Area

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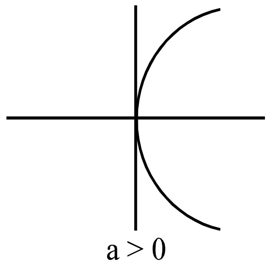
C1 Curve Tracing :

To find the approximate shape of a curve, the following procedure is adopted in order :

(a) Symmetry :

(i) Symmetry about x-axis :

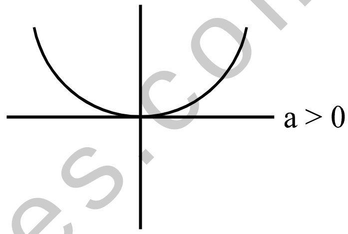
If all the powers of ‘y’ in the equation are even then the curve is symmetrical about the x-axis.



for example : $y^2 = 4ax$

(ii) Symmetry about y-axis :

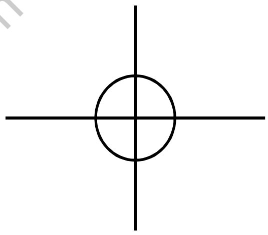
If all the powers of ‘x’ in the equation are even then the curve is symmetrical about the y-axis.



for example : $x^2 = 4ay$

(iii) Symmetry about both axis :

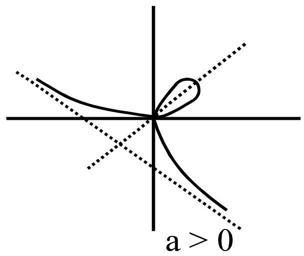
If all the powers of ‘x’ and ‘y’ in the equation are even, the curve is symmetrical about the axis of ‘x’ as well as ‘y’.



for example : $x^2 + y^2 = a^2$

(iv) Symmetry about the line y = x :

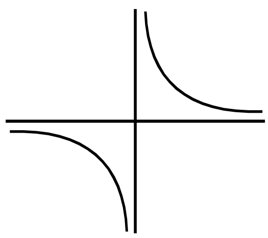
If the equation of the curve remains unchanged on interchanging ‘x’ and ‘y’, the the curve is symmetrical about the line y = x.



for example : $x^3 + y^3 = 3axy$

(v) Symmetry in opposite quadrants :

If the equation of the curve remains unaltered when ‘x’ and ‘y’ are replaced by–x and –y respectively then there is symmetry in opposite quadrants.



for example : $x\,y = c^2$

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(b) Find the points where the curve crosses the x-axis and also the y-axis.

(c) Find $\frac{dy}{dx}$ and equate it to zero to find the points on the curve where you have horizontal tangents.

(d) Examine if possible the intervals when f(x) is increasing or decreasing.

(e) Examine what happens to 'y' when $x \to \infty$ or $x \to -\infty$.

(f) **Asymptotes :**

Asymptote(s) is (are) line (s) whose distance from the curve tends to zero as point on curve moves towards infinity along branch of curve.

(i) If $\underset{x \to a}{\text{Lt}}\, f(x) = \infty$ **or** $\underset{x \to a}{\text{Lt}}\, f(x) = -\infty$ then x = a is asymptote of y = f(x).

(ii) If $\underset{x \to +\infty}{\text{Lt}}\, f(x) = k$ **or** $\underset{x \to -\infty}{\text{Lt}}\, f(x) = k$, then y = k is asymptote of y = f(x)

(iii) If $\underset{x \to \infty}{\text{Lt}}\, \frac{f(x)}{x} = m_1,\ \underset{x \to \infty}{\text{Lt}}\, (f(x) - m_1 x) = c,$ then $y = m_1 x + c_1$ is an asymptote.

