not there.

C13 Interference : Adding waves that differ in Phase only :

If the two waves have different amplitudes a_1 and a_2, respectively, the resultant amplitude is given by

$$\mathbf{A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\varphi}}$$

where ϕ is the constant phase difference.

If $\phi = 2n\pi$ where n = 0, 1, 2, ...

$A_{max} = a_1 + a_2$

If $\phi = (2n - 1)\pi$ where n = 1, 2, 3, ...

$A_{min} = a_1 - a_2$

Since intensity is proportional to the square of the amplitude. Therefore,

$$\mathbf{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\ \phi}$$

If $\phi = 2n\pi$ where n = 0, 1, 2,.....

$$\mathbf{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2}$$

If $\phi = (2n - 1)\pi$ where n = 1, 2, 3,....

$$\mathbf{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

Practice Problems :

1. **Can independent sources produce interference pattern ?**
2. **What is the result intensity if waves from incoherent sources are superimposed ?**
3. **Waves generated from two incoherent sources of intensity level 10dB and 90 dB are superimposed. What is the resultant intensity in dB ?**
4. **Two speakers placed 3.00 m apart are driven in phase by the same oscillator. A listeners originally at point O, which is located 8.00 m from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O before reaching the first cancellation of waves, resulting in a minimum in sound intensity. What is the frequency of the oscillator ?**
5. **Two waves travelling in the same direction along a stretched string. The waves are 90.0^0 out of phase. Each wave has an amplitude of 4.00 cm. Find the amplitude of the resultant wave.**
6. **Two sinusoidal waves are described by the wave functions y_1 = (5.00 m) sin [π(4.00x – 1200t)] and y_2 = (5.00 m) sin [π(4.00x – 1200t – 0.250)] where x, y_1, and y_2 are in meters t is in seconds. (a) What is the amplitude of the resultant wave ? (b) What is the frequency of the resultant wave ?**
7. **Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first, but at a later time. Determine the minium possible time interval between the starting moments of the two waves if the amplitude of the resultant wave is the same as that of each of the two initial waves.**

[Answers : (4) 1.3 kHz (5) (a) 9.24 m (b) 600 Hz]

C14 Beats : Adding Wave That Differ in Frequency Only :

If two or more waves of slightly different frequencies (frequency different should be very small as compared to the frequency of waves) are superimposed, the intensity of the resulting wave have alternate maxima and minima such that the intensity of the resultant wave varies periodically with time. The number of minima or maxima in one second is called the beat frequency.

Practice Problems :

1. **When two tuning forks A and B are sounded together x beat/s are heard. Frequency of A is n. Now when one prong of fork B is loaded with a little wax, the number of beat/s decreases. The frequency of fork B is**

   (a) n + x **(b)** n – x **(c)** $n - x^2$ **(d)** n – 2x

2. **The speed of sound in a gas in which two waves of wavelengths 50 cm and 50.4 cm produce 6 beats per second is**

   (a) 338 m/s **(b)** 350 m/s **(c)** 378 m/s **(d)** 400 m/s

3. **Consider two waves of slightly different frequencies f_1 and f_2 with equal amplitudes A. Derive for (a) frequency of the resultant wave (b) beat frequency (c) frequency of amplitude variation (d) frequency of intensity variation (e) intensity of the resultant wave as a function of time. Also draw the variation of intensity and amplitude of resultant wave with time. Also draw the variation of resultant displacement with time at a given position.**

[Answers : (1) a (2) c]

C15 Standing Waves : Adding Waves That Differ In Direction Only :

Consider two waves y_1 and y_2 that have the same amplitude, wavelength and frequency but travel in opposite directions.

$$y_1 = a\sin(kx - \omega t)$$

$$y_2 = a\sin(kx + \omega t)$$

Using superposition principle

$$y = y_1 + y_2 = a[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

or $$y = 2a\sin kx\cos\omega t$$

or $$y = A\cos\omega t \text{ where } A = 2a\sin kx$$

The above equation shows that the string executes simple harmonic motion such that every point on the string vibrates in same phase with same frequency but different amplitudes which depends on the position x of the point along the string.

This type of wave motion represented by equation is called a standing wave because it appears to travel neither to the left nor to the right.

