ELECTRIC POTENTIAL AND CAPACITANCE

2.1 Introduction :

Q. What is the nature of electrostatic force or Coulomb's force ?

Solution : Coulomb's force between two stationary charges is a conservative force.

Q. Does the work done by an electrostatic field depend on the path ? Explain.

Solution : As the electrostatic field is conservative field, i.e., the work done by electrostatic field is path independent.

Q. What is the work done by electrostatic force for closed path ?

Solution : Zero

Q. Can absolute value of potential energy be defined or significant ? Explain.

Solution : No, there is only the significance of difference of potential energy because it equals to the work done by the external force in bringing the charge from one point R to the point P in the presence of field due to any charge configuration i.e., $\Delta U = U_P - U_R = W_{RP} = \int_R^P \mathbf{F_{ext}} \cdot \mathbf{dr}$.

Q. Define electrostatic potential energy difference.

Solution : It equals to the work done by the external force in bringing the charge q from one point R to the point P in the presence of field due to any charge configuration i.e., $\Delta U = U_P - U_R = W_{RP} = \int_R^P \mathbf{F_{ext}} \cdot \mathbf{dr}$.

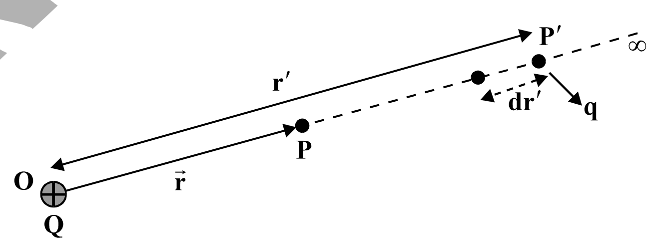
Q. Define electrostatic potential energy. Write down its units and dimensional formula.

Solution : Electrostatic potential energy of charge q at a point P (in the presence of field due to any charge configuration) is the work done by the external force (equal and opposite to the electric force) in bringing (infinitesimally slow constant speed) the charge q from infinity to that point. Mathematically, $W_{\infty P} = U_P - U_\infty = U_P$ (The potential energy at infinity is taken as zero).

The unit of electrostatic potential energy is joule and dimensional formula $[ML^2T^{-2}]$

Q. Consider a charge Q placed (fixed) at the origin. A test charge q is moved very slowly by an external agent from infinity to the point P at distance 'r' from Q. Derive the expression for work done by external agent in this process ? Derive the expression for electrostatic potential energy of charge q ?

Solution : Consider a point charge Q at the origin.



At some intermediate point $\mathbf{P'}$ on the path, the electrostatic force on charge q is $\dfrac{\mathbf{Q \times q}}{\mathbf{4\pi\varepsilon_0 r'^2}}\mathbf{\hat{r}'}$, where $\mathbf{\hat{r}'}$ is a unit vector along $\mathbf{OP'}$. Work done against this force from $\vec{\mathbf{r}}'$ **to** $\vec{\mathbf{r}}' + \mathbf{d}\vec{\mathbf{r}}'$ is

$$\mathbf{dW = -\frac{Q \times q}{4\pi\varepsilon_0 r'^2} dr'}$$

The negative sign appears because for $\mathbf{dr' < 0}$, dW is positive. Total work done (W) by the external force is obtained by integrating from $\vec{\mathbf{r}}_1$ (magnitude r_1) to $\vec{\mathbf{r}}_2$ (magnitude r_2).

$$W = -\int_{\infty}^{r} \frac{Qq}{4\pi\varepsilon_0 r'^2} dr' = \left.\frac{Qq}{4\pi\varepsilon_0 r'}\right|_{\infty}^{r} = \frac{Qq}{4\pi\varepsilon_0 r}$$

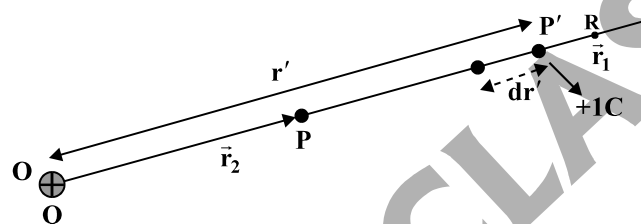
This work done W is the expression for electrostatic potential energy of charge q.

2.2 Electrostatic Potential :

Q. Define electrostatic potential difference. Find an expression for it due to a point charge ?

Solution : It equals to the work done by the external force in bringing the unit charge from one point R to the point P in the presence of field due to any charge configuration. Mathematically, $V_P - V_R = \frac{U_P - U_R}{q}$, where V_P and V_R are the electrostatic potentials at P and R, respectively.

Consider a point charge Q at the origin. To determine the potential difference between the points R (with position vector $\vec{r}_1$ from the origin) and P (with position vector $\vec{r}_2$ from the origin) due to this charge, we must calculate the work done in bringing a unit positive test charge from R to P.



At some intermediate point P' on the path, the electrostatic force on a unit positive charge is $\frac{Q \times 1}{4\pi\varepsilon_0 r'^2}\hat{r}'$, where $\hat{r}'$ is a unit vector along OP'. Work done against this force from $\vec{r}'$ **to** $\vec{r}' + d\vec{r}'$ is

$$dW = -\frac{Q}{4\pi\varepsilon_0 r'^2} dr'$$

The negative sign appears because for $dr' < 0$, dW is positive. Total work done (W) by the external force is obtained by integrating from $\vec{r}_1$ (magnitude r_1) to $\vec{r}_2$ (magnitude r_2).

$$W = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 r'^2} dr' = \left.\frac{Q}{4\pi\varepsilon_0 r'}\right|_{r_1}^{r_2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

This work done W equals to $V_P - V_R$

Q. Define electrostatic potential. Is it a vector or scalar quantity ? Write down its unit and dimensional formula.

Solution : The electrostatic potential (V) at any point due to a charge distribution in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. Mathematically, $V_P - V_\infty = \frac{U_P - U_\infty}{q} = \frac{U_P}{q}$, where V_P and V_∞ (= 0) are the electrostatic potentials at P and ∞, respectively.

It is a scalar quantity. The unit of electrostatic potential is joule/coulomb or volt. The dimensional formula is $[ML^2T^{-3}A^{-1}]$

Q. Can absolute value of potential be defined or significant ? Explain.

Solution : No, there is only the significance of difference of potential because it equals to the work done by the external force in bringing the unit charge from one point R to the point P in the presence of field due to any charge configuration.

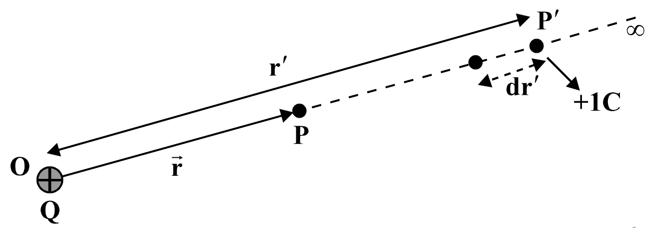
Q. While defining potential why is infinity taken as a reference point ?

Solution : Infinity is taken as a reference point because at infinity potential due to any charge configuration equals to zero.

2.3 Potential Due to a Point Charge :

Q. Derive the expression for electrostatic potential due to a point charge Q at the distance 'r'.

Solution : Consider a point charge Q at the origin. To determine the potential at any point P (with position vector $\vec{r}$ from the origin) due to this charge, we must calculate the work done in bringing a unit positive test charge from infinity to the point P.



At some intermediate point P' on the path, the electrostatic force on a unit positive charge is $\frac{Q \times 1}{4\pi\varepsilon_0 r'^2}\hat{r}'$,

where $\hat{r}'$ is a unit vector along OP'. Work done against this force from $\vec{r}'$ to $\vec{r}' + d\vec{r}'$ is

$$dW = -\frac{Q}{4\pi\varepsilon_0 r'^2}dr'$$

The negative sign appears because for $dr' < 0$, dW is positive. Total work done (W) by the external force is obtained by integrating from $r' = \infty$ to $r' = r$.

$$W = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2}dr' = \left.\frac{Q}{4\pi\varepsilon_0 r'}\right|_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

This, by definition is the potential at P due to the charge Q

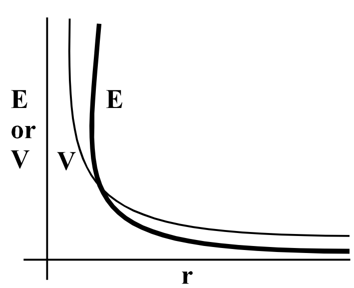
$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}.$$

Q. Why the electrostatic potential due to a single charge is spherically symmetric ?

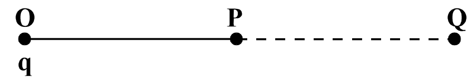
Solution : As the electrostatic potential due to a single charge is inversely proportional to r and at equal distance from the charge, value of v remains same.