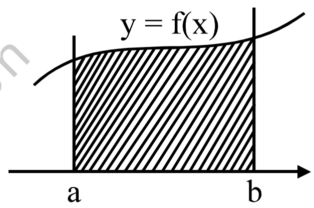
(inclined to right)

(iv) If $\underset{x \to -\infty}{\text{Lt}}\, \frac{f(x)}{x} = m_2,\ \underset{x \to -\infty}{\text{Lt}}\, (f(x) - m_2 x) = c_2,$ then $y = m_2 x + c_2$ is an asymptotes

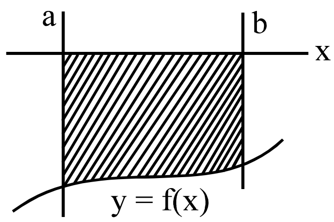
(inclined to left).

C2 Quadrature :

(a) If f(x) ≥ 0 for x ∈ [a, b], then area bounded by curve y = f(x), x-axis, x-axis, x = a and x = b is $\int_a^b f(x)dx$.

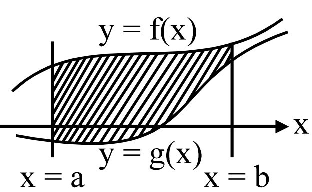


(b) If f(x) ≤ 0 for x ∈ [a, b], then area bounded by curve y = f(x), x-axis, x = a and x = b is $-\int_a^b f(x)dx$.

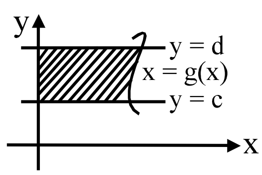


(c) If f(x) ≥ 0 for x ∈ [a, c] and f(x) ≤ 0 for x ∈ [c, b] (a < c < b) then area bounded by curve y = f(x) and x-axis between x = a and x = b is $\int_a^c f(x)dx - \int_c^b f(x)dx$.

(d) If f(x) ≥ g(x) for x ∈ [a, b] then area bounded by curves y = f(x) and y = g(x) between ordinates x = a and x = b is $\int_a^b (f(x) - g(x))dx$.



(e) If g(y) ≥ 0 for y ∈ [c, d] then area bounded by curve x = g(y) and y-axis between abscissa y = c and y = d is $\int_{y=c}^{d} g(y)dy$.



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<u>Practice Problems :</u>

1. **The area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4by$ is**

(a) $\frac{16}{3}ab$ (b) $\frac{18}{3}ab$ (c) $\frac{20}{3}ab$ (d) $\frac{22}{3}ab$

2. **The area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ is**

(a) 26 (b) 27 (c) 28 (d) 29

3. **The area of the region bounded by the curve $y = x \sin x$ and the x-axis between $x = 0$ and $x = 2\pi$ is**

(a) 2π (b) 3π (c) 4π (d) 5π

4. **The area bounded by the curve $y = \log_e x$, $x = 0$, $x = 1$ and $y = 0$ is**

(a) 1 (b) 2 (c) 3 (d) 4

5. **The area enclosed by $|x| + |y| = 1$ is**

(a) 5 (b) 4 (c) 3 (d) 2

6. **The area of the figure bounded by two branches of the curve $(y – x)^2 = x^3$ and the straight line $x = 1$ is**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

7. **The area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = 1$ and $x = b$ is $(b – 1) \sin (3b + 4)$ The function $f(x)$ is $a(x – 1) \cos (3x + 4) + \sin (3x + 4)$. The value of a is**

(a) 1 (b) 2 (c) 3 (d) 4

8. **The area enclosed by the curve $y = 2^x$ and $\max \{|x|, |y|\} = 1$ is**

(a) $3+\frac{1}{2\ln 2}$ (b) $4+\frac{1}{2\ln 2}$ (c) $5+\frac{1}{2\ln 2}$ (d) $6+\frac{1}{2\ln 2}$

9. **Area enclosed by the curve $|x – 2| + |y + 1| = 1$ is equal to**

(a) 4 sq. units (b) 6 sq. units (c) 2 sq. units (d) 8 sq. units

10. **Area bounded by $y = \tan^{-1}x$, $y = \cot^{-1}x$ and y-axis is equal to**

(a) $\ln \sqrt{2}$ sq. units (b) $\ln 4$ sq. units (c) $\ln 8$ sq. units (d) $\ln 2$ sq. units

[Answers : (1) a (2) b (3) c (4) a (5) d (6) d (7) c (8) a (9) c (10) d]

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