There are positions along the string for which the amplitude of oscillation is always zero (called nodes), and other positions where the amplitudes of oscillation is always 2a (called antinodes)

The distance between two successive node is $\frac{\lambda}{2}$. The distance between two successive antinodes is $\frac{\lambda}{2}$.

Also, nodes and antinodes occur alternatively and equally spaced from each other.

Practice Problems :

1. **For the stationary wave**

   $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ **, the distance between a node and the next antinode is**

   (a) 7.5 units **(b)** 15 units **(c)** 22.5 units **(d)** 30 units

2. **A standing wave having 3 nodes and 2 antinodes is formed between two atoms having a distance 1.21 Å between them. The wavelength of the standing wave is**

   (a) 1.21 Å **(b)** 6.08 Å **(c)** 3.63 Å **(d)** 2.42 Å

[Answers : (1) a (2) a]

C16 Standing Wave Pattern on the String

Standing waves on a string can be set up by reflection of traveling waves from the ends of the string. If an end is fixed, it must be the position of a node. This limits the frequencies at which standing waves will occur on a given string. Each possible frequency is a resonant frequency, and the corresponding standing wave pattern is a resonant frequency, and the corresponding standing wave pattern is an oscillation mode. For a stretched string of length L with fixed ends, the resonant frequences are $\mathbf{f = \frac{v}{\lambda} = n\frac{v}{2L}}$, for n = 1, 2, 3,.....

(where $v = \sqrt{\frac{T}{\mu}}$).

The oscillation mode corresponding to n = 1 is called the fundamental mode or the first harmonic; the mode corresponding to n = 2 is the second harmonic; and so on.

Practice Problems :

1. Two vibrating strings of the same material but length L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency v_1 and the other with frequency v_2. The ratio v_1/v_2 is given by

(a) 2 **(b)** 4 **(c)** 8 **(d)** 1

2. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning form forming three antinodes for the same positions of the bridges. The value of M is

(a) 25 kg **(b)** 5 kg **(c)** 12.5 kg **(d)** 1/25 kg

3. In order to double the frequency of the fundamental note emitted by a stretched string, the length is reduced to 3/4 th of the original length and the tension is changed. The factor by which the tension is to be changed is

(a) 3/8 **(b)** 2/3 **(c)** 8/9 **(d)** 9/4

[Answers : (1) d (2) a (3) d]

C17 Standing wave pattern in pipes

Standing sound wave pattern can be set up in pipes. A pipe open at both ends will resonate at frequencies

$\mathbf{f = \frac{v}{\lambda} = \frac{nv}{2L}}$, n = 1, 2, 3,....

where v is the speed of sound in the gas in the pipe. For a pipe closed at one end and open at the another, the resonant frequencies are $\mathbf{f = \frac{v}{\lambda} = \frac{nv}{4L}}$, n = 1, 3, 5, ...

Practice Problems :

1. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end correction, the air column in the pipe cannot resonate for sound of frequency

(a) 80 Hz **(b)** 240 Hz **(c)** 320 Hz **(d)** 400 Hz

2. If the fundamental frequency of a pipe closed at one end is 512 Hz, the fundamental frequency of a pipe of the same dimensions but open at both ends will be

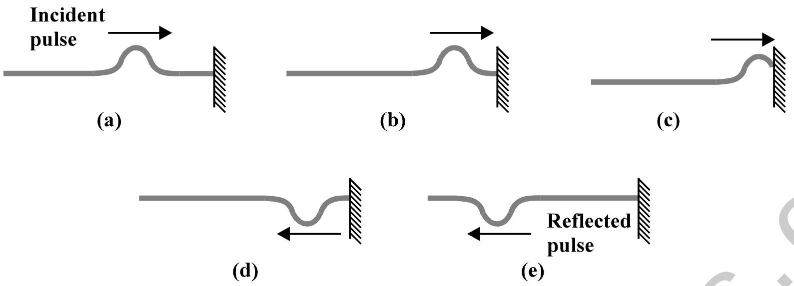
(a) 1024 Hz **(b)** 512 Hz **(c)** 256 Hz **(d)** 128 Hz

[Answers : (1) c (2) a]

C18 Reflection and Transmission of Waves :

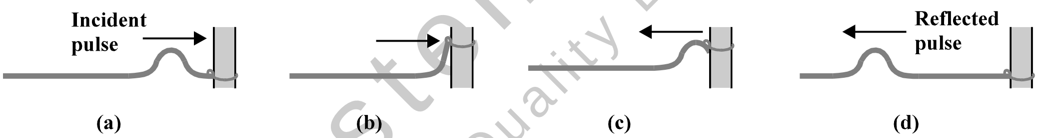
When a traveling pulse reaches a boundary, part of all of the pulses is reflected. Any part not reflected is said to be transmitted through the boundary.