Q. Draw the graph for the variation of electrostatic potential and electrostatic field due to a point charge with distance 'r'.

Solution :



Q. A point charge 'q' is placed at O. Is $V_P - V_Q$, positive or negative when (i) $q > 0$, (ii) $q < 0$? Justify your answer.



Solution : $V_P - V_Q$ equals to the work done by an external agent to move a unit positive charge from Q to P in the presence of other charges. (i) when $q > 0$, the work done by an external agent to move a unit positive charge from Q to P is positive hence $V_P - V_Q$ is positive. (ii) when $q < 0$, the work done by an external agent to move a unit positive charge from Q to P is negative hence $V_P - V_Q$ is negative.

Q. If the electrostatic potential at any point due to certain charge configuration equals to V then find the work done by the external agent in bringing a charge q from infinity to this point without acceleration (very slowly) ?

Solution : The electrostatic potential (V) at any point in a region with electrostatic field is the work done in bringing a unit positive charge (without acceleration) from infinity to that point. Hence the work done by the external agent in bringing a charge q from infinity to this point without acceleration (very slowly) equals to qV.

Q. Define one volt.

Solution : Electrostatic potential at a point is said to be one volt, when one joule of work is done in moving one coulomb of positive charge from infinity to that point by an external agent (against the electrostatic force of the field) without acceleration.

Q. (a) Calculate the potential at a point P due to a charge of 4×10^{-7}C located 9 cm away.

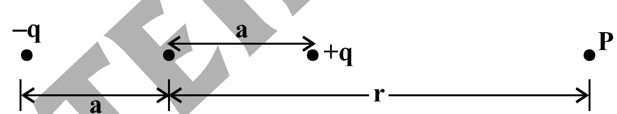
(b) Hence obtain the work done in bringing a charge of 2×10^{-9}C from infinity to the point P. Does the answer depend on the path along which the charge is brought ? [NCERT solved example 2.1]

Solution : (a) 4×10^4 V (b) 8×10^{-5} J, work done will be path independent.

2.4 Potential Due to an Electric Dipole :

Q. Derive an expression for the electric potential at any point along (i) axial line (ii) equatorial line of an electric dipole.

Solution : Electric potential due to dipole along axial line



The potential due to '+q' at P is $\dfrac{q}{4\pi \in_0 (r-a)}$.

The potential due to '–q' at P is $\dfrac{-q}{4\pi \in_0 (r+a)}$

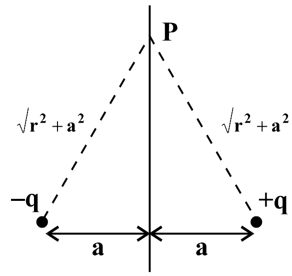
From principle of superposition, the net potential at P is

$$V = \frac{q}{4\pi \in_0 (r-a)} - \frac{q}{4\pi \in_0 (r+a)} = \frac{q(2a)}{4\pi \in_0 (r^2 - a^2)} = \frac{p}{4\pi \in_0 (r^2 - a^2)}$$

Here p is the dipole moment of dipole equals to q(2a).

For short dipole, r >> a $\quad V \cong \dfrac{p}{4\pi \in_0 r^2}$

Potential at equatorial line

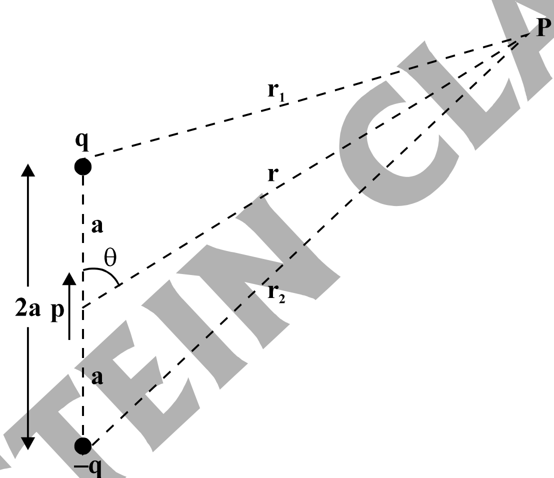


The electrostatic potential at point P due to +q is $\frac{q}{4\pi \epsilon_0 \sqrt{r^2+a^2}}$ and due to '–q' is $\frac{-q}{4\pi \epsilon_0 \sqrt{r^2+a^2}}$.

Hence, from principle of superposition, the net potential at P equals to zero.

Q. Derive an expression for the electric potential at any point due to electric dipole. Discuss this result for axial line and equatorial line.

Solution :



Let us take the origin at the centre of the dipole. We have to calculate electric potential at any point P where $\overrightarrow{OP} = \vec{r}$.

Let the distance of P from charge –q be r_2 and distance of P from charge +q be r_1.

Electrostatic Potential at P due to –q charge is $V_2 = \frac{-q}{4\pi\varepsilon_0 r_2}$

and Electrostatic potential at P due to +q charge is $V_1 = \frac{q}{4\pi\varepsilon_0 r_1}$

Therefore, potential at P due to the dipole,

$$V = V_2 + V_1$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad ...(1)$$

Now, by geometry $r_1^2 = r^2 + a^2 - 2\,ar\cos\theta$, and

$r_2^2 = r^2 + a^2 + 2\,ar\cos\theta$

We may rewrite $$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

If a < < r, $\frac{a}{r}$ is small, $\frac{a^2}{r^2}$ can be neglected.

$\therefore$ $$r_1^2 = r^2\left(1 - \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 - \frac{2a}{r}\cos\theta\right)^{1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Using Binomial approximation, $(1 + x)^n = 1 + nx$ for x << 1

$$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right]$$

Similarly, $$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$$

Putting these values in (1)

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{a}{r}\cos\theta - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$V = \frac{q \times 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

As p cos θ = $\vec{p}.\hat{r}$, where $\hat{r}$ is unit vector along the position vector $\overrightarrow{OP} = \vec{r}$,

$\therefore$ Electrostatic potential at P due to a short dipole (a < < r) is $V = \frac{\vec{p}.\hat{r}}{4\pi\varepsilon_0 r^2}$

On the dipole axis, θ = 0^0 or π $\therefore$ $V = \pm\frac{p}{4\pi\varepsilon_0 r^2}$

Positive sign for θ = 0^0 and negative sign for θ = π.

At any point in the equatorial plane, $\theta = \frac{\pi}{2}; \cos\theta = \cos\frac{\pi}{2} = 0$. $\therefore$ V = 0

i.e., electrostatic potential at any point in the equatorial plane of dipole is zero.

Q. What is the difference between the electric potential of a dipole from that due a single charge ?

Solution : (i) The potential due to a dipole depends not just on r but also on the angle between the position vector $\vec{r}$ and the dipole moment vector $\vec{p}$.

(ii) The electric dipole potential falls off, at large distance, as $1/r^2$, not as 1/r, characteristic of the potential due to a single charge.

Q. What do you mean by axially symmetric about dipole moment of potential due to an electric dipole ?

Solution : If you rotate the position vector $\vec{r}$ about $\vec{p}$, keeping θ (between $\vec{r}$ and p) fixed, the points corresponding to P on the cone so generated will have the same potential as at P.

Q. What do you mean by cylindrical symmetry of electrical potential due to an electric dipole ?

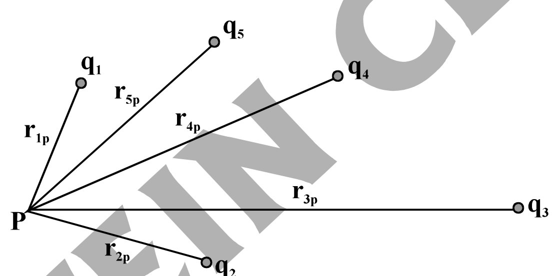
Solution : It means potential due to electric dipole will be same at every point on the surface of a right circular cylinder imaginated with the electric dipole as the axis.

2.5 Potential due to a system of Charges :

Q. Which principle is used to find the potential due to system of charges and derive the potential due to system of charges ? or Consider a system of charges q_1, q_2, q_3 ...... q_n with position vectors $\vec{r}_1, \vec{r}_2 ....\vec{r}_n$. Find the expression for the potential at point P of position vector $\vec{r}_P$.

Solution : Superposition principle is used to find the potential due to system of charges.

Consider a system of charges q_1, q_2, q_3 ...... q_n with position vectors $\vec{r}_1, \vec{r}_2 ....\vec{r}_n$ related to some origin O.



The potential V_1 at P due to the charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}} (r_{1P} = |\vec{r}_P - \vec{r}_1|)$$, where r_{1P} is the distance between q_1 and P.

Similarly, the potential V_2 at P due to q_2 and V_3 due to q_3 are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}} (r_{2P} = |\vec{r}_P - \vec{r}_2|), \quad V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}} (r_{3P} = |\vec{r}_P - \vec{r}_3|)$$

where r_{2P} and r_{3P} are the distances of P from charges q_2 and q_3, respectively; and so on for the potential due to other charges. By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges

$$V = V_1 + V_2 + ... + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... + \frac{q_n}{r_{nP}}\right)$$

Q. What is the electrostatic potential due to uniformly charged spherical shell ?

Solution : For a uniformly charged spherical shell, the electric field outside the shell is as if the point charge is concentrated at the centre. Thus, the potential outside the shell is given by

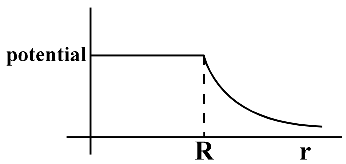
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} (r \geq R)$$

where q is the total charge on the shell and R its radius. The electric field inside the shell is zero. This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell), and, therefore, equals its value at the surface, which is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}(r < R)$$

Q. Draw the graph for the variation of potential due to uniformly charged spherical shell with the distance from the center.

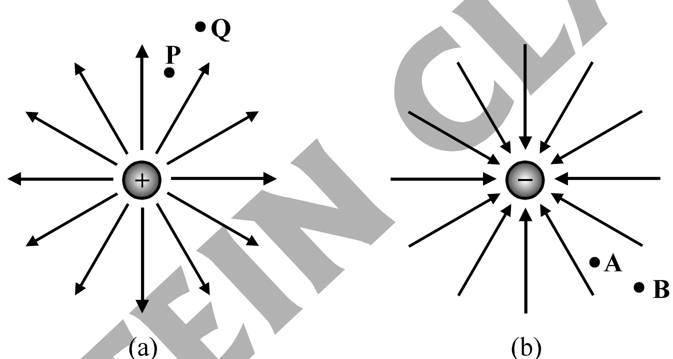
Solution :



Q. Two charges 3×10^{-8} C and -2×10^{-8}C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero. [NCERT solved example 2.2]

Solution : Electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge.

Q. Figures (a) and (b) show the field lines of a positive and negative point charge respectively.



(a) Give the signs of the potential difference $V_P - V_Q$; $V_B - V_A$.

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A ? [NCERT solved example 2.3]

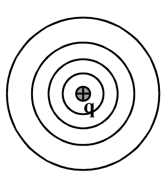
Solution : (a) +, + (b) +, + (c) – (d) + (e) decrease.

2.6 Equipotential Surface :

Q. Define equipotential surface.

Solution : An equipotential surface is a surface with a constant value of potential at all points on the surface.

Q. Draw equipotential surface for a point charge. What is the shape of equipotential surface for point charge ?

Solution : 

The shape of equipotential surface for a point charge is spherical.

Q. Why equipotential surface through a point is normal to the electric field at that point for any charge configuration ?

Solution : If the field were not normal to the equipotential surface, it would have non-zero component along the surface. To move a unit test charge against the direction of the component of the field, work would have to be done. But this is in contradiction to the definition of an equipotential surface : there is no potential difference between any two points on the surface and no work is required to move a test charge on the surface. The electric field must, therefore, be normal to the equipotential surface at every point.

Q. Can two equipotential surface ever intersect each other ? Comment

Solution : No, they cannot intersect at any point. If they will intersect then at the point of intersection the potential for each surface will be the same which is not possible.

Q. If an electric field is along the x-axis then what will be the shape of equipotential surface ?

Solution : If an electric field is along the x-axis then the shape of equipotential surface is planes parallel to the yz plane.

Q. What will be the work done by an external agent to move a charge q_0 from one point of an equipotential surface of potential V_1 to another point of another equipotential surface of potential V_2 ?

Solution : $q_0 (V_2 - V_1)$

Q. What will be the work done by an external agent to move a charge from one point to another point on the same equipotential surface ? Give reason.

Solution : The work done by an external agent to move a charge from one point to another point on the same equipotential surface equals to zero because on an equipotential surface, the potential at each point remain same.

Q. State main characteristics of an equipotential surface.

Solution : The main characteristics of an equipotential surface are as given below :

(i) An equipotential surface is a surface with a constant value of electric potential at all points on the surface.

(ii) No work is required to be done to move a charge on an equipotential surface.

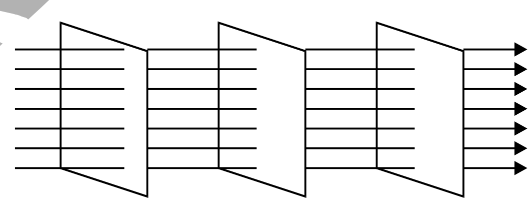
(iii) For any charge configuration, equipotential surface through a point is normal to the electric field at that point.

(iv) No two equipotential surfaces can ever intersect each other.

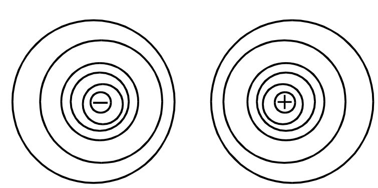
(v) Relative closeness of equipotential surfaces having a given potential difference means a region of stronger electric field.

Q. Draw the equipotential surface for (a) uniform electric field (b) dipole (c) two identical positive charges.

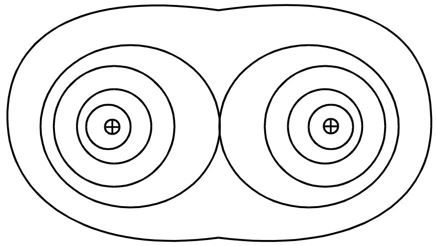
Solution : (a)



(b)

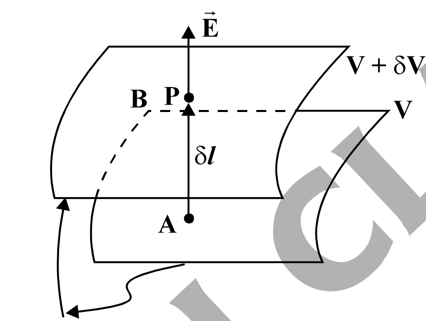


(c)



Q. What is the relation between electric field and electric potential ? Derive it.

Solution : Consider two closely spaced equipotential surfaces A and B with potential values V and V + δV, where δV is the change in V in the direction of the electric field $\vec{E}$. Let P be a point on the surface B. δl is the perpendicular distance of the surface A from P. Imagine that a unit positive charge is moved along this perpendicular from the surface B to surface A against the electric field. The work done in this process is $|\vec{E}|\ \delta l$.



This work equals the potential difference $V_A - V_B$.

Thus,

$$|\vec{E}|\ \delta l = V - (V + \delta V) = -\delta V$$

$$\text{i.e., } |\vec{E}| = -\frac{\delta V}{\delta l}$$

Since δV is negative, $\delta V = -|\delta V|$, we can rewrite as $|\vec{E}| = -\frac{\delta V}{\delta l} = +\frac{|\delta V|}{\delta l}$

Q. Write down two conclusion concerning the relation between $\vec{E}$ and potential 'OR' what is the physical meaning of potential gradient ? Is it a scalar or vector quantity.

Solution : We arrive at two important conclusions concerning the relation between electric field and potential :

(i) Electric field is in the direction in which the potential decreases steepest.

(ii) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

Potential gradient is a vector quantity.

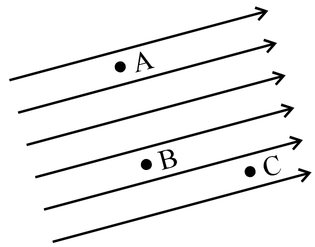
Q. In a particular region in space the electric potential has a constant high value. What is the value of electric field there ?

Solution : As potential is constant, hence, value of electric field strength is zero in that region in space in that direction.

Q. Is electrostatic potential necessarily zero at a point where electric field strength is zero ? Illustrate your answer.

Solution : No, if the electric field in the space is zero then the potential in that space may have some non-zero constant value. For e.g., inside the coductor electric field is zero but the potential will exists.

Q. At which of the points A, B and C in a uniform electrostatic field as shown, will the electric potential be (a) minimum (b) maximum

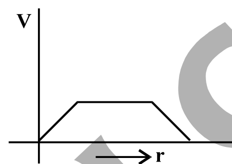


Solution : $V_A = V_B > V_C$

Q. In a space the electric potential is changing with distance r according to $V = kr^2$. Find the electric field.

Solution : $-2kr$

Q. The electrostatic potential V is changing with distance r according to the following graph.



Draw the graph for the variation of electric field with distance r.

Solution :

Q. The electric field in a space due to certain charged distribution is inversely proportional to distance. How the electrostatic potential depends on distance ?