When the pulse reaches the fixed boundary, it is reflected. The reflected pulse has exactly the same amplitude as the incoming pulse but is inverted. The inversion can be explained as follows. The pulse is created initially with an upward and then downward force on the free end of the string.As the pulse arrives at the fixed end of the string, the string first produces an upward force on the support. By Newton's third law, the support exerts an equal and opposite reaction force on the string. Thus, the positive shape of the pulse results in a downward and then upward force on the string as the entirety of the pulse encounters the rigid end. Thus, reflection at a rigid end causes the pulse to invert on reflection.



Reflection of the pulse at the fixed boundary or rigid boundary

Now consider a second idealized situation in which reflection is total and transmission is zero. In this simplification model, the pulse arrives at the end of a string that is perfectly free to move vertically. The tension at the free end is maintained by tying the string to a ring of negligible mass that is free to slide vertically on a frictionless post. Again, the pulse is reflected, but this time it is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward. In the process, the ring reaches the top of its motion and is then returned to its original position by the downward component of the tension force. Thus, the ring experiences the same motion as if it were raised and lowered by hand. This produces a reflected pulse that is not inverted and whose amplitude is the same as that of the incoming pulse.



Reflection of the pulse at the free end

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases; that is, it is neither completely rigid nor completely free. In this case, part of the wave is transmitted and part is reflected. For instance, suppose a string is attached to a more dense string as in figure. When a pulse traveling on the first string reaches the boundary between the two strings, part of the pulse is reflected and inverted and part is transmitted to the more dense string. Both the reflected and transmitted pulses have smaller amplitude than the incident pulse. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary. As the pulse travels from the initial string to the more dense string, the junction acts more like a rigid end than a free end. Thus, the reflected pulse is inverted.

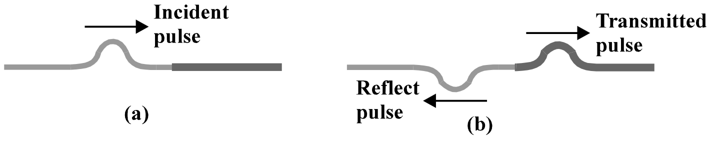


Figure : (a) A pulse travelling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string.

When a pulse traveling on a dense string strikes the boundary of a less dense string, as in figure, again part is reflected and part transmitted. This time, however, the reflection pulse is not inverted. As the pulse travels from the dense string to the less dense one, the junction acts more like a free end than a rigid end.

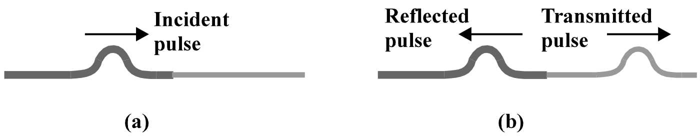


Figure : (a) A pulse traveling to the right on a heavy string attached to a lighter string. (b) The incident pulse is partially reflected and partially transmitted. In this case, the reflected pulse is not inverted.

The above observations may be stated as follows :

1. If the wave will incident on a fixed boundary or rigid boundary then the reflected wave will be inverted that means there is a phase change of π. If the wave will incident on a loose boundary or free boundary then the reflected wave will not be inverted that means there is no phase change.

2. If a wave enters a region from higher wave velocity (rarer medium) to smaller wave velocity (denser medium), the reflected wave is inverted that means there is a phase change of π. If a wave enters a region from lower wave velocity (denser medium) to larger wave velocity (rarer medium), the reflected wave is not inverted that means there is no phase change.

Reflection of sound wave expressed in terms of pressure wave : In case of reflection of longitudinal pressure waves, it suffers a phase change of π from a free or open end and no change in phase from rigid boundaries. When a sound wave gets reflected from a rigid boundary, a compression pulse reflects as a compression pulse and a rarefaction pulse reflect as a rarefaction pulse.