Solution : proportional to $ln r$

Q. A metal sphere with a charge Q is surrounded by an uncharged concentric thin spherical shell. The potential difference between them is V. If the shell is now given an additional charge Q, what is the new potential difference between them ?

Solution : V

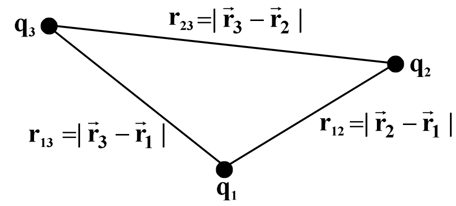
Q. We know that electric field is discontinuous across the surface or a charged conductor. Is electric potential also discontinuous there ?

Solution : No

2.7 Potential Energy of a System of Charges :

Q. Derive the expression for potential energy of a system of three point charges q_1, q_2 and q_3 placed at the position $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ relative to some origin O ?

Solution : Consider a system of three point charges q_1, q_2 and q_3 placed at the position $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$ relative to some origin O. A system of three charges q_1, q_2 and q_3 located at $\vec{r}_1, \vec{r}_2, \vec{r}_3$ respectively.



To bring q_1 first from infinity to $\vec{r}_1$, no work is required. Next we bring q_2 from infinity to $\vec{r}_2$. As before, work done in this step is $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$.

Work done next in bringing q_3 from infinity to the point $\vec{r}_3$ is $\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$

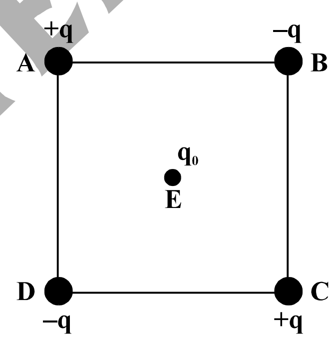
The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

Q. The potential energy is characteristic of the present state of configuration, and not the way the state is achieved. Comment.

Solution : The conservative nature of electrostatic force (or equivalently, the path independence of work done), the expression for potential energy, is independent of the manner in which the configuration is assembled i.e., the potential energy is characteristic of the present state of configuration, and not the way the state is achieved.

Q. Four charges are arranged at the corners of a square ABCD of side d, as shown (a) Find the work required to put together this arrangement. (b) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners.



How much extra work is needed to do this ? [NCERT solved example 2.4]

Solution : (a) $\frac{-q^2}{4\pi\varepsilon_0 d}\left(4-\sqrt{2}\right)$ (b) Zero

2.8 Potential Energy in an External Field :

Q. What is the potential energy of a charge q in a given field ?

Solution : The work done in bringing the charge 'q' from infinity to the point P, at which the potential is V due to the field, is qV and this work is stored in the form of potential energy. If the point P has position vector $\vec{r}$ relative to some origin, we can write : Potential energy of q at $\vec{r}$ in an external field = qV ($\vec{r}$) where V($\vec{r}$) is the external potential at the point $\vec{r}$.

Q. Define one electron volt.

Solution : If an electron with charge q = e = 1 = 1.6×10^{-19} C is accelerated by a potential difference of ΔV = 1 volt, it would gain energy of $q\Delta V = 1.6 \times 10^{-19}$ J. This unit of energy is defined as 1 electron volt or 1eV, i.e., 1 eV = 1.6×10^{-19} J.

Q. What is the value of 1 keV, 1 MeV, 1 GeV & 1 TeV in joule ?

Solution : 1 keV = 10^3eV = 1.6×10^{-16}J, 1 MeV = 10^6eV = 1.6×10^{-13}J, 1 GeV = 10^9eV = 1.6×10^{-10} J and 1 TeV = 10^{12} eV = 1.6×10^{-7} J.

Q. What is the potential energy of a system of two charges q_1 and q_2 located at $\vec{r}_1$ and $\vec{r}_2$, respectively, in an external field ?

Solution : First, we calculate the work done in bringing the charge q_1 from infinity to $\vec{r}_1$. Work done in this step is $q_1 V(\vec{r}_1)$. Next, we consider the work done in bringing q_2 to $\vec{r}_2$. In this step, work is done not only against the external field $\vec{E}$ but also against the field due to q_1.

Work done on q_2 against the external field = $q_2 V(\vec{r}_2)$

Work done on q_2 against the field due to q_1 = $\dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

where r_{12} is the distance between q_1 and q_2. By the superposition principle for fields, we add up the work done on q_2 against the two fields ($\vec{E}$ and that due to q_1) = $q_2 V(\vec{r}_2) + \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

Thus, Potential energy of the system

= the total work done in assembling the configuration = $q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2) + \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

Q. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 µC and –2 µC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.

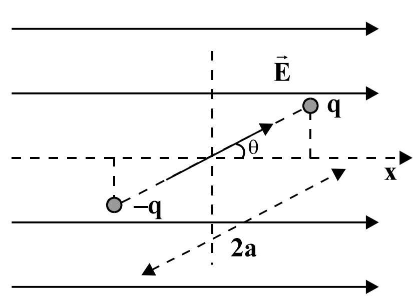
(b) How much work is required to separate the two charges infinitely away from each other ?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5$ C m^{-2}. What would the electrostatic energy of the configuration be ? [NCERT Solved Example 2.5]

Solution : (a) –0.7 J (b) 0.7 J (c) 49.3 J

Q. Derive the expression for potential energy of a dipole in an external electric field $\vec{E}$. 'OR' Derive an expression for the total work done in rotating the dipole through an angle 'θ' in uniform electric field 'E'.

Solution : Consider a dipole charges q_1 = +q and q_2 = –q placed in a uniform electric field $\vec{E}$, as shown.



Suppose an external torque $\vec{\tau}_{ext}$ is applied in such a manner that it just neutralises the torque $\vec{p} \times \vec{E}$ and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and without angular acceleration. The amount of work done by the external torque will be given by

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta_1} pE\sin\theta d\theta = pE(\cos\theta_0 - \cos\theta_1)$$

This work is stored as the potential energy of the system. We can then associate potential energy U(θ) with an inclination θ of the dipole. Similar to other potential energies, there is a freedom in choosing the angle where the potential energy U is taken to be zero. A natural choice is to take $\theta_0 = \pi/2$. We can then write,

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right) = -pE\cos\theta = -\vec{p}.\vec{E}$$

Q. A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude 10^6 V m^{-1}. The direction of the field is suddenly changed by an angle of 60^0. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of the sample.

Solution : 3 J, So, there is loss in potential energy. This must be the energy released by the substance in the form of heat in aligning its dipoles.

2.9 Electrostatics of Conductors :

Q. What is the electrostatic field inside a conductor ?

Solution : Electrostatic field is zero inside a conductor at electrostatic equilibrium.

Q. What is the direction of electrostatic field at the surface of a conductor at electrostatic equilibrium or static situation ? Give reasons.

Solution : Electrostatic field at the surface of a charged conductor must be normal to the surface at every point. If $\vec{E}$ were not normal to the surface, it would have some non-zero components along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, $\vec{E}$ should have no tangential component.

Q. There is no net charge at any point inside the conductor, and any excess charge must reside at the surface. Explain.

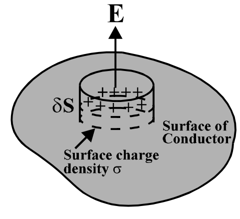
Solution : A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation. This follows from the Gauss's law. Consider any arbitrary volume element v inside a conductor. On the closed surface S bounding the volume element v, electrostatic field is zero. Thus the total electric flux through S is zero. Hence, by Gauss's law, there is no net charge enclosed by S. But the surface S can be made as small as you like, i.e., the volume v can be made vanishingly small. This means there is no net charge at any point inside the conductor, and any excess charge must reside at the surface.

Q. Why the electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface ?

Solution : Since $\vec{E} = 0$ inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the surface of the conductor.

Q. Derive the expression for the electric field at the surface of a charge conductor of surface charge density σ.

Solution :



Choose a pill box (a short pill box (a short cylinder) as the Gaussian surface about any point P on the surface. The pill box is partly inside and partly outside the surface of the conductor. It has a small area of cross section δS and negligible height.

Just inside the surface, the electrostatic field is zero; just outside, the field is normal to the surface with magnitude E. Thus, the contribution to the total flux through the pill box comes only from the outside (circular) cross-section of the pill box. This equals $\pm E\delta S$ (positive for $\sigma > 0$, negative for $\sigma < 0$), since over the small area δS, $\vec{E}$ may be considered constant and $\vec{E}$ and δS are parallel or antiparallel. The charge enclosed by the pill box is $\sigma\delta S$.

By Gauss's law

$$E\delta S = \frac{|\sigma|\delta S}{\varepsilon_0}$$

$$E = \frac{|\sigma|}{\varepsilon_0}$$

Including the fact that electric field is normal to the surface, we get the vector relation, equation, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

which is true for both signs of σ. For $\sigma > 0$, electric field is normal to the surface outward; for $\sigma < 0$, electric field is normal to the surface inward.