If a_i is the amplitude of the incident wave in the medium 1, then the amplitudes of the reflected (a_r) and transmitted (a_t) waves in the medium 1 and 2 respectively, are given by

$$a_r = a_i\left(\frac{v_2 - v_1}{v_2 + v_1}\right) \text{ and } a_t = a_i\left(\frac{2v_2}{v_2 + v_1}\right).$$

<u>**Practice Problems :**</u>

1. Consider a wave which is represented by y = a sin ($\omega t - k_1x$) is incident from rarer medium to denser medium. If amplitude of reflected wave is a_r, amplitude of transmitted wave is a_t and wave number of the transmitted wave is k_2 then find (a) equation of reflected wave (b) equation of transmitted wave (c) a_r (d) a_t.

2. Consider a wave which is represented by y = a sin ($\omega t - k_1x$) is incident from denser medium to rarer medium. If amplitude of reflected wave is a_r, amplitude of transmitted wave is a_t and wave number of the transmitted wave is k_2 then find (a) equation of reflected wave (b) equation of transmitted wave (c) a_r (d) a_t.

3. Consider a wave which is represented by y = a sin (ωt – kx) is incident on a rigid boundary. If there is no transmission of waves then what is the equation of the reflected wave ?

4. Consider a wave which is represented by y = a sin (ωt – kx) is incident on a free boundary. If there is no transmission of waves then what is the equation of the reflected wave ?

5. Consider a pressure wave which is represented by $\Delta P = \Delta P_0$ sin (ωt – kx) is incident on a rigid boundary. If there is no transmission of waves then what is the equation of the reflected wave ?

6. Consider a wave which is represented by $\Delta P = \Delta P_0$ sin (ωt – kx) is incident on a free boundary. If there is no transmission of waves then what is the equation of the reflected wave ?

7. A man standing in front of a mountain at a certain distance beats a drum at regular intervals. The drumming rate is gradually increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 m and finds the echo is again not heard when the drumming rate becomes 60 per minute. Calculate (a) the distance between the mountain and the initial position of the man (b) the velocity of sound.

[Answers : (1) (a) $y_r = -a_r\sin(\omega t + k_1x)$ (b) $y_t = a_t\sin(\omega t - k_2x)$ (c) $\frac{k_2 - k_1}{k_1 + k_2}a$ (d) $\frac{2k_1}{k_1 + k_2}a$

(2) (a) $y_r = a_r\sin(\omega t + k_1x)$ (b) $y_t = a_t\sin(\omega t - k_2x)$ (c) $\frac{k_1 - k_2}{k_1 + k_2}a$ (d) $\frac{2k_1}{k_1 + k_2}a$ (3) –asin(ωt + kx)

(4) asin(ωt + kx) (5) ΔP_0 sin(ωt + kx) (6) – ΔP_0 sin(ωt + kx) (7) (a) 270 m (b) 360 m/s]

C19 Doppler Effect :

The Doppler effect is a change in the observed frequency of a wave when the source or the detector moves relative to the transmitting medium (such as air). For sound the observed frequency f' is given in terms of the source frequency f by $f' = f\frac{v \pm v_0}{v \mp v_S}$, where v_0 is the speed of the detector relative to the medium, v_S is that of the source, and v is the speed of sound in the medium. The signs are chosen such that f' tends to be greater for motion (of detector of source) "toward" and less for motion "away".

Wind Effect :

The above formulae can be modified by taking the wind effects into account. The velocity of sound should be taken as : $v + v_w$ or $v - v_w$ if the wind is blowing in the same or opposite direction as source to observer.

Practice Problems :

1. **A vehicle with a horn of frequency n is moving with a velocity of 30 m/s in a direction perpendicular to the straight line joining the observer and the vehicle. The observer perceives the sound to have a frequency n + n_1. Then (if the sound velocity in air is 300 m/s)**

   (a) $n_1 = 10$ n (b) $n_1 = 0$ (c) $n_1 = -0.1$ n (d) $n_1 = 0.1$ n

2. **A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of sound is 340 m/s then the ratio f_1/f_2 is**

   (a) 18/19 (b) 1/2 (c) 2 (d) 19/18

3. **A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is**

   (a) 242/252 (b) 2 (c) 5/6 (d) 11/6

[Answers : (1) b (2) d (3) b]