Q. Consider a conductor with a cavity of any type of size and shape, with no charges inside the cavity. What is the electric field inside the cavity ?

Solution : The electric field inside the cavity is zero.

Q. Consider a conductor with a cavity of any type of size and shape, with no charges inside the cavity. Does the electric potential remain constant inside the cavity and why ?

Solution : The electric potential inside the cavity remains constant because inside the cavity electric field equals to zero.

Q. What is electrostatic shielding ? What is the use of this ?

Solution : Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence; the field inside the cavity is always zero. This is known as electrostatic shielding. The effect can be made use of in protecting sensitive instruments from outside electrical influence.

Q.(a) A comb runs through one's dry hair attracts small bits of paper. Why ? What happens if the hair is wet or it it is a rainy day ? (Remember, a paper does not conduct electricity)

(b) Ordinary rubber is an insulator. But special rubber tyres of aircraft are made slightly conducting. Why is this necessary ?

(c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why ?

(d) A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why ? [NCERT Solved Example 2.7]

Solution : (a) This is because the comb gets charged by friction. The molecules in the paper gets polarised by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.

(b) To enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire.

(c) Reason similar to (b).

(d) Current passes only when there is difference in potential.

2.10 Dielectrics And Polarisation :

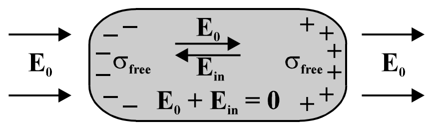
Q. What is the difference between dielectric and conductor ?

Solution : Conductor contain mobile charge carriers. In metallic conductors, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal.

Dielectrics are non-conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers.

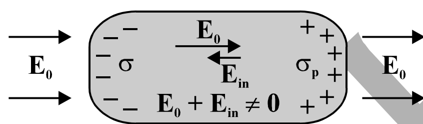
Q. What is the difference in behaviour of a conductor and a dielectric in an external field ?

Solution : The free charge carriers (electrons) in conductor drift against the direction of the external electric field.



Conductor

The charge distribution in the conductor adjusts itself in such a way that the electric field due to induced charges opposes the external field within the conductor. This happens until, in the static situation, the two fields cancel each other and the net electrostatic field in the conductor is zero.



Dielectric

In a dielectric, this free movement of charges is not possible. It turns out that the external field induces dipole moment by stretching or re-orienting molecules of the dielectric. The collective effect of all the molecular dipole moments is net charges on the surface of the dielectric which produce a field that opposes the external field. Unlike in a conductor, however, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric.

Q. What is the difference between polar and non-polar molecules ?

Solution : polar molecules (e.g., HCl, H_2O) have a permanent dipole moment whereas non-polar molecules (e.g., O_2, H_2) have no permanent dipole moment.

Q. Is there a net dipole moment of the dielectric consists of non-polar molecules in the presence of external electric field ? Explain.

Solution : In an external electric field, the positive and negative charges of a non-polar molecule are displaced in opposite directions. The displacement stops when the external force on the constituents charges of the molecule is balanced by the restoring force (due to internal fields in the molecule). The non-polar molecules thus develops an induced dipole moment.The dielectric is said to be polarised by the external field. The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field.

Q. What is linear isotropic dielectrics ?

Solution : The induced dipole moment is in the direction of the field and is proportional to the field strength. Substances for which this assumption is true are called linear isotropie dielectrics.

Q. Why the net dipole moment of a substance of polar molecules equal to zero in the absence of electric field ?

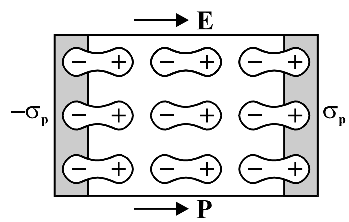
Solution : In the absence of any external field, the different permanent dipoles are oriented randomly due to thermal agiration; so the total dipole moment is zero.

Q. What is polarisation ? How does it depend on the external electric field for linear isotropic dielectrics ?

Solution : A polar or non-polar dielectric developes a net dipole moment in the presence of external electric field. The dipole moment per unit volume is called polarisation denoted by $\vec{P}$. For linear isotropic dielectrics, $\vec{P} = \chi_e \vec{E}$ where χ_e is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium.

Q. How does the polarised dielectric modify the original external field inside it ?

Solution : If a dielectric is placed in an external electric field, the electric field induces a net dipole moment in the dielectric. Although the net charge in the dielectric is zero. However, at the surfaces of the dielectric normal to the electric field, there is evidently a net charge density. As seen in figure, the positive ends of the dipoles remain unneutralised at the right surface and the negative ends at the left surface. The unbalanced charges are the induced charges due to the external field.

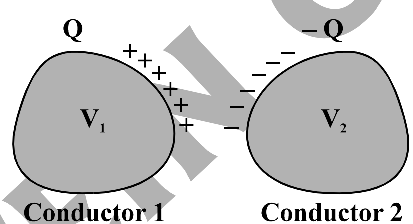


Thus the polarised dielectric is equivalent to two charged surfaces with induced surface charge densities, say σ_p and $-\sigma_p$. Clearly, the field produced by these surface charges opposes the external field. The total field in the dielectric is, thereby, reduced from the case when no dielectric is present. The electric field in the dielectric is E/K where K is the dielectric constant of the dielectric.

2.11 Capacitors and Capacitance :

Q. What is Capacitors ?

Solution : A capacitors is a system of two conductors separated by an insulator.



In practice, the two conductors have charges Q and –Q with potential difference $V = V_1 - V_2$ between them. Q is called the charge on the capacitor.

Q. What is the total charge on the capacitor ?

Solution : Zero

Q. Why the potential difference V between the conductors of the capacitors is proportional to charge Q on the capacitor ?

Solution : The electric field in the region between the conductors is proportional to the charge Q. Now, potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field. Consequently, V is also proportional to Q.

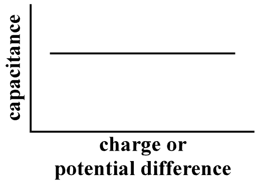
Q. Define capacitance ?

Solution : The potential difference V between the conductors of the capacitors is proportional to charge Q on the capacitor, and the ratio Q/V is a constant : $C = \frac{Q}{V}$. The constant C is called the capacitance of the capacitor.

Q. Does the capacitance of a capacitor depend on Q or V ?

Solution : No

Q. Draw the graph for the variation of capacitance with charge on the capacitor or potential difference across the capacitor.

Solution : 

Q. On what factors does the capacitance of capacitor depend ?

Solution : The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors, it also depends on the nature of the insulator (dielectric) separating the two conductors.

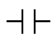
Q. What is the SI unit of capacitance ?

Solution : The SI unit of capacitance is 1 farad (= 1 coulomb volt^{-1}) or 1 F = 1 C V^{-1}.

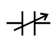
Q. Define 1 farad ?

Solution : 1 farad (1F) capacitance of a capacitor is defined, when a charge of 1 coulomb raises the potential difference by 1V.

Q. Give the symbol of capacitance ?

Solution : 

Q. Give the symbol of variable capacitance ?

Solution : 

Q. What is the advantage of large value of capacitance of a capacitor ?

Solution : A capacitor with large capacitance can hold large amount of charge at a relatively small potential difference.

Q. How the charge of the capacitor leakes away due to the reduction in insulating power of the intervening medium ?

Solution : High potential difference between the conductors of a capacitor implies strong electric field around the conductors. A strong electric field can ionised the surrounding air and accelerate the charges so produced to the oppositely charged plates, thereby neutralizing the charges on the capacitor plates, at least partly. This is the way the charge of the capacitor leakes away due to the reduction in insulating power of the intervening medium.

Q. Define dielectric strength of a medium or material ?

Solution : The maximum electric field that a dielectric medium can withstand without breakdown (of its insulating property) is called its dielectric strength.

Q. Is there a limit to the amount of charge that can be stored on a given capacitor ? Explain.

Solution : Yes, there is a limit to the amount of charge that can be stored on a given capacitor. For a capacitor to store a large amount of charge without leaking, its capacitance should be high enough so that the potential difference and hence the electric field do not exceed the break-down limits.

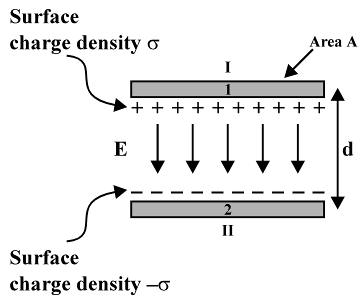
Q. What is the use of capacitor ?

Solution : (i) Capacitor stores charges and hence electric energy within it. (ii) It is a key element of most A.C. circuit. (iii) Capacitor are used in many electrical appliances for e.g., oscillator circuit, fan, motors etc.

2.12 The Parallel Plate Capacitor :

Q. What is parallel plate capacitor ? Derive the expression for the capacitance of parallel plate capacitor ?

Solution : A parallel plate capacitor consists of two large parallel conducting plates separated by a small distance.



We first take the intervening medium between the plates to be vacuum. Let A be the area of each plate and d the separation between them. The two plates have charges Q and –Q. Since d is much smaller than the linear dimension of the field by an infinite plane sheet of uniform surface charge density. Plate 1 has a surface charge density $\sigma = Q/A$ and plate 2 has a surface charge density $-\sigma$.

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving $\mathbf{E} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$.

Now for uniform electric field, potential difference is simply the electric field times the distance between the plates, that is, $\mathbf{V = Ed} = \frac{1}{\varepsilon_0}\frac{Qd}{A}$.

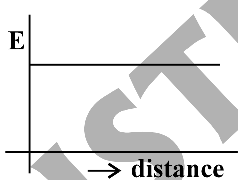
The capacitance C of the parallel plate capacitor is then $\mathbf{C} = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$.

Q. What is the nature of electric field inside the parallel plate capacitor ?

Solution : Uniform.

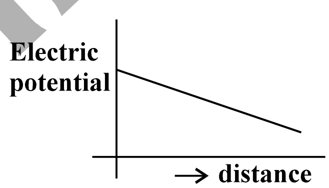
Q. Draw the graph for variation of electric field with distance from positive plate to negative plate in parallel plate capacitor.

Solution :



Q. Draw the graph for variation of electric potential with distance from positive plate to negative plate in parallel plate capacitor.

Solution :



Q. At one point in parallel plate capacitor the electric field is E_1 and at another point the field is E_2. What is the ratio $E_1 : E_2$?

Solution : 1 : 1

Q. What is fringing of the electric field in parallel plate capacitor ?

Solution : For plates with finite area the field lines bend outward at the edges – an effect called 'fringing of the field'.

Q. Comment on the following statement : The capacitance 1F of a capacitor is too big a unit in practice.

Solution : The 'bigness' of 1F is to calculate the area of the plates needed to have C = 1F for a separation of, say 1 cm : $\mathbf{A = \frac{Cd}{\varepsilon_0} = \frac{1F \times 10^{-2} m}{8.85 \times 10^{-12} C^2 N^{-1} m^{-2}} = 10^9 m^2}$, which is a plate about 30 km in length and breadth !

2.13 Effect Of Dielectric On Capacitance :

Q. How the capacitance of a parallel plate capacitor is modified when a dielectric is introduced between the plates ? Explain.

Solution : If a dielectric is inserted between the plates fully occupying the intervening region then the dielectric is polarised by the field, the effect is equivalent to two charged sheets (at the surface of the dielectric normal to the field) with surface charge densities σ_p and $-\sigma_p$. The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$. That is $\mathbf{E = \frac{\sigma - \sigma_P}{\varepsilon_0}}$, so that the potential difference across the plates is $\mathbf{V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0} d}$.

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ. Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write $\mathbf{\sigma - \sigma_P = \frac{\sigma}{K}}$, where K is a constant characteristic of the dielectric. Clearly, K > 1. We then have $\mathbf{V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K}}$. The capacitance C, with dielectric between the plates, is then $\mathbf{C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d}}$.

Q. Define dielectric constant in terms of capacitance of capacitors ?

Solution : $\mathbf{K = \frac{C}{C_0}}$.

The dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

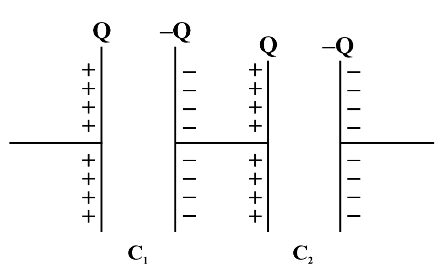
Q. A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4)d, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates ? [NCERT Solved Example 2.8]

Solution : $\mathbf{\frac{4K}{K+3} C_0}$

2.14 Combination Of Capacitance :

Q. Consider two capacitors of capacitance C_1 and C_2 are combined in series combination. Derive the expression for the effective capacitance ?

Solution : Figures shows capacitors C_1 and C_2 combined in series.



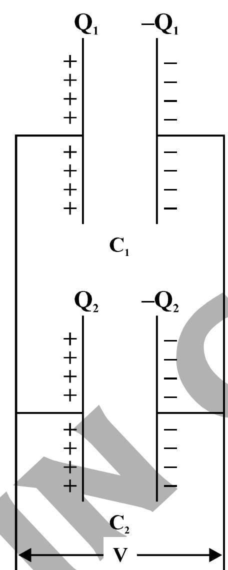
The left plate of C_1 and right plate of C_2 are connected to two terminals of a battery and have charges Q and –Q, respectively. It then follows that the right plate of C_1 has charge –Q and the left plate of C_2 has charge Q. If this was not so, the net charge on each capacitor would not be zero. This would result in an electric field in the conductor connecting C_1 and C_2. Charge would flow until the net charge on both C_1 and C_2 is zero and there is no electric field in the conductor connecting C_1 and C_2. Thus, in the series combination, charges on the two plates (+Q) are the same on each capacitor. The total potential drop V across the combination is the sum of the potential drops V_1 and V_2 across C_1 and C_2, respectively.

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \text{i.e.,} \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Now we can ragard the combination as an effective capacitor with charge Q and potential difference V. The effective capacitance of the combination is $C = \frac{Q}{V}$. Hence we obtain $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Q. Consider two capacitors of capacitance C_1 and C_2 are combined in parallel combination. Derive the expression for the effective capacitance ?

Solution : Figure shows two capacitors arranged in parallel.



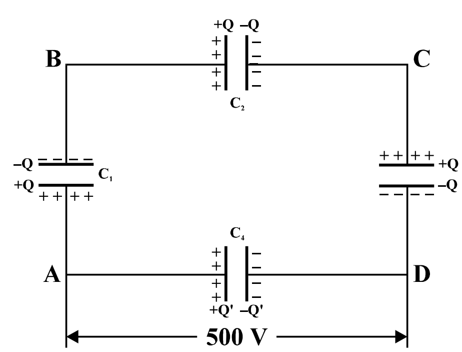
In this case, the same potential difference is applied across both the capacitors. But the plate charges ($+Q_1$) on capacitor 1 and the plate charges ($+Q_2$) on the capacitor 2 are not necessarily the same :

$Q_1 = C_1V$, $Q_2 = C_2V$

The equivalent capacitor is one with charge $Q = Q_1 + Q_2$ and potential difference V.

Hence $Q = CV = C_1V + C_2V$. The effective capacitance C is $C = C_1 + C_2$

Q. A network of four 10 μF capacitors is connected to a 500 V supply, as shown. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential). [NCERT Solved Example 2.9]

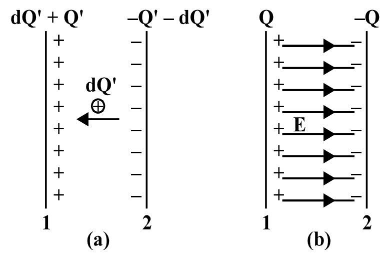


Solution : (a) 13.3 μF (b) 5.0×10^{-3} C

2.15 Energy Stored In A Capacitor :

Q. Derive the expression for energy stored in a capacitor ?

Solution : To determine the energy stored in this configuration, consider initially two uncharged conductors 1 and 2. Imagine next a process of transferring charge from conductor 2 to conductor 1 bit by bit, so that at the end, conductor 1 gets charge Q. By charge conservation, conductor 2 has charge –Q at the end.



In transferring positive charge from conductor 2 to conductor 1, work will be done externally, since at any stage conductor 1 is at a higher potential than conductor 2. To calculate the total work done, we first calculate the work done in a small step involving transfer of an infinitesimal (i.e., vanishingly small) amount of charge. Consider the intermediate situation when the conductors 1 and 2 have charges $\mathbf{Q'}$ and $-\mathbf{Q'}$ respectively. At this stage, the potential difference $\mathbf{V'}$ between conductors 1 to 2 is $\mathbf{Q'/C}$, where C is the capacitance of the system. Next imagine that a small charge $\delta\mathbf{Q'}$ is transferred from conductor 2 to 1. Work done in this step $(\mathbf{dW'})$, resulting in charge $\mathbf{Q'}$ on conductor 1 increasing to $\mathbf{Q'+dQ'}$, is given by

$$\mathbf{dW = V'dQ' = \frac{Q'}{C}dQ'}$$

Total work done equals to the energy stored in capacitor, which will be obtain by integration as

$$\mathbf{W = \int_0^Q \frac{Q'}{C}dQ' = \frac{1}{C}\frac{Q'^2}{2}\Big|_0^Q = \frac{Q^2}{2C}}$$

We can write the final result, in different ways

$$\mathbf{W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV} \quad (Q = CV)$$

Q. Show that electric field E itself is a source of energy with energy density $\frac{1}{2}\varepsilon_0E^2Jm^{-3}$.

Solution : Since electrostatic force is conservative, this work is stored in the form of potential energy of the system. To see this, consider for simplicity, a parallel plate capacitor of area A (of each plate) and separation d between the plates.

Energy stored in the capacitor $\mathbf{= \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A}}$

The surface charge density σ is related to the electric field E between the plates, $\mathbf{E = \frac{\sigma}{\varepsilon_0}}$

Hence, we get energy stored in the capacitor U = (1/2) ε_0E^2 × Ad

Note that Ad is the volume of the region between the plates (where electric field alone exists). If we define energy density as energy stored per unit volume of space, then energy density of electric field u = (1/2)ε_0E^2

Though we derived above equation for the case of a parallel plate capacitor, the result on energy density of an electric field is, in fact, very general and holds true for electric field due to any configuration of charges.

Q. (a) A 900 pF capacitor is charged by 100 V battery. How much electrostatic energy is stored by the capacitor ? (b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system ? [NCERT Solved Example 2.10]

Solution : (a) 4.5 μJ (b) 2.25 μJ

2.16 Van De Graaff Generator :

Q. What is Van De Graaff Generator ?

Solution : This is a machine that can build up the voltage in the order of million (10^6) volts.

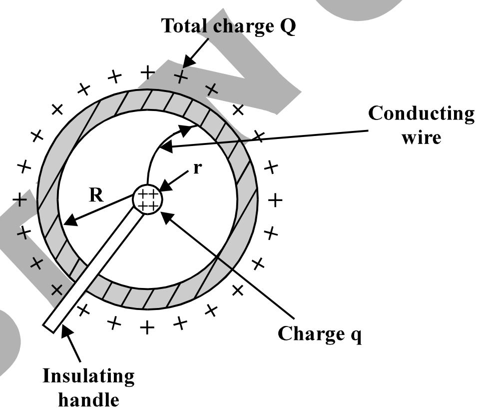
Q. What is the use of high voltages developed by Van De Graaff Generator ?

Solution : The high voltages and hence large electric fields, produced by this generator, are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter.

Q. Explain the basic principle, construction and working of a Van de Graaff generator with the help of a diagram.

Solution : Principle, suppose we have a large spherical conducting shell of radius R, on which we place a charge Q. This charge spreads itself uniformly all over the sphere. As we know, the field outside the sphere is just that of a point charge Q at the centre; while the field inside the sphere vanishes. So the potential outside is that of a point charge; and inside it is constant. We thus have : Potential inside conducting spherical shell of radius R carrying charge Q = constant = $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$.

Now, as shown, let us suppose that in some way we introduce a small sphere of radius r, carrying some charge q, into the large one, and place it at the centre. The potential due to this new charge clearly has the following values at the radii indicated :



Taking both charges q and Q into account, the potential of outer sphere is V(R) and potential of inner sphere V(r) is given by

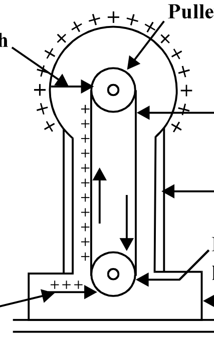
$$V(R) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right), \quad V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

Hence potential difference between them equals to, $V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$

Assume now that q is positive. We see that, independent of the amount of charge Q that may have accumulated on the larger sphere and even if it is positive, the inner sphere is always at a higher potential: the difference V(r) – V(R) is positive.

This means that if we now connect the smaller and larger sphere by a wire, the charge q will immediately flow to larger sphere. The natural tendency is for positive charge to move from higher to lower potential. Thus, provided we are somehow able to introduced the small charged sphere into the larger one, we can in this way keep piling up larger and larger amount of charge on the latter. The potential at the outer sphere would also keep rising, at least until we reach the breakdown field of air.

Construction



A schematic diagram of the Van de Graaff generator is given in figure. A large spherical conducting shell (of few metres radius) is supported at a height several meters above the ground on an insulating column. A long narrow endless belt of insulating material, like rubber or silk, is wound around two pulleys - one at ground level, one at the centre of the shell. This belt is kept continuously moving by a motor driving the lower pulley. It continuously carries positive charge, sprayed on ot it by a brush at ground level, to the top. There it transfers its positive charge to another conducting brush connected to the large shell. Thus positive charge is transferred to the shell, where it spreads out uniformly on the outer surface. In this way, voltage differences of as much as 6 to 8 million volts (with respect to ground) can be built up.

NCERT EXERCISE

2.1 Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.

2.2 A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

2.3 Two charges 2 μC and –2 μC are placed at points A and B 6 cm apart.

(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface ?

2.4 A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field.

(a) inside the sphere

(b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere ?

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1 pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6 ?

2.6 Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination ?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply ?

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination ?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor ?

2.9 Explain what would happen if in the capacitor in the above problem, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.

(i) while the voltage supply remained connected.

2.10 A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor ?

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ? (ii) after the supply was disconnected.

<u>ADDITIONAL EXERCISES</u>

2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P(0, 0, 3 cm) to a point Q(0, 4 cm, 0) via a point R(0, 6 cm, 9 cm).

2.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field :

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this mid-point in a plane normal to the line and passing through the mid-point.

2.15 A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surface of the shell ?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape ? Explain.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $(\vec{E}_2 - \vec{E}_1)\cdot\hat{n} = \frac{\sigma}{\varepsilon_0}$. where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2). Hence show that just outside a conductor, the electric field is $\sigma\hat{n}/\varepsilon_0$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders ?

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å.

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a) ?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation ?

2.19 If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

2.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres ? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

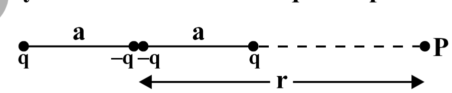
2.21 Two charges –q and +q are located at points (0, 0, –a) and (0, 0, a) respectively.

(a) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0) ?

(b) Obtain the dependence of potential on the distance r at a point from the origin when r >> a.

(c) How much work is done in moving a small test charge from a point (5, 0, 0) to (–7, 0, 0) along the x-axis ? Does the answer change if the path of the test charge between the same points is not along the the x-axis ?

2.22 Figure shows a charge array known as an electric quadrupole.

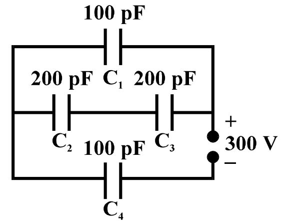


For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).

2.23 An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors. Also find the minimum number of capacitors.

2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm ? [You will realise from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

2.25 **Obtain the equivalent capacitance of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor.**



2.26 **The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.**

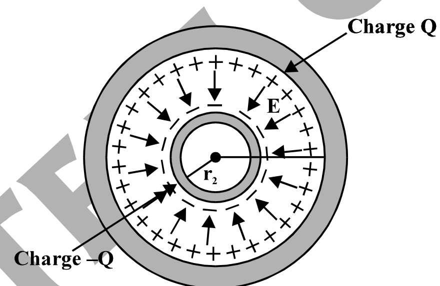
(a) **How much electrostatic energy is stored by the capacitor ?**

(b) **View this energy as stored in the electrolytic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.**

2.27 **A 4μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation ?**

2.28 **Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.**

2.29 **A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports.**



Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ **where r_1 and r_2 are the radii of outer and inner spheres, respectively.**

2.30 **A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.**

(i) **Determine the capacitance of the capacitor ?**

(ii) **What is the potential of the inner sphere ?**

(iii) **Find the ratio of the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.**

2.31 **Answer carefully :**

(a) **Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres ?**

(b) **If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true ?**

(c) **A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point ?**

(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron ? What if the orbit is elliptical ?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there ?

(f) What meaning would you give to the capacitance of a single conductor ?

(g) Guess a possible reason why water has a much greater dielectric constant (= 80) then say, mica (= 6).

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 V m^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown i.e., without starting to conduct electricity through partial ionisation). For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF ?

2.34 Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the z-direction.

(b) a field that uniformly increases in magnitude but remains a constant (say, z) direction.

(c) A single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

2.35 In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required ? (you will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential).

2.36 A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

2.37 Answer the following :

(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open ? (Assume the house to be a steel cage so there is no field inside !)

(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area $1m^2$. Will he get an electric shock if he touches the metal sheet next morning ?

(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral ? In other words, what keeps the atmosphere charged ?

(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning ?

[Hint : The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amout of negative charge on the earth.)

ANSWERS

2.1 10 cm, 40 cm away from the positive charge on the side of the negative charge.

2.2 2.7×10^6 V

2.3 (a) The plane normal to AB and passing through its mid-point has zero potential everywhere.

(b) Normal to the plane in the direction AB.

2.4 (a) Zero (b) 10^5 N C^{-1} (c) 4.4×10^4 N C^{-1}

2.5 96 pF

2.6 (a) 3 pF (b) 40 V

2.7 (a) 9 pF (b) 2×10^{-10} C, 3×10^{-10} C, 4×10^{-10} C

2.8 18 pF, 1.8×10^{-9} C

2.9 (a) V = 100 V, C = 108 pF, Q = 1.08×10^{-8}C (b) Q = 1.8×10^{-9}C, C = 108 pF, V = 16.6 V

2.10 1.5×10^{-8} J

2.11 6×10^{-6} J

2.12 1.2 J; the point R is irrelevant to the answer

2.13 Potential = $4q/(\sqrt{3}\pi\varepsilon_0 b)$; field is zero, as expected by symmetry.

2.14 (a) 2.4×10^5 V; 4.0×10^5 Vm^{-1} from charge 2.5 μC to 1.5 μC.

(b) 2.0×10^5 V; 6.6×10^5 Vm^{-1} in the direction that makes an angle of about 69^0 to the line joining charge 2.5 μC to 1.5 μC.

2.15 (a) $-q/(4\pi r_1^2)$, $(Q + q)/(4\pi r_2^2)$

(b) By Gauss's law, the net charge on the inner surface enclosing the cavity (not having any charge) must be zero. For a cavity of arbitrary shape, this is not enough to claim that the electric field inside must be zero. The cavity may have positive and negative charges with total charge zero. To dispose of this possibility, take a closed loop, part of which is inside the cavity along a field line and the rest inside the conductor. Since field inside the conductor is zero, this gives a net work done by the field in carrying a test charge over a closed loop. We know this is impossible for an electrostatic field. Hence, there are no field lines inside the cavity (i.e., no field), and no charge on the inner surface of the conductor, whatever be its shape.

2.17 $\lambda/(2\pi\varepsilon_0 r)$, where r is the distance of the point from the common axis of the cylinders. The field is radial, perpendicular to the axis.

2.18 (a) –27.2 eV (b) 13.6 eV (c) –13.6 eV, 13.6 eV, Note in the latter choice the total energy of the hydrogen atom is zero.

2.19 –19.2 eV; the zero of potential energy is taken to be at infinity.

2.20 The ratio of electric field of the first to the second is (b/a). A flat portion may be equated to a spherical surface of large radius, and a pointed portion to one of small radius.

2.21 (a) On the axis of the dipole, potential is $(\pm 1/4\pi\varepsilon_0)\, p/(x^2 - a^2)$ where p = 2qa is the magnitude of the dipole moment; the + sign when the point is closer to q and the – sign when it is closer to –q. Normal to the axis, at points (x, y, 0), potential is zero.

(b) The dependence on r is $1/r^2$ type

(c) Zero. No, because work done by electrostatic field between two points is independent of the path connecting the two points.

2.22 For large r, quadrupole potential goes like $1/r^3$, dipole potential goes like $1/r^2$, monopole potential goes like 1/r.

2.23 Eighteen 1 μF capacitors arranged in 6 parallel rows, each row consisting of 3 capacitors in series.

2.24 1130 km^2

2.25 Equivalent capacitance = (200/3) pF.

$Q_1 = 10^{-8}$ C, $V_1 = 100$ V ; $Q_2 = Q_3 = 10^{-8}$ C

$V_2 = V_3 = 50$ V

$Q_4 = 2.55 \times 10^{-8}$ C, $V_4 = 200$ V

2.26 **(a) 2.55×10^{-6} J (b) u = 0.113 J m^{-3}, u = (½) $\varepsilon_0 E^2$**

2.27 **2.67×10^{-2} J**

2.28 **Hint : Suppose we increase the separation of the plates by Δx. Work done (by external agency) = F Δx. This goes to increase the potential energy of the capacitor by u a Δx where u is energy density. Therefore, F = u a which is easily seen to be (1/2) QE, using u = (1/2) $\varepsilon_0 E^2$. The physical origin of the factor 1/2 in the force formula lies in the fact that just outside the conductor, field is E, and inside it is zero. So, the average value E/2 contributes to the force.**

2.30 **(a) 5.5×10^{-9} F (b) 4.5×10^2 V (c) 1.3×10^{-11} F**

2.31 **(a) No, because charge distributions on the spheres will not be uniform.**

(b) No

(c) Not necessarily. (True only if the field line is a straight line.) The field lines gives the direction of acceleration, not that of velocity, in general.

(d) Zero, no matter what the shape of the complete orbit.

(e) No, potential is continuous.

(f) A single conductor is a capacitor with one of the 'plates' at infinity.

(g) A water molecule has permanent dipole moment. However, detailed explanation of the value of dielectric constant requires microscopic theory and is beyond the scope of the book.

2.32 **1.2×10^{-10} F, 2.9×10^4 V**

2.33 **19 cm^2**

2.34 **(a) Planes parallel to x-y plane.**

(b) Same as in (a), except that planes differing by a fixed potential get closer as field increases.

(c) Concentric spheres centred at the origin.

(d) A periodically varying shape near the grid which gradually reaches the shape of planes parallel to the grid at far distances.

2.35 **30 cm**

2.36 **Hint : By Gauss's law, field between the sphere and the shell is determined by q_1 alone. Hence, potential difference between the sphere and the shell is independent of q_2. If q_1 is positive, this potential difference is always positive.**

2.37 **(a) Our body and the ground form an equipotential surface. As we step out into the open, the original equipotential surfaces of open air change, keeping our head and the ground at the same potential.**

(b) Yes. The steady discharging current in the atmosphere charges up the aluminium sheet gradually and raises its voltage to an extent depending on the capacitance of the capacitor (formed by the sheet, slab and the ground).

(c) The atmosphere is continually being charged by thunderstorms and lightning all over the globe and discharged through regions of ordinary weather. The two opposing currents are, on an average, in equilibrium.

(d) Light energy involved in lightning; heat and sound energy in the accompanying thunder.

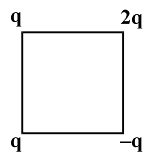
ADDITIONAL QUESTIONS AND PROBLEMS

Q. **What does positive and negative potential energy of a system of two charges mean ?**

Q. **In the following cases find the total potential energy of the system :**

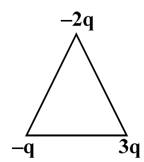
(a) **Three charges each of value +q are placed at the vertices of an equilateral triangle of side length a.**

(b) **Charges q, –q, 2q and q are placed at the vertices of a square of side length a as shown in figure**



A. (a) $\frac{3kq^2}{a}$ (b) $\frac{kq^2}{\sqrt{2}a}$

Q. **Charges –q, 3q, –2q are placed at the vertices of an equilateral triangle of side length a.**



What is the work done by the external agent to move all the charges on an equilateral triangle of very large length that means to move them by infinite distance.

A. $\frac{7kq^2}{a}$

Q. **A point charge –q goes around another point charge Q in a semi circular path of radius R. What will be the change in electrostatic potential energy of the system of charge ?**

A. zero

Q. **What is the work done by the field of a nucleus in a complete circular orbit of the electron ? What if the orbit is elliptical ?**

A. zero

Q. **The electric potential due to a point charge may be negative or positive. Does it mean that it is a vector quantity. Comment.**

Q. **Obtain the energy in joules acquired by an electron beam when accelerated through a potential difference of 2000 V. Also find the speed of the particle.**

A. 2.67×10^7 m/s

Q. **How the charge can be transferred from one sphere to another sphere completely ?**

Q. **An electric dipole of length 2 cm is placed with its axis making an angle of 60^0 to a uniform electric field of 10^5 N/C. If it experiences a torque of $8\sqrt{3}$ Nm, calculate the**

(i) **magnitude of the charge on the dipole, and**

(ii) **potential energy of the dipole.**

A. (i) 8 mC (ii) –8 J

Q. **Why Van de Graaff generator cannot produce of potential greater than mega volt ?**

Q. **Find the electric field between two metal plate 3 mm. apart, connected to a 12 V battery.**

A. 4000 V/m

Q. **An infinite plane sheet of charge density σ, is held in air. In this situation how far apart are two equipotential surfaces, whose p.d. is V ?**

A. $\frac{2\varepsilon_0 V}{\sigma}$

Q. **A charge of +10μC is given to a hollow metallic sphere of radius 0.1 m. Find the potential at the (i) outer surface, and (ii) centre of the sphere.**

A. (i) 9×10^5V (ii) 9×10^5 V